UNIVERSITÉ DU LUXEMBOURG

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Nonequilibrium Thermodynamics of Small Systems: Classical and Quantum Aspects

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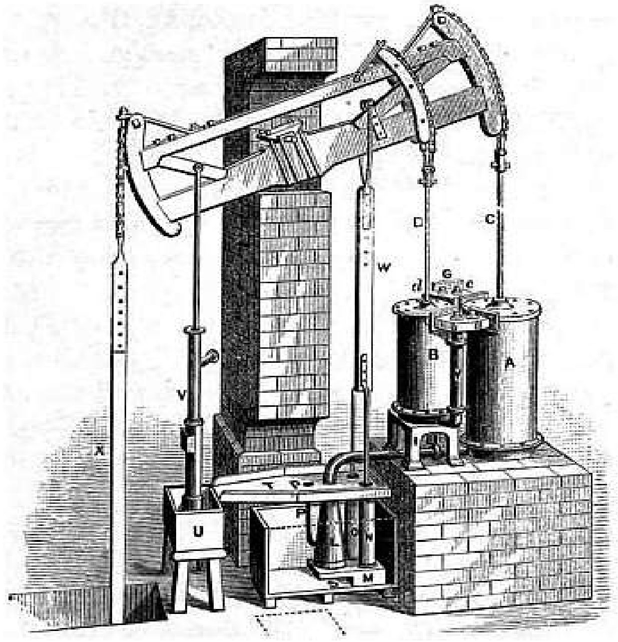
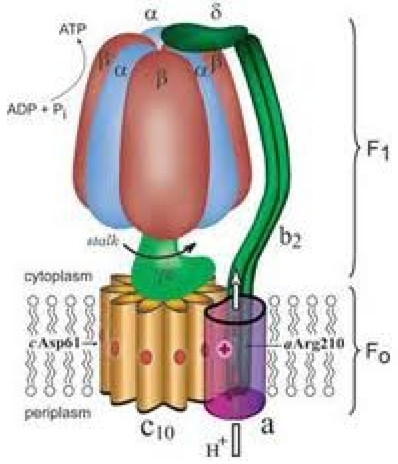
Paris – May 9-11, 2017

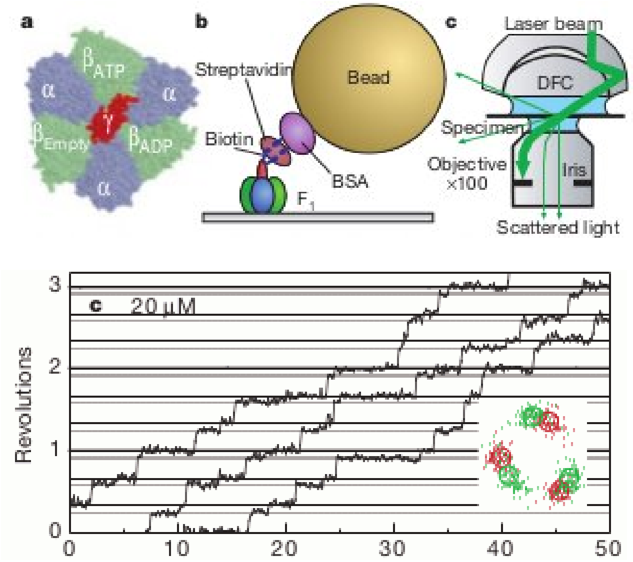
Introduction

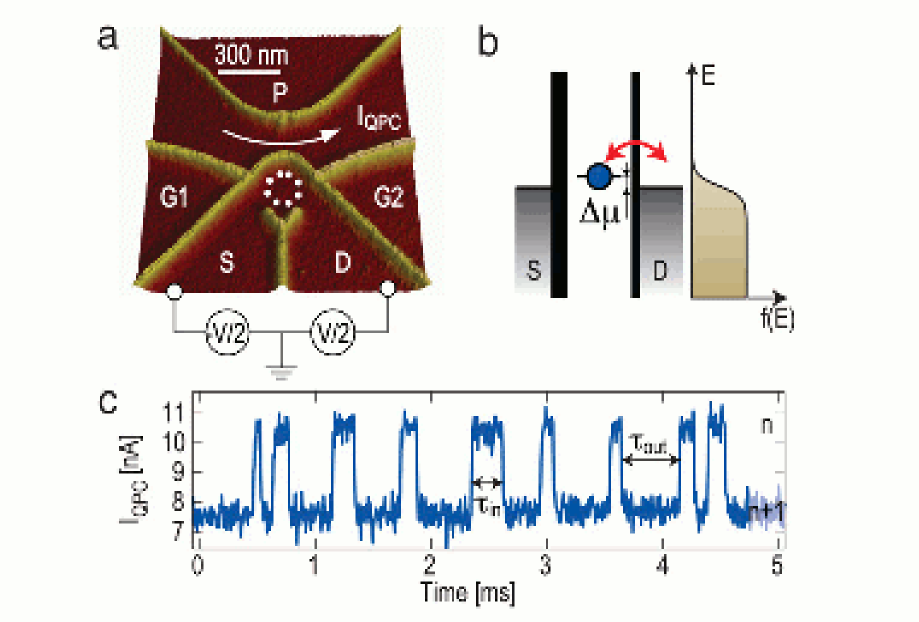


Thermodynamics in the 19th century:

Thermodynamics in the 21th century:







Challenges when dealing with small systems

Small systems

Large fluctuations

Average behavior
contains limited information

Higher moments or full
probability distribution is needed

Far from equilibrium
(large surf/vol ratio)

Linear response
theories fail

Nonlinear response and
nonperturbative methods
need to be developed

Quantum effects
(low temperatures)

Traditional stochastic
descriptions fail

Stochastic descriptions
taking coherent effects
into account are needed

Stochastic thermodynamics

Outline

Part I: Stochastic Thermodynamics:

From fluctuation theorems to stochastic efficiencies

Part II: Thermodynamics of Information Processing

Part III: Quantum Thermodynamics

Part I: Stochastic Thermodynamics: From fluctuation theorems to stochastic efficiencies

1) Stochastic thermodynamics

2) Universal fluctuation relation

3) Finite-time thermodynamics

4) Efficiency fluctuations

1) Stochastic thermodynamics

Esposito and Van den Broeck, Phys. Rev. E **82**, 011143 (2010)

Esposito, Phys. Rev. E **85**, 041125 (2012)

Markovian master equation:

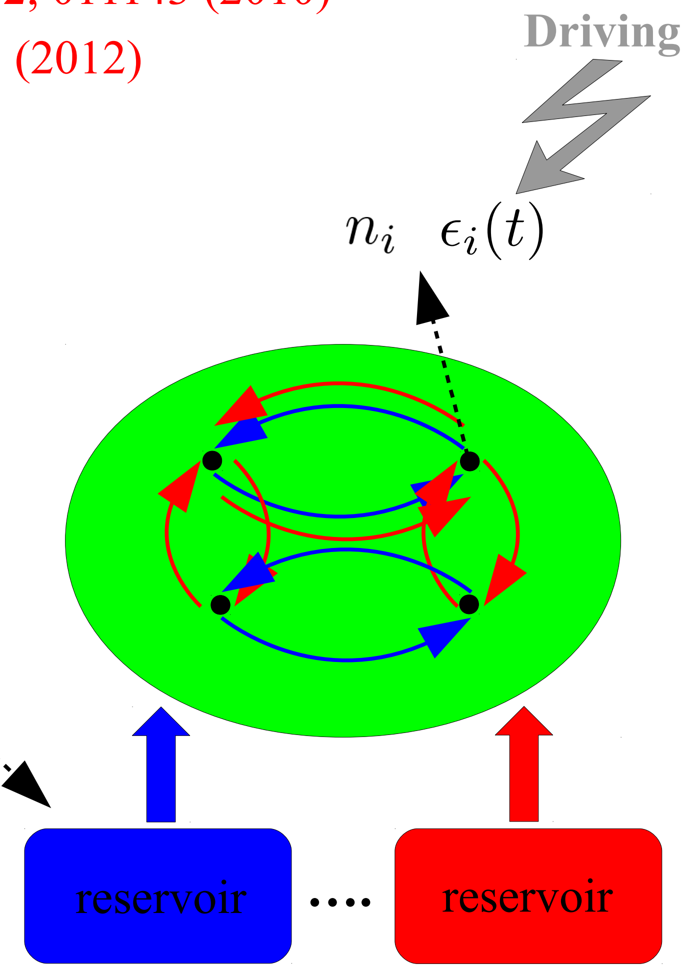
$$d_t p_i = \sum_j W_{ij} p_j = \sum_j \left(W_{ij} p_j - W_{ji} p_i \right)$$

$$W_{ij} = \sum_\nu W_{ij}^{(\nu)}$$

Energy and Matter currents

$$I_E^{(\nu)} = \sum_{i,j} W_{ij}^{(\nu)} p_j (\epsilon_i - \epsilon_j)$$

$$I_M^{(\nu)} = \sum_{i,j} W_{ij}^{(\nu)} p_j (n_i - n_j)$$



Different reservoirs $T^{(\nu)}$ $\mu^{(\nu)}$

Local detailed balance:

$$\frac{W_{ij}^{(\nu)}}{W_{ji}^{(\nu)}} = \exp\left(- \frac{(\epsilon_i - \epsilon_j) - \mu^{(\nu)}(n_i - n_j)}{k_b T^{(\nu)}} \right)$$

Energy Particle number Shannon entropy

$$E = \sum_i \epsilon_i p_i \quad , \quad N = \sum_i n_i p_i \quad , \quad S = \sum_i \underbrace{[-k_b \ln p_i]}_{s_i} p_i$$

1st law: Energy balance

$$\boxed{d_t E = \dot{W}_m + \dot{W}_c + \sum_\nu \dot{Q}^{(\nu)}}$$

Particle balance

$$\boxed{d_t N = \sum_\nu I_M^{(\nu)}}$$

2nd law: Entropy balance

$$\boxed{\dot{S}_{\mathbf{i}} = d_t S - \underbrace{\sum_\nu \frac{\dot{Q}_\nu}{T_\nu}} \geq 0}$$

Entropy production

Entropy change in the reservoirs

$$\dot{S}_{\mathbf{i}} = k_b \sum_{\nu,i,j} \left(W_{ij}^{(\nu)} p_j - W_{ji}^{(\nu)} p_i\right) \ln \frac{W_{ij}^{(\nu)} p_j}{W_{ji}^{(\nu)} p_i} \geq 0$$

$\dot{S}_{\mathbf{i}} = 0$ iff $W_{ij}^{(\nu)} p_j = W_{ji}^{(\nu)} p_i$ (detailed balance)

Mechanical work

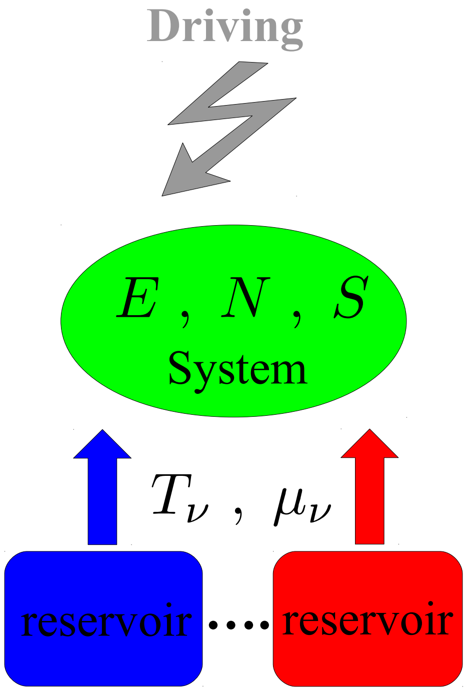
$$\dot{W}_m = \sum_i d_t \epsilon_i \, p_i$$

Chemical work

$$\dot{W}_c = \sum_\nu \mu^{(\nu)} I_M^{(\nu)}$$

Heat flow

$$\dot{Q}^{(\nu)} = I_E^{(\nu)} - \mu^{(\nu)} I_M^{(\nu)}$$

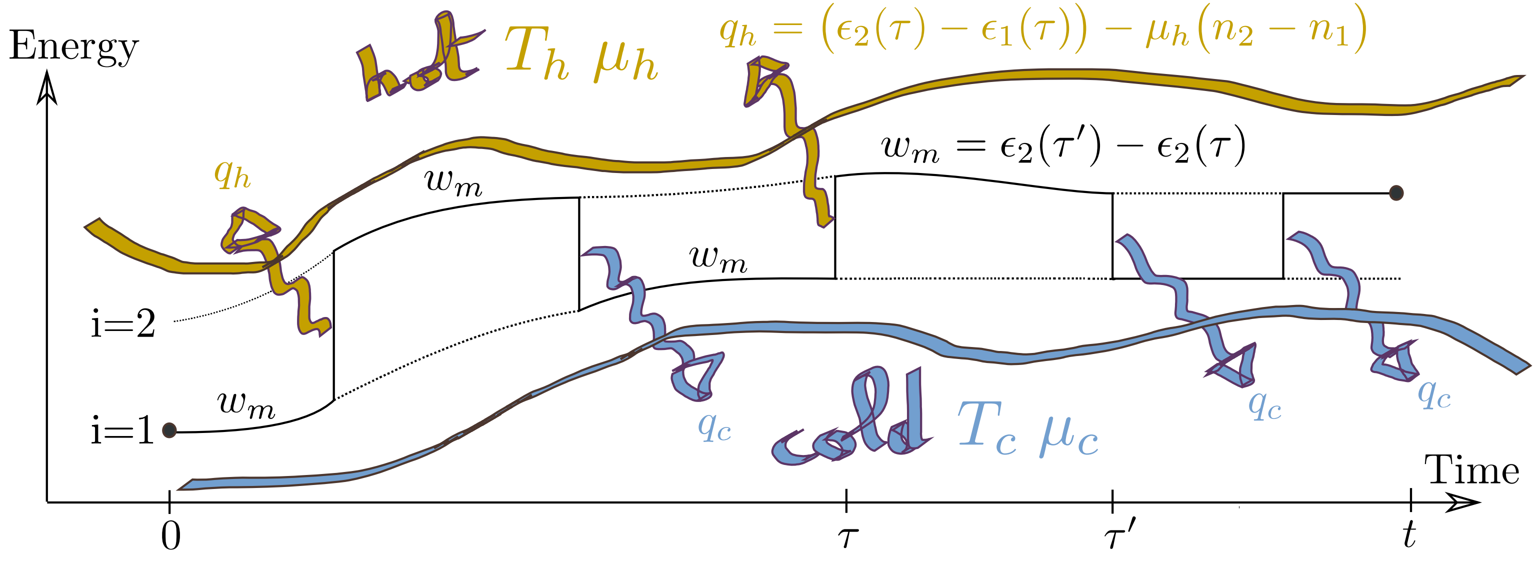


Slow driving with 1 reservoir: $\dot{S}_{\mathbf{i}} \approx 0$

equilibrium thermo

$$T d_t S = d_t E - \dot{W}_m - \mu d_t N$$

2) Universal Fluctuation Relation



Energy balance: $$\epsilon_{i_t}(\lambda_t) - \epsilon_{i_0}(\lambda_0) = w_m\left[\Gamma|\lambda\right] + \underline{w_c\left[\Gamma|\lambda\right]} + \sum_{\nu=1}^{N} \underbrace{\left(\Delta\epsilon_\nu\left[\Gamma|\lambda\right] \underline{- \mu_\nu \Delta n_\nu\left[\Gamma|\lambda\right]}\right)}_{q_\nu\left[\Gamma|\lambda\right]}$$

Particle balance: $$n_{i_t} - n_{i_0} = \sum_{\nu=1}^{N} \Delta n_\nu\left[\Gamma|\lambda\right]$$

Entropy balance: $$\Delta_{\mathrm{i}} s\left[\Gamma|\lambda\right] = \ln\frac{P[\Gamma,\lambda]}{\tilde{P}[\bar{\Gamma},\lambda]} = \underbrace{\ln p_{i_0}(0) - \ln p_{i_t}(t)}_{\Delta s\left[\Gamma|\lambda\right]} - \sum_{\nu=1}^{N} \beta_\nu q_\nu\left[\Gamma|\lambda\right]$$

Time-reversed driving
Time-reversed trajectory

not a physical observable

Integral fluctuation theorem: $$\boxed{\langle \mathrm{e}^{-\Delta_{\mathrm{i}} s}\rangle = 1} \Longrightarrow \langle \Delta_{\mathrm{i}} s\rangle \geq 0$$

Detailed FT
for entropy production

$$\boxed{\ln \frac{P(\Delta_{\rm i}s)}{\tilde{P}(-\Delta_{\rm i}s)} = \Delta_{\rm i}s}$$

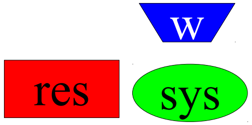
if

Involution

$$\Delta_{\rm i}s\,[\Gamma|\lambda] = -\tilde{\Delta_{\rm i}}s\,[\bar{\Gamma}|\lambda]$$

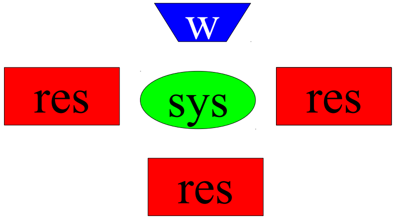
Seifert, PRL **95** 040602 (2005)

Fluctuation theorem for physical observable?



Driving + 1 reservoir + start at equilibrium: Work FT (Crooks FT)

No driving + multiple reservoirs + longtime limit $t \to \infty$: Current FT



Driving + multiple reservoirs + start at equilibrium vs reservoir $\nu = 1$:

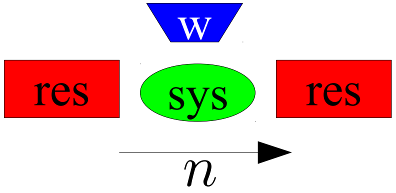
$$\boxed{\ln \frac{P(w_m, \{\Delta\epsilon_\nu\}, \{\Delta n_\nu\})}{\tilde{P}(-w_m, \{-\Delta\epsilon_\nu\}, \{-\Delta n_\nu\})} = \overbrace{\beta_1\left(w_m - \Delta\Phi_1^{eq}\right)}^{\text{Work FT}} + \overbrace{\sum_{\nu=2}^{N}\left(\underbrace{A^\epsilon_\nu}_{=\beta_1-\beta_\nu}\Delta\epsilon_\nu + \underbrace{A^n_\nu}_{=\beta_\nu\mu_\nu-\beta_1\mu_1}\Delta n_\nu\right)}^{\text{Current FT}}}$$

Bulnes Cuetara, Esposito, Imparato, PRE **89**, 052119 (2014)

Isothermal example: driven junction

Bulnes Cuetara, Esposito, Imparato, PRE **89**, 052119 (2014)

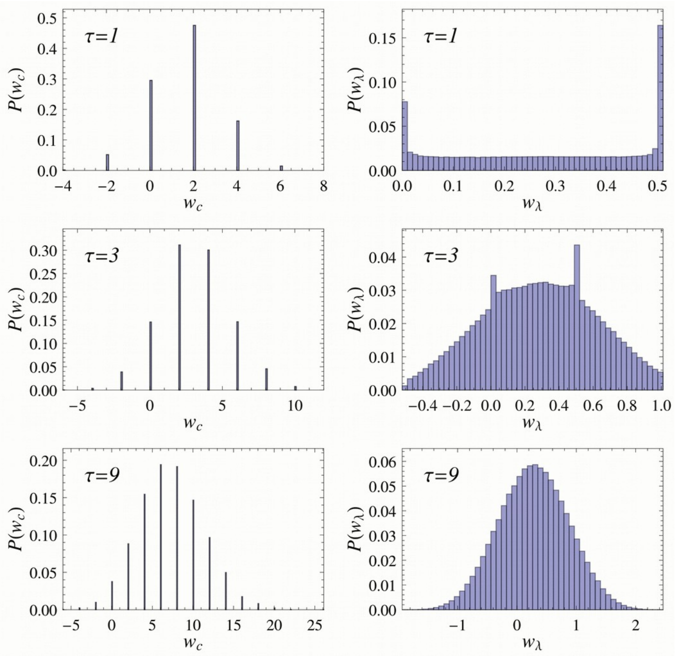
Setup:

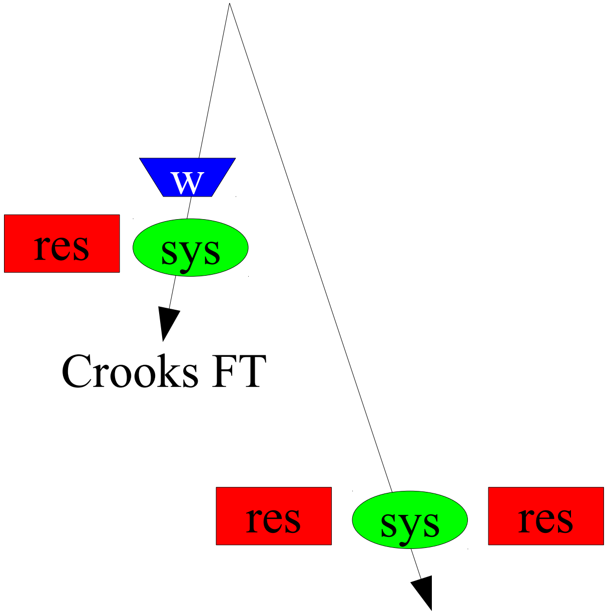


Initial condition: equilibrium vs a reference reservoir

Mechanical work: w_m Chemical work: $w_c = n\Delta\mu$

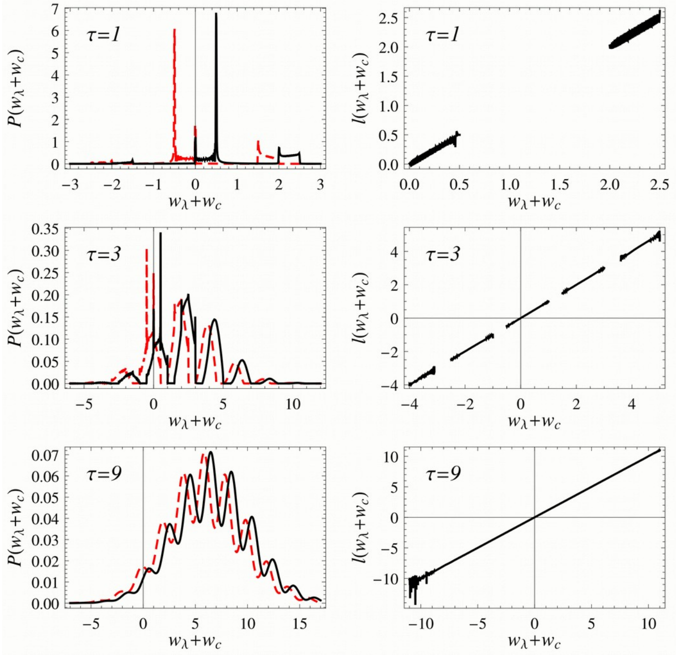
$$\frac{P_F(w_m + w_c)}{P_B(-w_m - w_c)} = \frac{P_F(w_m, w_c)}{P_B(-w_m, -w_c)} = \exp\left\{\frac{w_m + w_c - \Delta\Phi_1}{k_B T}\right\}$$





Crooks FT

Current FT



Fluctuation Relation: Synthesis

Fluctuations in small out-of-equilibrium systems satisfy a universal symmetry

Everything can also be done for:
- Fokker-Planck dynamics
- Open quantum systems (weak coupling)

FT can be used: to derive Onsager reciprocity relations and generalizations

to derive fluctuation-dissipation relations and generalizations

to check the consistency of a transport theory

to calculate free energy differences

…

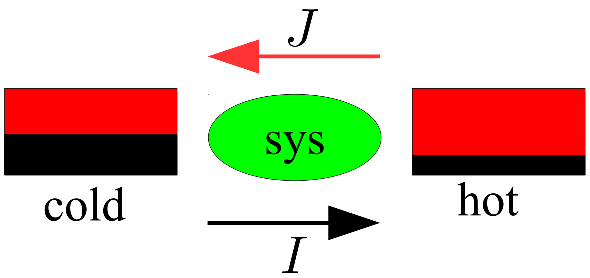
Nonequilibrium fluctuations, fluctuation theorems and counting statistics in quantum systems, Esposito, Harbola, Mukamel, Rev. Mod. Phys. **81**, 1665 (2009)

Ensemble and Trajectory Thermodynamics: A Brief Introduction, Van den Broeck and Esposito, Physica A **418**, 6 (2015)

3) Finite-time thermodynamics

a) Steady state energy conversion

- *Thermoelectricity:*



Reservoir entropy change:

$$dS_r = \frac{1}{T_r} dE_r - \mu_r dN_r$$

Entropy production (entropy change in the reservoirs):

$$\sigma = J(\frac{1}{T_c} - \frac{1}{T_h}) + I\frac{\Delta\mu}{T_c} \geq 0$$

Efficiency: $\eta = \frac{-W}{\eta_C Q_h} \leq 1$

Thermoelectric effect if: $I < 0$

Power: $\mathcal{P} = -I\Delta\mu$

- *General formulation:*

$$\sigma = \underbrace{J_1 A_1}_{\substack{\sigma_1 > 0 \\ \text{intput}}} + \underbrace{J_2 A_2}_{\substack{\sigma_2 < 0 \\ \text{ouput}}} \geq 0$$

$$\eta = -\frac{\sigma_2}{\sigma_1} = 1 - \frac{\sigma}{\sigma_1} \leq 1$$

$$\mathcal{P} = -\sigma_2$$

b) Energy conversion in the linear regime

Linear regime: $J_1 = L_{11}A_1 + L_{12}A_2 \qquad J_2 = L_{21}A_1 + L_{22}A_2$

$$\sigma = L_{11}A_1^2 + 2L_{12}A_1A_2 + L_{22}A_2^2 \geq 0$$

Maximum efficiency: $\partial_{A_2}\eta = 0$

$$\eta^* = \frac{Det[L] + L_{11}L_{22} - 2\sqrt{Det[L]L_{11}L_{22}}}{L_{11}L_{22} - Det[L]} \leq 1$$

Maximum is reached at *tight coupling*: $Det[L] = 0$ $(J_1 \propto J_2)$ vanishing power! $\mathcal{P} \to 0$

Efficiency at maximum power: $\partial_{A_2}\mathcal{P} = 0$

$$\eta^* = \frac{1}{2} - \frac{Det[L]}{L_{11}L_{22} + Det[L]} \leq \frac{1}{2}$$

c) Efficiency at maximum power beyond linear regime

Phenomenological models

I. I. Novikov & P. Chambadal (1957).
F. Curzon and B. Ahlborn, Am. J. Phys. **43**, 22 (1975).

$$\eta_{CA} = 1 - \sqrt{1 - \eta_C} \approx \eta_C/2 + \eta_C^2/8 + \eta_C^3/16 + \dots$$

Linear
(In case of tight coupling)

Van den Broeck, Phys. Rev. Lett. **95**, 190602, (2005)

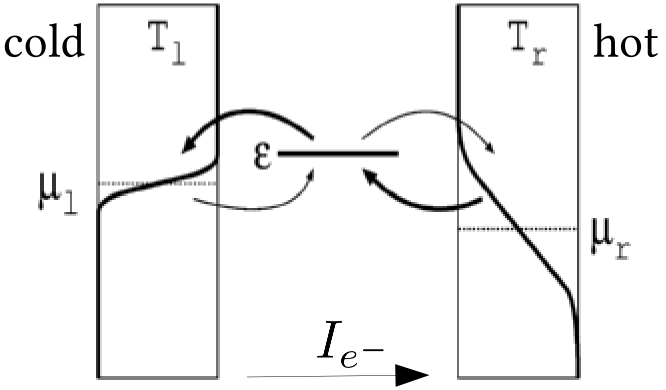
Nonlinear
(In presence of a left-right symmetry)

Esposito, Lindenberg, Van den Broeck, Phys. Rev. Lett. **102**, 130602 (2009)

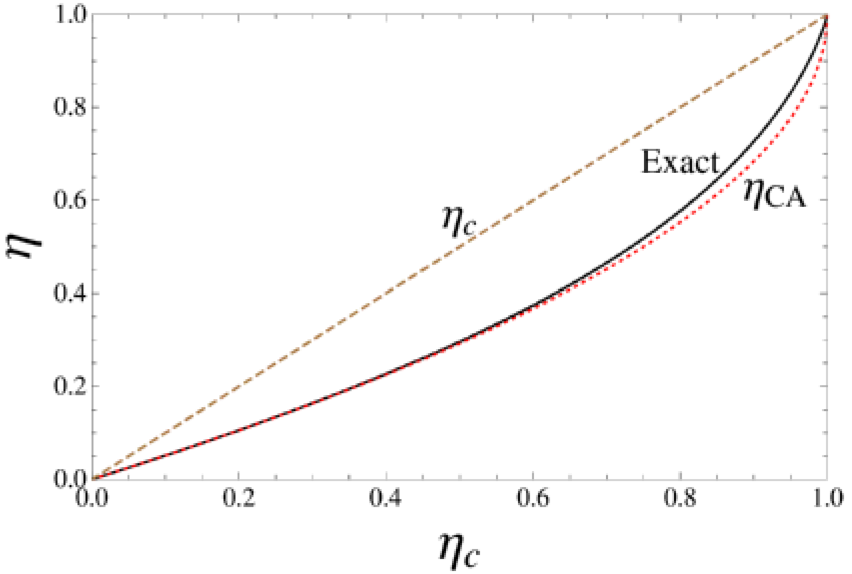
Exactly solvable models using stochastic thermodynamics

Thermoelectric quantum dot

Esposito, Lindenberg, Van den Broeck,
EPL **85**, 60010 (2009)

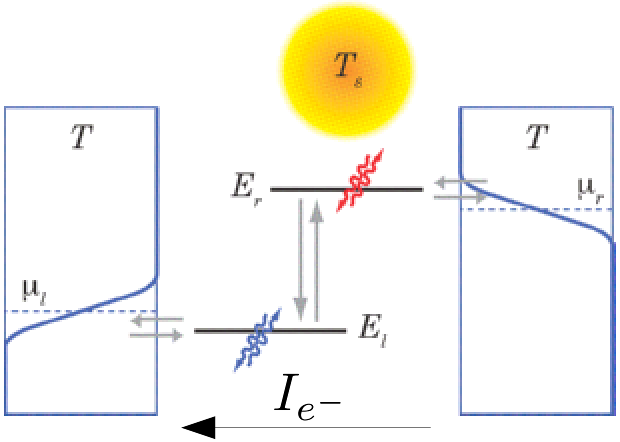


$$\dot{\mathcal{W}} = (\mu_l - \mu_r) I_{e^-}$$
$$\dot{\mathcal{Q}}_r = (\varepsilon - \mu_r) I_{e^-}$$
$$\eta = \frac{-\dot{\mathcal{W}}}{\dot{\mathcal{Q}}_r}$$

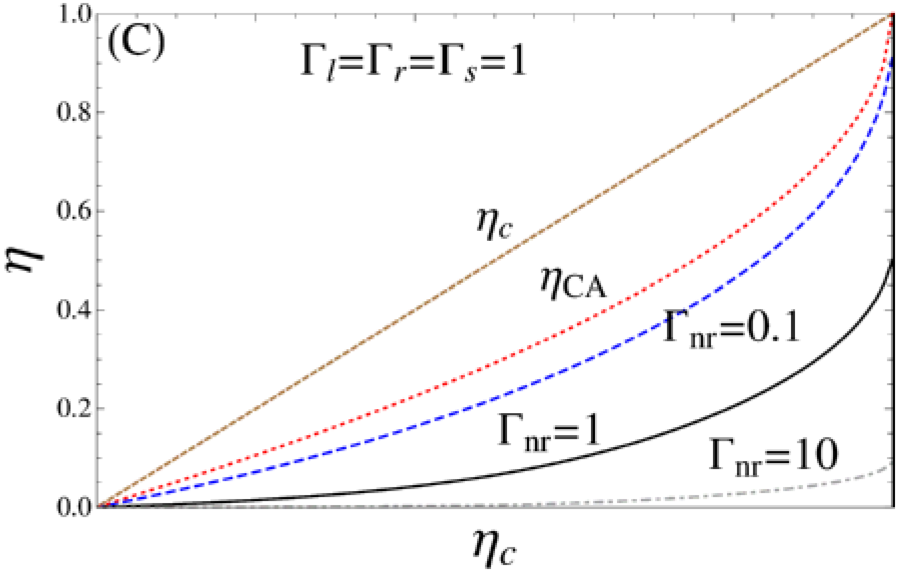


Photoelectric nanocell

Rutten, Esposito, Cleuren,
Phys. Rev. B **80**, 235122 (2009)



$$\dot{\mathcal{W}} = (\mu_r - \mu_l) I_{e^-} \quad \eta = \frac{-\dot{\mathcal{W}}}{\dot{\mathcal{Q}}_s}$$



Finite-Time Thermodynamics: Synthesis

Stochastic thermodynamics naturally combines kinetics and thermodynamics

↓

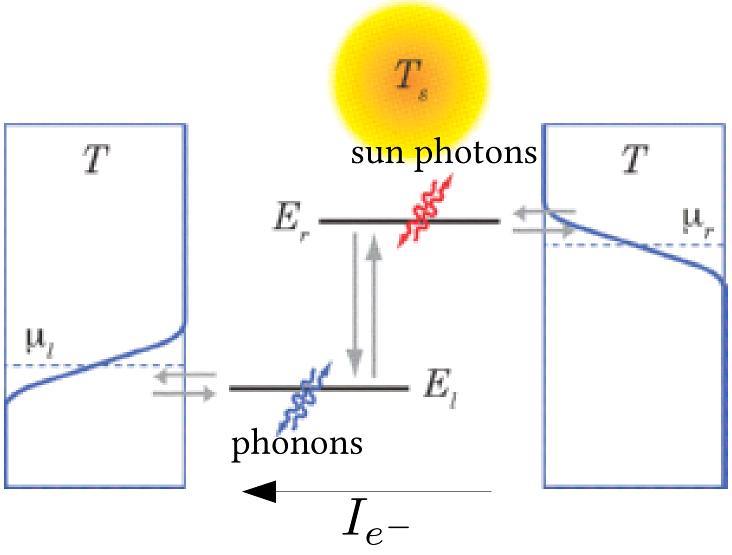
Powerful formalism to study energy transduction at the nanoscale

It allows to:

- Unambiguously define thermodynamic efficiencies (connected to EP)
- Distinguish the system specific features from the universal ones
- View very different devices (bio., chem., meso.) from the same global perspective
- …

4) Efficiency fluctuations

Verley, Esposito, Willaert, Van den Broeck, *The unlikely Carnot efficiency*, Nature Communications **5**, 4721 (2014)



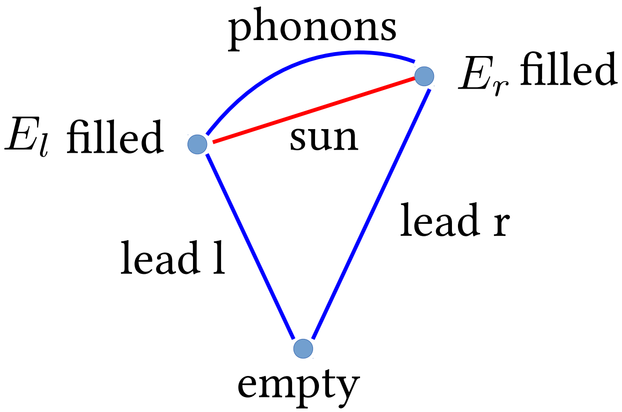
Ensemble averaged description:

$$\langle w \rangle = (\mu_r - \mu_l)\langle I_{e^-} \rangle$$

$$\langle q_h \rangle = (E_r - E_l)\langle I_{ph}^{sun} \rangle$$

$$T\langle \sigma \rangle = \langle w \rangle + \eta_C \langle q_h \rangle \geq 0$$

$$\bar{\eta} = \frac{-\langle w \rangle}{\eta_C \langle q_h \rangle} \leq 1$$



phonons

E_r filled

E_l filled

sun

lead r

lead l

empty

At the trajectory level:

$$T\sigma = w + \eta_C q_h$$

$$\eta = \frac{-w}{\eta_C q_h}$$

Fluctuation theorem: $\dfrac{P(\sigma)}{P(-\sigma)} = \exp \sigma$

What can we say about $P(\eta)$?

a) Long time efficiency fluctuations

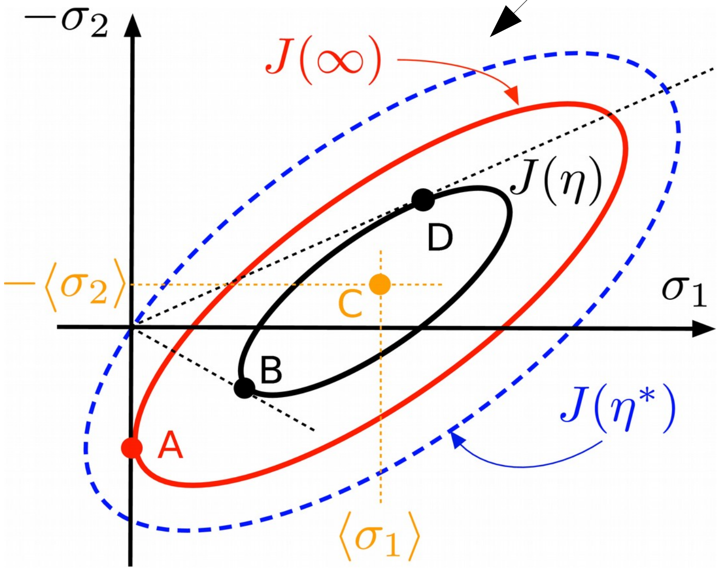
$$\sigma = \eta_C q_h/T + w/T$$

$$\eta = \frac{-\sigma_2}{\sigma_1} = \frac{-w}{\eta_C q_h}$$

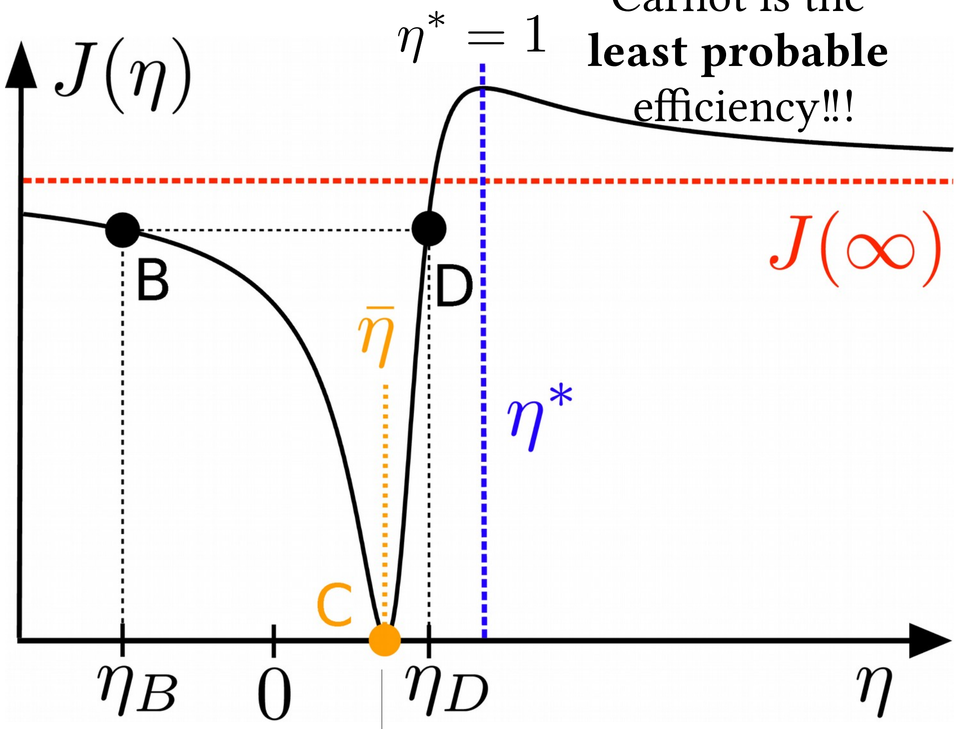
$$P_t(\eta) \asymp \exp\{-tJ(\eta)\}$$

$$J(\eta) = \min_{\sigma_1} I(\sigma_1, -\eta\sigma_1)$$

$$P_t(\sigma_1, \sigma_2) \asymp \exp\{-tI(\sigma_1, \sigma_2)\}$$



Carnot is the **least probable** efficiency!!!



FT: $J(1) = I(0,0)$

➡ $J(\eta) \leq J(1)$

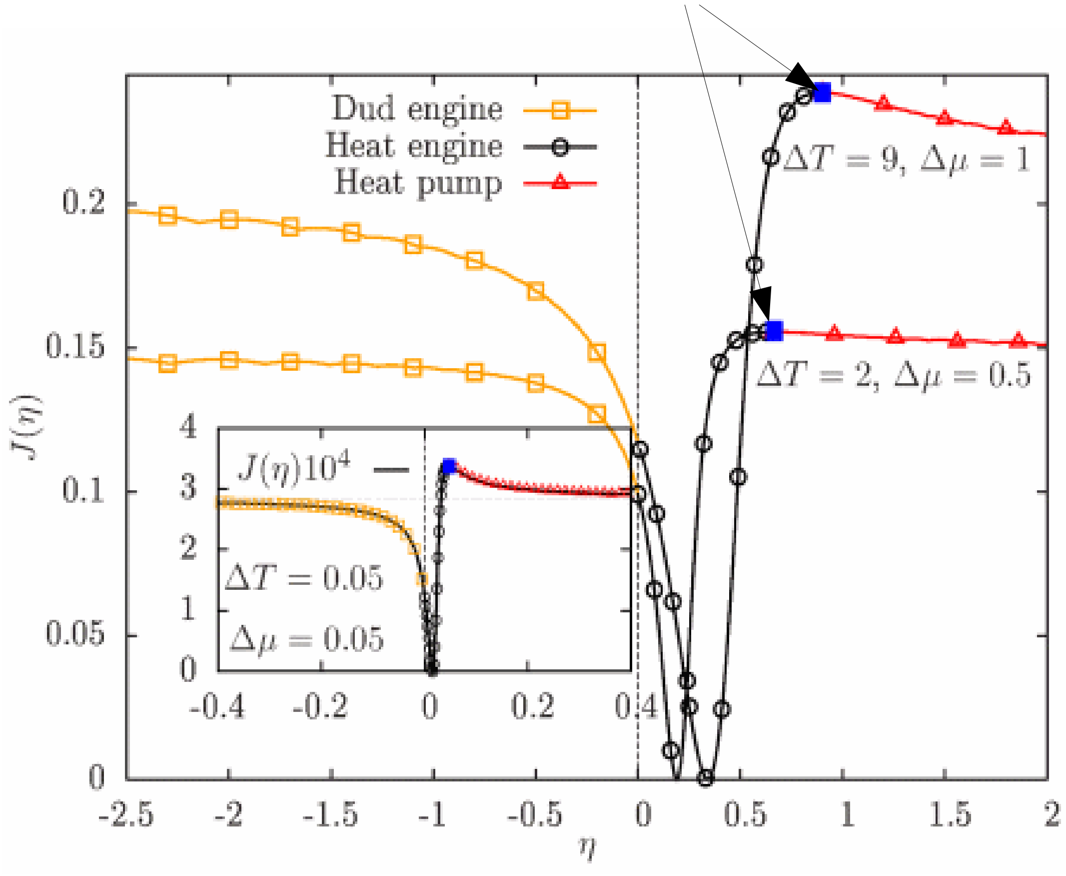
Macroscopic efficiency is the **most probable** efficiency $\bar{\eta} = \frac{-\langle\sigma_2\rangle}{\langle\sigma_1\rangle} \leq 1$

Verley, Esposito, Willaert, Van den Broeck, *The unlikely Carnot efficiency*, Nature Communications **5**, 4721 (2014)

Verley, Esposito, Willaert, Van den Broeck, *The unlikely Carnot efficiency*, Nature Communications **5**, 4721 (2014)

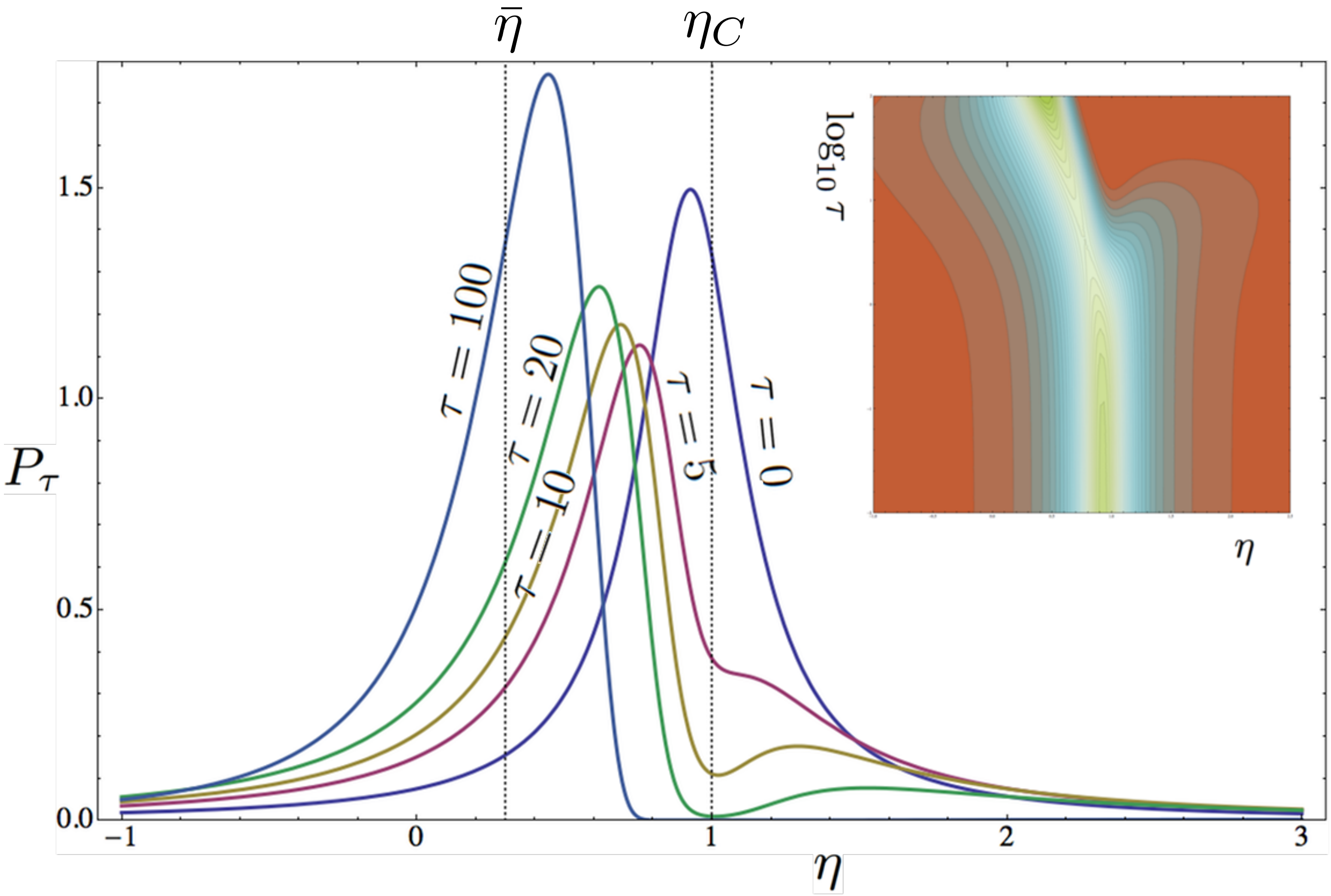
The least likely efficiency is the Carnot efficiency: $\boxed{\eta^* = \bar{\eta}_{rev}}$ Consequence of FT!

Carnot efficiency



b) Finite-time efficiency fluctuations

Polettini, Verley, Esposito, *Finite-time efficiency fluctuations: Enhancing the most likely value*, PRL **114**, 050601 (2015)



- At $\tau = 0$: Lorentzian with max $\eta_0 = -L_{12} f_1 / (L_{22} f_2)$: $1 \geq \eta_0 \geq \bar{\eta}$
- After critical time, the distribution becomes bimodal:
  - local min goes to $\eta_C = 1$ - local max goes to infinity - global max goes to $\bar{\eta}$
- $P_t(\eta < 1) = P_t(\sigma > 0)$ and $P_t(\eta > 1) = P_t(\sigma < 0)$
- The distribution has no moments: $P_t(\eta \to \pm\infty) \propto \eta^{-2}$
- Tight coupling: no efficiency fluctuations $P_t(\eta) = \delta(\eta - \eta_C)$

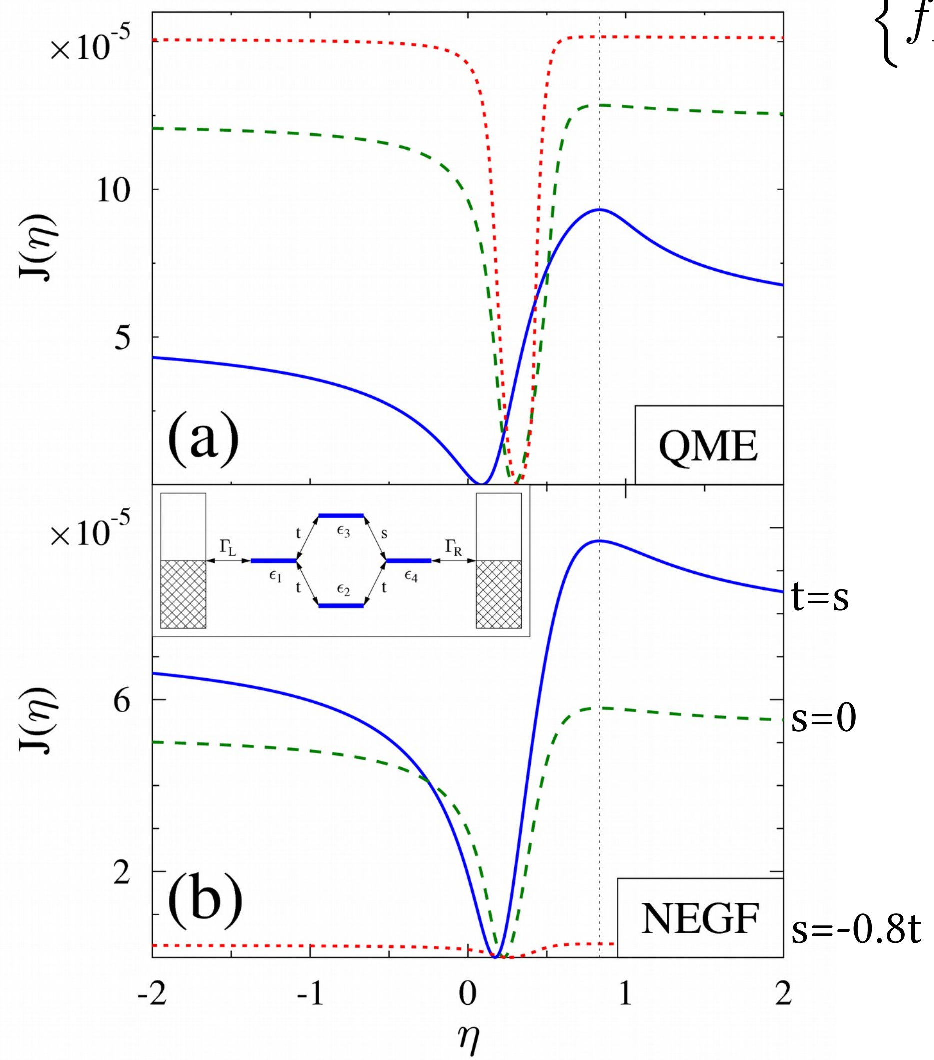
c) Long-time efficiency fluctuations in quantum systems

Esposito, Ochoa, Galperin, Efficiency fluctuation in quantum thermoelectric devices, PRB **91**, 115717 (2015)

Cumulant GF (heat & work) $\phi(\gamma,\lambda) = \int \frac{dE}{2\pi} \ln \Bigg(1 + T(E)$

$$\Big\{ f_L(E)[1 - f_R(E)][e^{-([E-\mu_R]\lambda - [\mu_L-\mu_R]\gamma)} - 1]$$

$$+ f_R(E)[1 - f_L(E)][e^{+([E-\mu_R]\lambda - [\mu_L-\mu_R]\gamma)} - 1] \Big\} \Bigg)$$



Fluctuation relation

$$\phi\Big(\gamma,\lambda\Big) = \phi\Big(-\frac{1}{T_L} - \gamma, \frac{1}{T_R} - \frac{1}{T_L} - \lambda\Big)$$

$$J(\eta) = -\min_{\gamma_2} \phi(\gamma_2\eta, \gamma_2)$$

Efficiency fluctuations: Synthesis

Finite-time thermodynamics at the fluctuating level

Accurate characterization of energy transduction at the nanoscale

- Experimental verification: Martinez, Roldan, Dinis, Petrov, Parrondo, Rica, *Brownian Carnot engine*, Nature Physics DOI: 10.1038/NPHYS3518 (2015)

  Proesmans, Dreher, Gavrilov, Bechhoefer, Van den Broeck, *Brownian duet: A novel tale of thermodynamic efficiency*, Phys. Rev. X 6, 041010 (2016)

- The long time results can be generalized:

  - to time-asymmetric drivings

    Verley, Willaert, Van den Broeck, Esposito, *Universal theory of efficiency fluctuations*, PRE **90**, 052145 (2014)

    Gingrich, Rotskoff, Vaikuntanathan, and Geissler, *Efficiency and Large Deviations in Time-Asymmetric Stochastic Heat Engines*, NJP **16**, 102003 (2014)

  - to quantum systems (NEGF approach)

    Esposito, Ochoa, Galperin, Efficiency fluctuation in quantum thermoelectric devices, PRB **91**, 115717 (2015)

    Agarwalla, Jiang, Segal, *Full counting statistics of vibrationally-assisted electronic conduction: transport and fluctuations of the thermoelectric efficiency*, PRB **92**, 245418 (2015)

- The finite-time behavior:

Polettini, Verley, Esposito, *Finite-time efficiency fluctuations: Enhancing the most likely value*, PRL **114**, 050601 (2015)

Proesman, Cleuren, Van den Broeck, *Stochastic efficiency for effusion as a thermal engine*, EPL **109**, 20004 (2015)

Jiang, Agarwalla, Segal, *Efficiency Statistics and Bounds for Systems with Broken Time-Reversal Symmetry*, PRL **115**, 040601 (2015)

Part II: Thermodynamics of Information Processing

1) Stochastic thermodynamics

  - Nonequilibrium thermodynamics
  - Landauer principle
  - Nonequilibrium state as a resource

2) Measurement and feedback

  - Szilard engine
  - Erasure with feedback

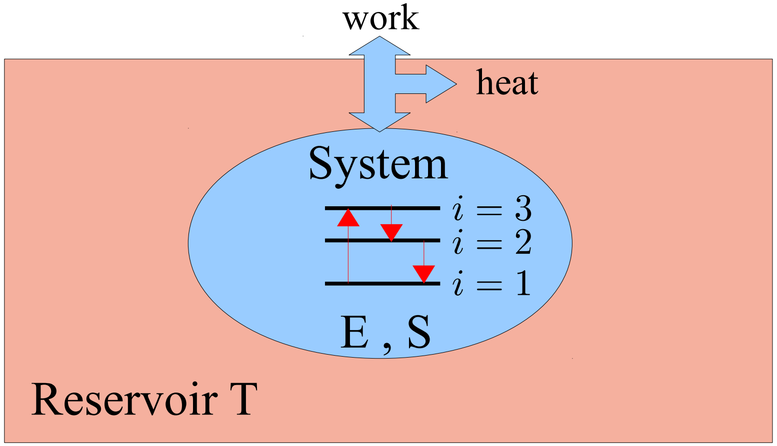
3) Bipartite perspective

  - Nonautonomous (measurement and feedback)
  - Autonomous (information flow)

4) Conclusions and perspectives

1) Stochastic Thermodynamics

Open system dynamics



Master equation: $$d_t p_i = \sum_j W_{ij} p_j$$

may depend on time

Local detailed balance: $$\ln \frac{W_{ij}}{W_{ji}} = -\frac{(\epsilon_i - \epsilon_j)}{k_b T}$$

0th law Equilibrium: $$p_i^{\text{eq}} = \exp\{-\frac{(\epsilon_i - F^{\text{eq}})}{k_b T}\}$$

Nonequilibrium Thermodynamics

Energy: $E = \sum_i \epsilon_i p_i$

Entropy: $S = \sum_i [-k_b \ln p_i] p_i$

1st law

$$\boxed{d_t E = \sum_i d_t \epsilon_i \; p_i + \sum_i \epsilon_i \; d_t p_i}$$

Energy change

Work $\dot{W}$

Heat $\dot{Q}$

2nd law

$$\boxed{\dot{S}_{\mathbf{i}} = d_t S - \frac{\dot{Q}}{T} \geq 0}$$

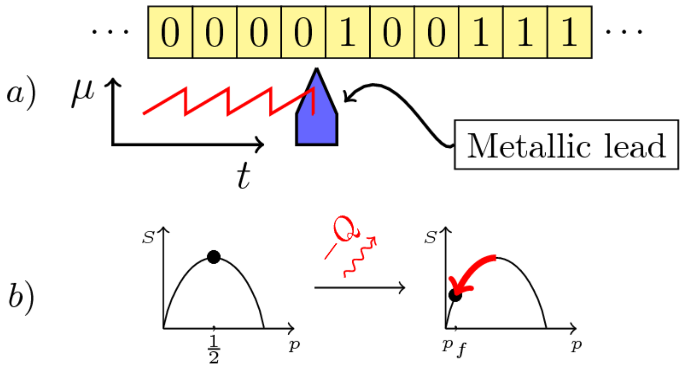
Entropy production

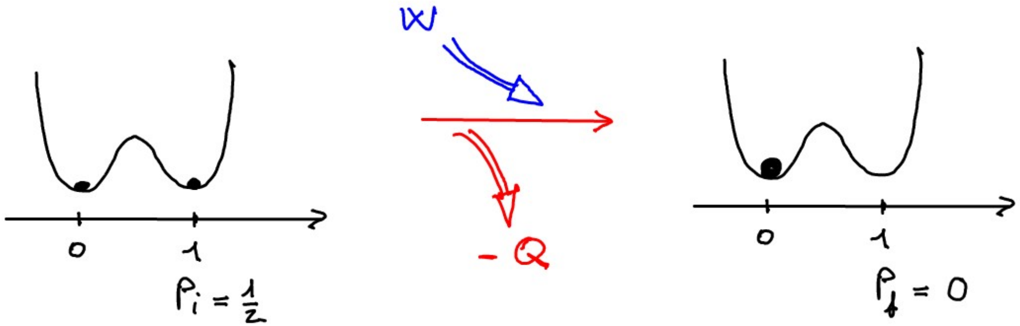
Entropy change

Entropy change in the reservoir

$$\dot{S}_{\mathbf{i}} = k_b \sum_{i,j} \left(W_{ij} p_j - W_{ji} p_i\right) \ln \frac{W_{ij} p_j}{W_{ji} p_i} \geq 0$$

Landauer principle



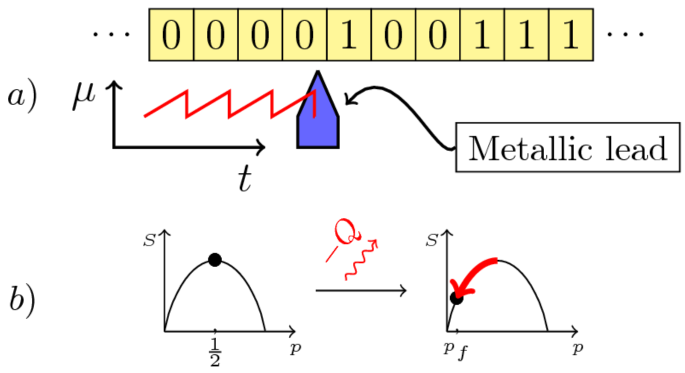


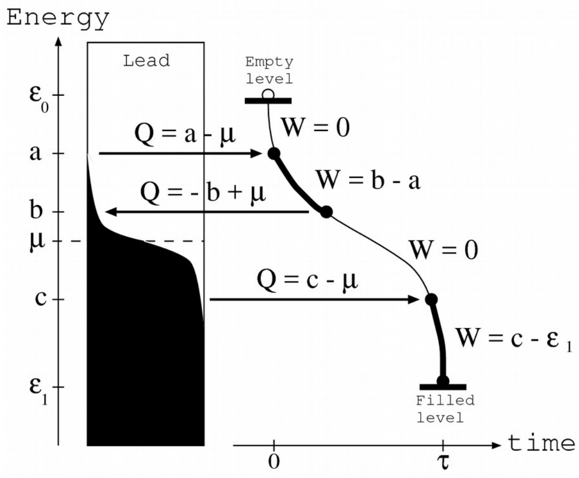
$$\Delta E = 0$$

Heat expelled: $-Q \geq -T\Delta S = TS_i - TS_f = k_b T \ln 2$

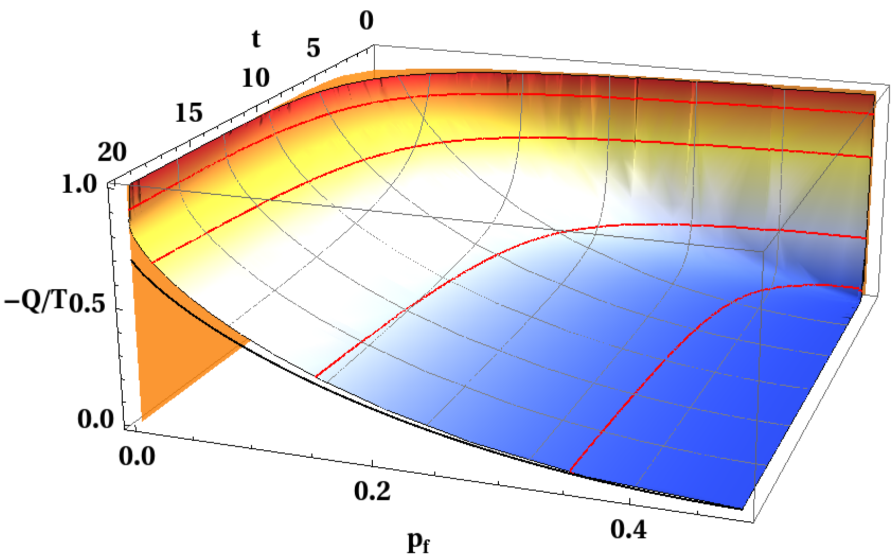
Work needed: $W = -Q = k_b T \ln 2$

Optimal erasure in finite time



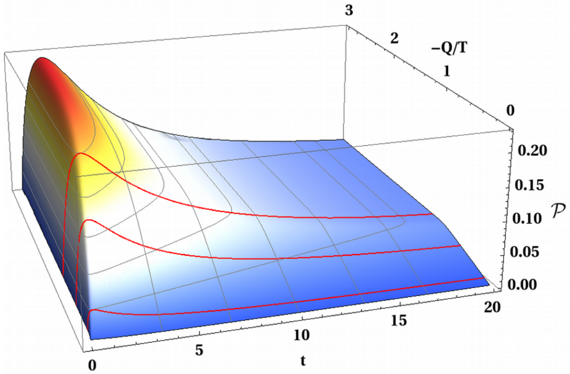


Accuracy-dissipation trade-offs:



Power:

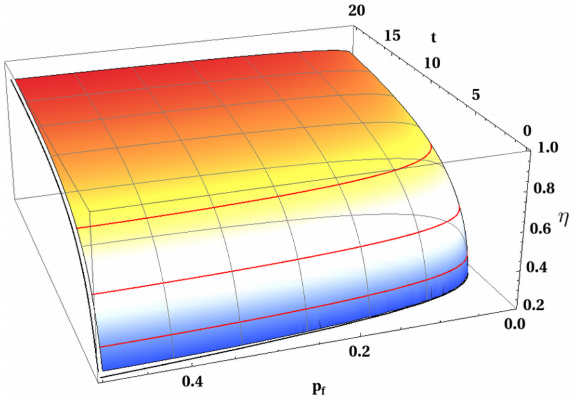
$$\mathcal{P}(Q,t) = \frac{-\Delta S}{t}$$



Efficiency:

$$\eta = \frac{-\Delta S}{-Q/T}$$

$$= 1 - \frac{\Delta_\mathbf{i} S}{-Q/T} \leq 1$$



Finite-time erasing of information stored in fermionic bits, Diana, Bagci, Esposito, Phys. Rev. E **85**, 041125 (2012)

Nonequilibrium state as a resource

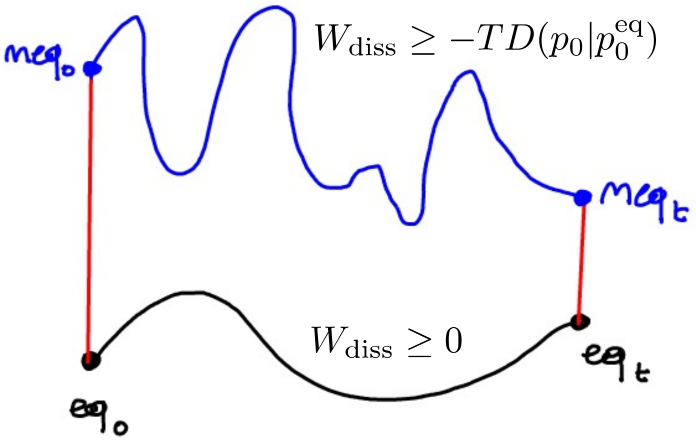
Nonequilibrium free energy $F \equiv E - TS$

$$F - F^{\mathrm{eq}} = TD(p|p^{\mathrm{eq}}) \geq 0$$

$$\left\{ \begin{aligned} & D(p|p') \equiv k_b \sum_i p_i \ln \frac{p_i}{p'_i} \geq 0 \\ & p_i^{\mathrm{eq}} = \exp\{-\frac{(\epsilon_i - F^{\mathrm{eq}})}{k_b T}\} \end{aligned} \right.$$

1st law + 2nd law : $T\Delta_{\mathbf{i}} S = W - \Delta F \geq 0$

$$W_{\mathrm{diss}} \equiv \underbrace{W - \Delta F^{\mathrm{eq}}}_{\substack{\text{if eq. to eq.} \\ \geq 0}} = \underset{\geq 0}{T\Delta_{\mathbf{i}} S} + \underset{\geq 0}{TD(p_t|p_t^{\mathrm{eq}})} \boxed{- \underset{\geq 0}{TD(p_0|p_0^{\mathrm{eq}})}}$$



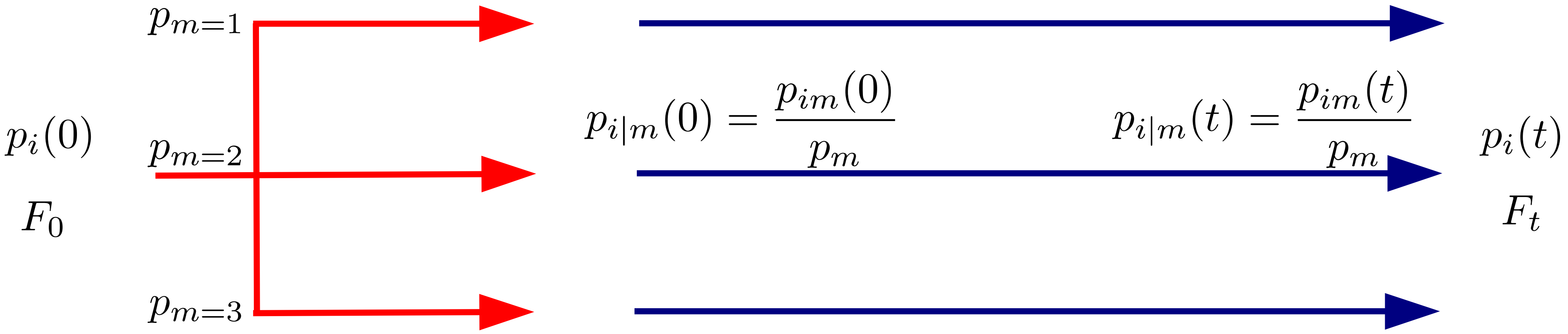
$$TD(p_0|p_0^{\mathrm{eq}}) = -W_{\mathrm{diss}} + T\Delta_{\mathbf{i}} S + TD(p_t|p_t^{\mathrm{eq}})$$

Pure waist:	0	x	0
Optimal extraction:	x	0	0

Second law and Landauer principle far from equilibrium, Esposito and Van den Broeck, EPL **95**, 40004 (2011)

2) Measurement and feedback

Phenomenological approach



$p_i(0)$

F_0

$p_{m=1}$

$p_{m=2}$

$p_{m=3}$

$p_{i|m}(0) = \frac{p_{im}(0)}{p_m}$

$p_{i|m}(t) = \frac{p_{im}(t)}{p_m}$

$p_i(t)$

F_t

Measurement

Feedback

$$\begin{aligned}\delta F_{\text{meas}} &= \sum_m p_m F^{|m} - F \\ &= TS - T\sum_m p_m S^{|m} \\ &= \underbrace{TD(p_{i,m}|p_i p_m)}_{\equiv I} \geq 0\end{aligned}$$

Mutual Information

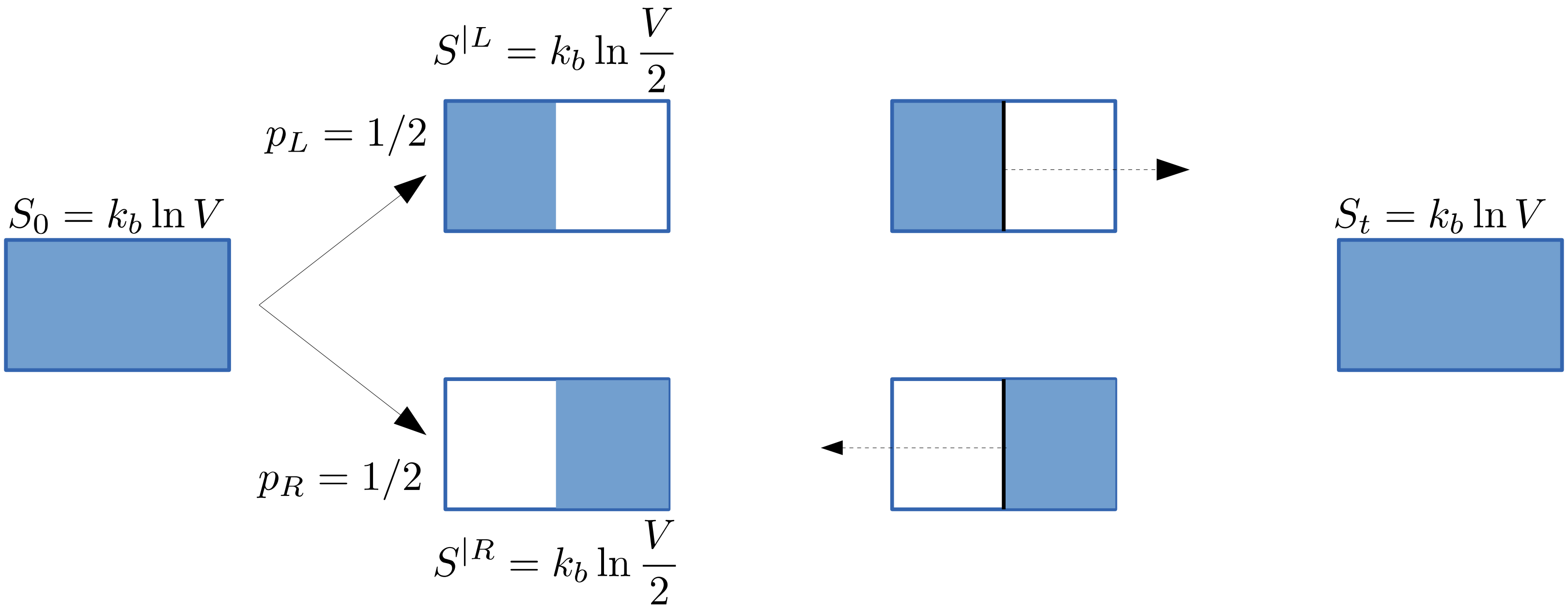
$$T\Delta_{\mathbf{i}} S^{|m} = W^{|m} - \Delta F^{|m} \geq 0$$

$$\sum_m p_m \cdot \quad \Big\downarrow$$

$$\boxed{\underbrace{W - (F_t - F_0)}_{\substack{\text{without feedback} \\ \geq 0}} \geq \boxed{T(I_t - I_0) \geq -TI_0}}$$

Ex1: Szilard engine

Energy plays no role: $\Delta F = -T\Delta S$



$S^{|L} = k_b \ln \frac{V}{2}$

$p_L = 1/2$

$S_0 = k_b \ln V$

$S_t = k_b \ln V$

$p_R = 1/2$

$S^{|R} = k_b \ln \frac{V}{2}$

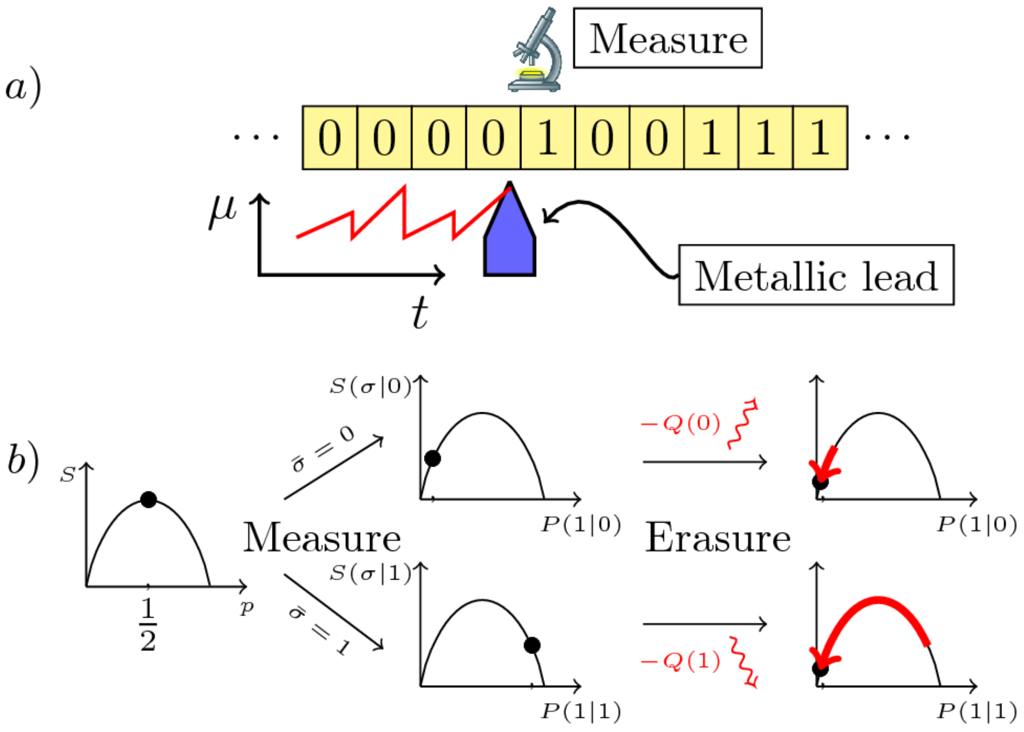
Measurement

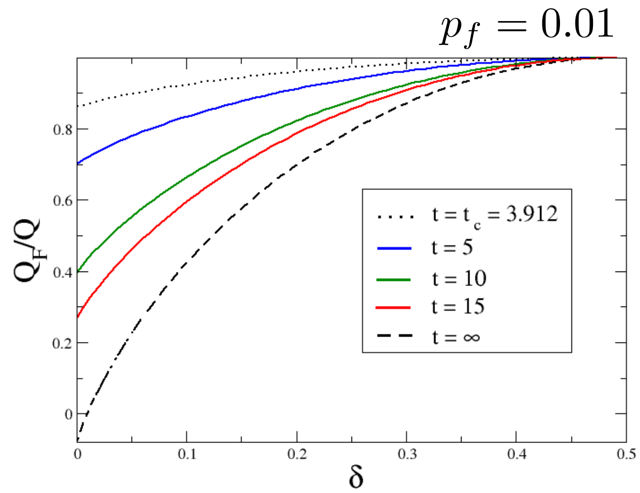
$$\delta F_{\text{meas}} = TI = k_b T \ln 2$$

Feedback

$$W = -k_b T \ln 2$$

Ex2: Erasure with feedback in finite time



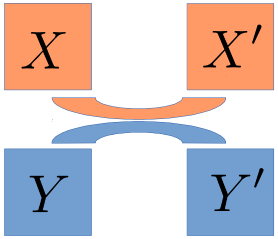


$$T(S_t - S_0) - Q \geq T(I_t - I_0) \geq -TI_0$$

Finite-time erasing of information stored in fermionic bits, Diana, Bagci, Esposito, Phys. Rev. E **85**, 041125 (2012)

3) Bipartite perspective

Non-autonomous systems (measurement and feedback)



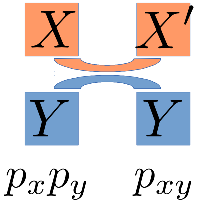
Mutual Information $I \equiv S_X + S_Y - S_{XY} = D(p_{xy}|p_x p_y) \geq 0$

$$\Delta E_{XY} = \Delta E_X + \Delta E_Y$$

$$\Delta F_{XY} = \Delta F_X + \Delta F_Y - T\Delta I$$

$$T\Delta_{\mathbf{i}} S = W - \Delta F_X - \Delta F_Y \boxed{- T\Delta I} \geq 0$$

Measurement



$$\Delta E_X = 0$$

$$\Delta E_Y = \Delta S_Y = 0$$

$$\boxed{W_{\text{meas}} - \Delta F_X \geq TI}$$

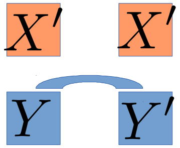
Perfect measurement

$p_{xy} = p_y \delta_{xy}$

$I = S_X = S_Y = S_{XY}$

$W_{\text{meas}} \geq 0$

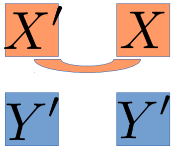
Feedback



$$\Delta E_X = \Delta S_X = 0$$

$$\boxed{W_{\text{feed}} - \Delta F_Y \geq T\Delta I \geq -TI}$$

Resetting memory



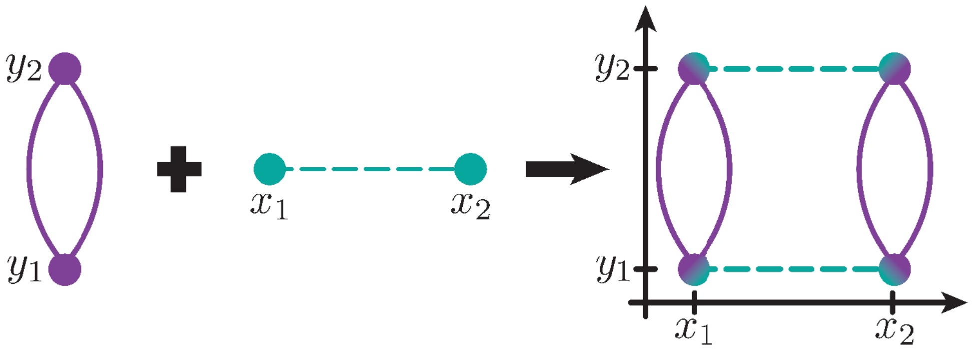
$$\Delta E_Y = \Delta S_Y = 0$$

$$W_{\text{reset}} + \Delta F_X \geq 0$$

$$W_{\text{meas}} + W_{\text{feed}} - \Delta F_X - \Delta F_Y \geq 0 \qquad W_{\text{meas}} + W_{\text{reset}} \geq TI$$

Sagawa, Ueda PRL **102**, 250602 (2009)

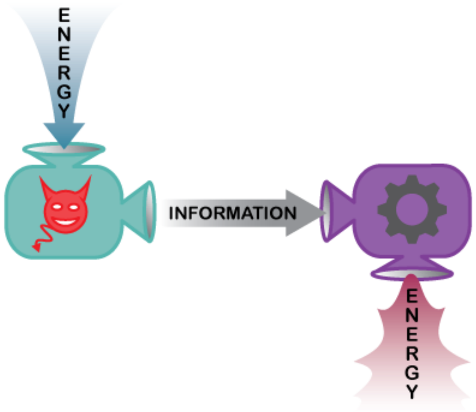
Autonomous systems (continuous information flow)



$$I=\sum_{x,y}p(x,y)\ln\frac{p(x,y)}{p(x)p(y)}\geq 0, \qquad d_tI=\dot{I}^X+\dot{I}^Y$$

$$\dot{S}_{\mathbf{i}}=\dot{S}_{\mathbf{i}}^X+\dot{S}_{\mathbf{i}}^Y$$

$$\boxed{\begin{aligned}\dot{S}_{\mathbf{i}}^X&=d_tS^X+\dot{S}_{\mathbf{r}}^X-\dot{I}^X\geq 0\\ \dot{S}_{\mathbf{i}}^Y&=d_tS^Y+\dot{S}_{\mathbf{r}}^Y-\dot{I}^Y\geq 0\end{aligned}}$$



Steady state: $d_tI=0$ $\qquad \dot{\mathcal{I}}=\dot{I}^X=-\dot{I}^Y$

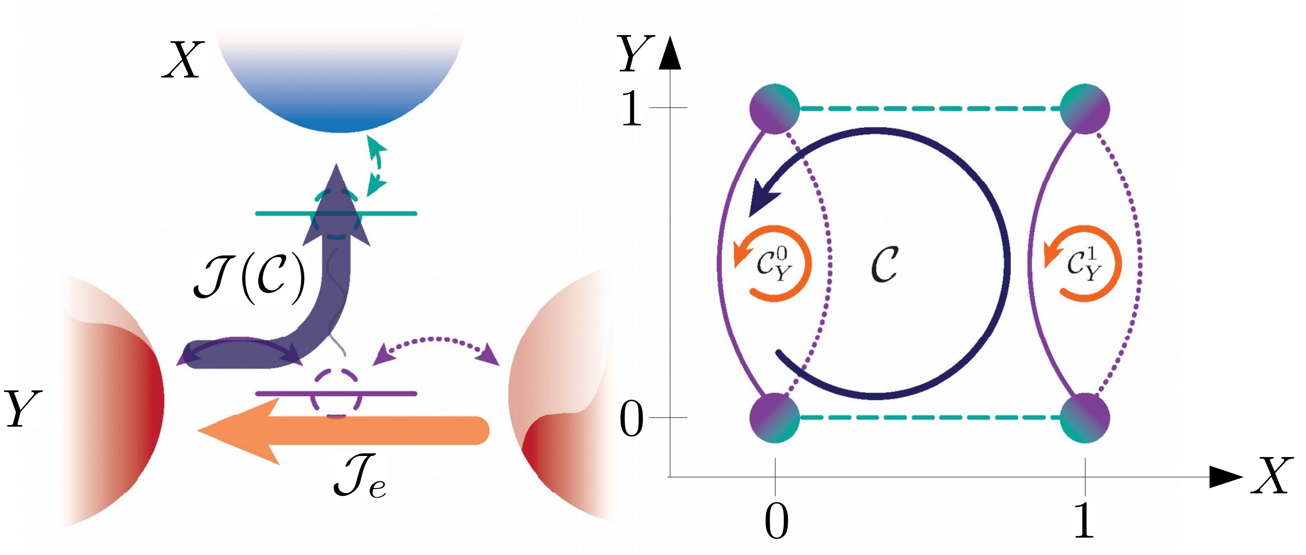
$$\dot{\mathcal{S}}_{\mathbf{i}}^X=\dot{\mathcal{S}}_{\mathbf{r}}^X-\dot{\mathcal{I}}\geq 0$$

$$\dot{\mathcal{S}}_{\mathbf{i}}^Y=\dot{\mathcal{S}}_{\mathbf{r}}^Y+\dot{\mathcal{I}}\geq 0$$

Thermodynamics with continuous information flow, Horowitz and Esposito, Phys. Rev. X **4**, 031015 (2014)

At steady state see also Hartich, Barato, Seifert, JSM P02016 (2014)

Ex: Two coupled quantum dots



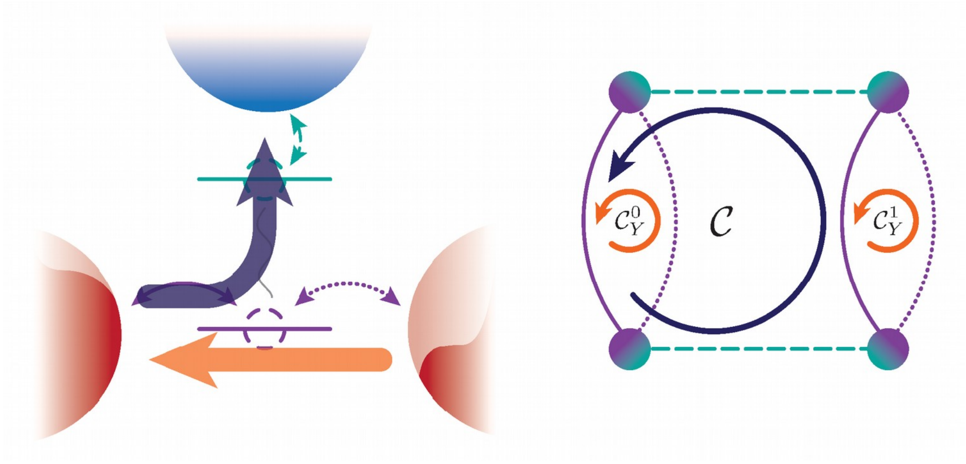
$$\dot{\mathcal{S}}_{\mathbf{i}} = -\mathcal{J}_e \overset{>0}{\frac{\Delta\mu}{T}} + \mathcal{J}(\mathcal{C}) \overset{>0}{\left(\frac{U}{T_D} - \frac{U}{T} \right)} \geq 0 \qquad \mathcal{J}_e = \mathcal{J}(\mathcal{C}_Y^0) + \mathcal{J}(\mathcal{C}_Y^1)$$

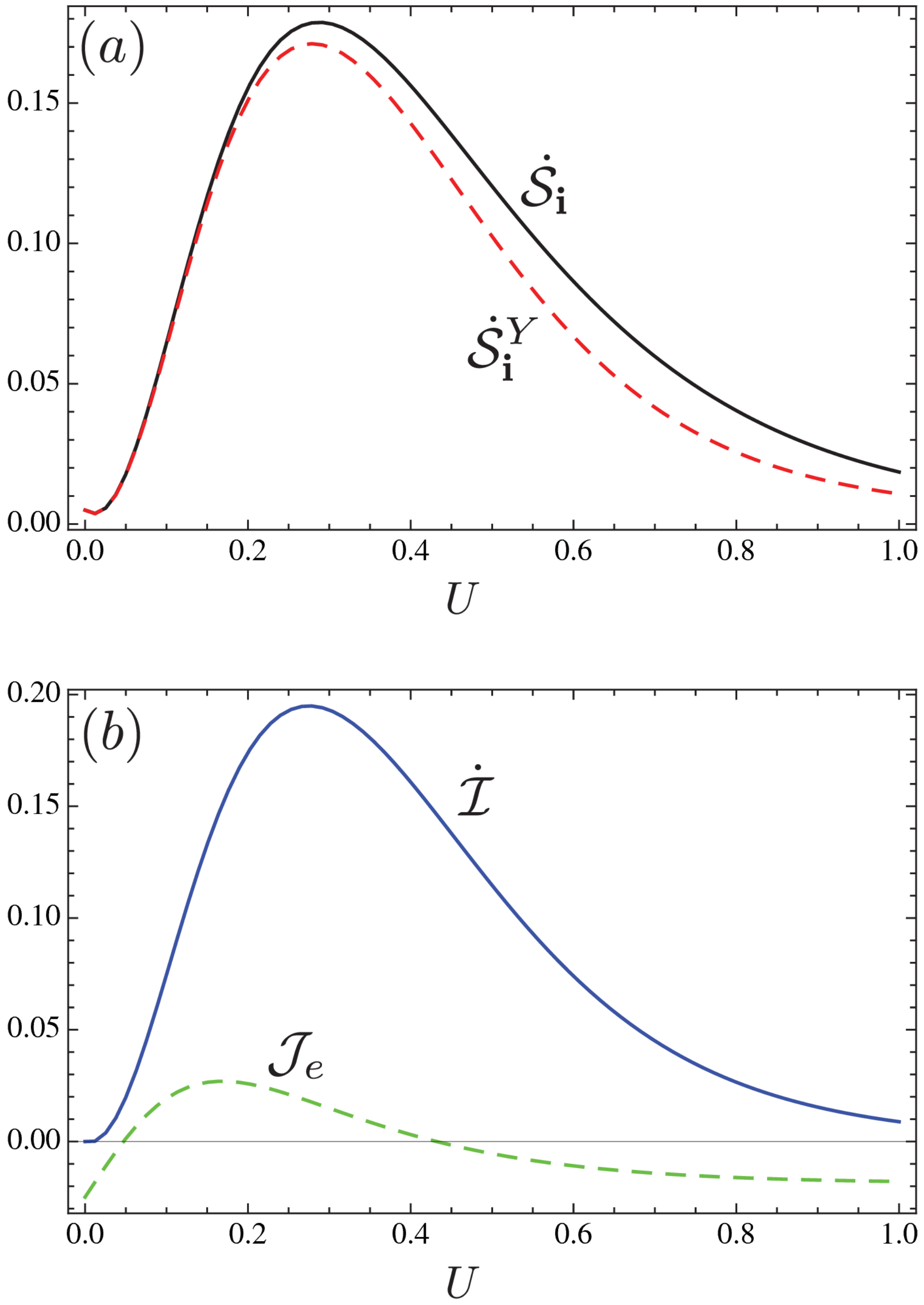
$$\dot{\mathcal{I}} = \mathcal{J}\mathcal{F}^I > 0 \quad \text{where} \quad \mathcal{F}^I(\mathcal{C}) = \ln \frac{p(x=1|y=0)p(x=0|y=1)}{p(x=1|y=1)p(x=0|y=0)}$$

$$\dot{\mathcal{S}}_{\mathbf{i}}^X = \mathcal{J}(\mathcal{C}) \left[\frac{U}{T_D} - \mathcal{F}^I(\mathcal{C}) \right] \geq 0 \qquad \dot{\mathcal{S}}_{\mathbf{i}}^Y = -\mathcal{J}_e \frac{\Delta\mu}{T} + \mathcal{J}(\mathcal{C}) \left[\mathcal{F}^I(\mathcal{C}) - \frac{U}{T} \right] \geq 0$$

Maxwell demon limit: $U \to 0 \qquad T_D \to 0 \qquad U/T_D = \mathrm{Cte}$

Thermodynamics of a physical model implementing a Maxwell demon,
Strasberg, Schaller, Brandes, Esposito, Phys. Rev. Lett. **110**, 040601 (2013)





Thermodynamics with continuous information flow, Horowitz and Esposito, Phys. Rev. X **4**, 031015 (2014)

4) Conclusions and perspectives

Thermodynamics of information, Parrondo, Horowitz, Sagawa, Nature Physics **11**, 131 (2015)

- Many other approaches
  Esposito and Schaller, EPL **99**, 30003 (2012)
  Mandal, Jarzynski, PNAS **109**, 11641 (2012)
  Barato, Seifert, PRL **112** 09061 (2014)
  Horowitz, Sandberg, NJP **16**, 125007 (2012)

- Experiments
  Toyabe & al., Nature Physics **6**, 988 (2010)
  Bérut & al., Nature **483**, 187 (2012)
  Jun, Gavrilov, Bechhoefer, PRL **113**, 190601 (2014)
  Koski & al. PRL **115**, 260602 (2015)

- Biology (sensing, proofreading, chemotaxis, chemical computing…)

Part III: Quantum Thermodynamics

1) Phenomenological thermodynamics

2) A Hamiltonian formulation

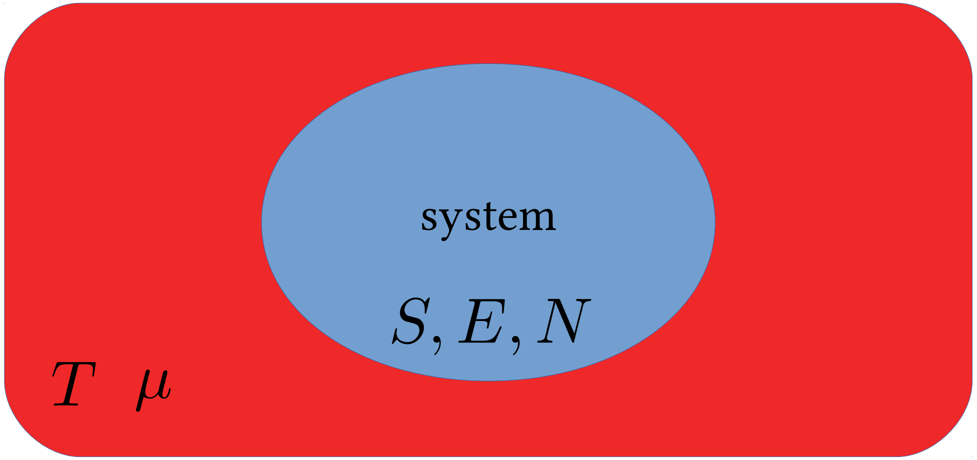
3) Born-Markov-Secular Quantum Master Equation (QME)

4) Landau-Zener QME

5) Repeated interactions

6) More...

1) Phenomenological Nonequilibrium Thermodynamics



Zeroth law: System dynamics with an equilibrium

1st law: $$d_t E = \dot{W} + \dot{Q}$$

Slow transformation

2nd law: $$\dot{\Sigma} = d_t S - \frac{\dot{Q}}{T} \geq 0$$ $$d_t S \approx \frac{\dot{Q}}{T}$$

$\dot{\Sigma}$ → Entropy production
(dissipation)

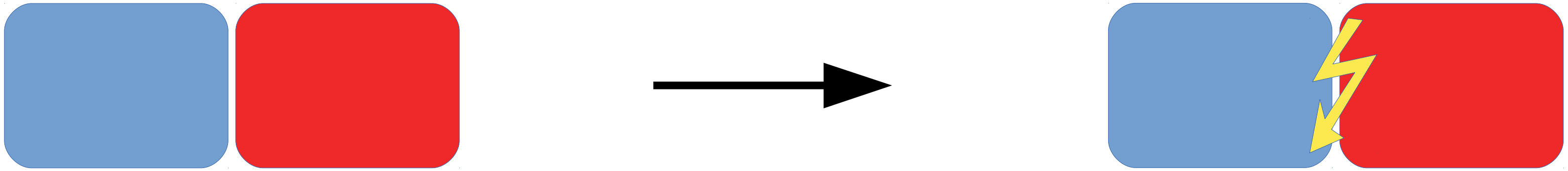
2) Hamiltonian formulation

System X – System Y

$$H_{\text{tot}}(t) = H_X(t) + H_Y(t) + H_{XY}(t)$$

$$\rho_{XY}(0) = \rho_X(0)\rho_Y(0)$$

$$\rho_{XY}(\tau) = U_\tau \rho_X(0)\rho_Y(0)U_\tau^\dagger$$



$$d_t E_{XY}(t) = \text{tr}_{XY}\left\{\rho_{XY}(t) d_t H_{\text{tot}}(t)\right\} \equiv \dot{W}(t)$$

$$I_{X:Y}(t) \equiv S_X(t) + S_Y(t) - S_{XY}(t)$$

$$S_{XY}(t) \equiv -\text{tr}_{XY}\{\rho_{XY}(t) \ln \rho_{XY}(t)\}$$

$$= \Delta S_X(\tau) + \Delta S_Y(\tau) \;= D[\rho_{XY}(t)||\rho_X(t)\rho_Y(t)] \geq 0$$

System X – Reservoir R

$$H_{\rm tot}(t) = H_X(t) + H_R + H_{XR}(t)$$

Assumption: $\rho_{XR}(0) = \rho_X(0)\rho_\beta^R$ $\qquad \rho_\beta^R \equiv \dfrac{e^{-\beta H_R}}{Z_R}$

$$E_X(t) \equiv {\rm tr}_{XR}\{[H_X(t) + H_{XR}(t)]\rho_{XR}(t)\}$$

1st law: $d_t E_X(t) = \dot{W}(t) + \dot{Q}(t)$ $\qquad \left\{ \begin{array}{l} \dot{W}(t) = d_t E_{XY}(t) \\ \dot{Q}(t) \equiv -{\rm tr}_R\left\{H_R d_t \rho_R(t)\right\} \end{array} \right.$

2nd law: $\Sigma(\tau) \equiv \Delta S_X(\tau) - \beta Q(\tau) = D[\rho_{XR}(\tau)||\rho_X(\tau)\rho_\beta^R]$

$$= D[\rho_R(\tau)||\rho_\beta^R] + I_{X:R}(\tau) \geq 0$$

[Esposito, Lindenberg, & Van den Broeck, NJP **12**, 013013 (2010)]

Ideal reservoir

Another identity: $TD[\rho_R(\tau)||\rho_\beta^R] = -Q(\tau) - T\Delta S_R(\tau) \geq 0$

$$\rho_R(\tau) = \rho_\beta^R + \epsilon\sigma_R \qquad D[\rho_R(\tau)||\rho_\beta^R] = \mathcal{O}(\epsilon^2)$$

$$\Downarrow$$

$$\Delta S_R(\tau) = -\beta Q(\tau)$$

$$\Sigma(\tau) = I_{X:R}(\tau)$$

$$\text{Summary:} \quad \begin{cases} 1^{\text{st}},\ 2^{\text{nd}} \text{ law, strong coupling} \\ \text{no } 0^{\text{th}} \text{ law, } \ \Sigma \geq 0 \text{ but not } \dot{\Sigma} \end{cases}$$

3) Born-Markov-Secular QME

$$H_{\mathrm{tot}}(t) = H_X \underset{\text{slow}}{(t)} + H_R + \sum_k \underset{\text{weak}}{A_k \otimes B_k}$$

Effective dynamics

$$d_t \rho_X(t) = -i[H_X(t), \rho_X(t)] + \mathcal{L}_\beta(t)\rho_X(t) \equiv \mathcal{L}_X(t)\rho_X(t),$$

$$\mathcal{L}_\beta(t)\rho(t) = \sum_\omega \sum_{k,\ell} \gamma_{k\ell}(\omega) \left(A_\ell(\omega)\rho(t)A_k^\dagger(\omega) - \frac{1}{2}\{A_k^\dagger(\omega)A_\ell(\omega), \rho(t)\} \right)$$

$$A_k(\omega) \equiv \sum_{\epsilon-\epsilon'=\omega} \Pi_\epsilon A_k \Pi_{\epsilon'} \qquad \gamma_{k\ell}(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \mathrm{tr}_R\{B_k(t)B_\ell(0)\rho_\beta^R\}$$

Local detailed balance: $\gamma_{k\ell}(-\omega) = e^{-\beta\omega}\gamma_{\ell k}(\omega)$

$$\mathcal{L}_\beta(t)\rho_\beta^X(t) = 0, \quad \rho_\beta^X(t) = \frac{e^{-\beta H_X(t)}}{Z_X(t)}$$

Thermodynamics

Energy: $E_X(t) = -\mathrm{tr}_X\{H_X(t)\rho_X(t)\}$

Entropy: $S_X(t) = -\mathrm{tr}_X\{\rho_X(t)\ln\rho_X(t)\}$

1st law $$d_t E_X(t) = \dot{W}(t) + \dot{Q}(t)$$

$$\dot{W}(t) = \mathrm{tr}_X\,\{\rho_X(t) d_t H_X(t)\}$$

$$\dot{Q}(t) = \mathrm{tr}_X\,\{H_X(t) d_t \rho_X(t)\} = \mathrm{tr}_X\{H_X(t)\mathcal{L}_X(t)\rho_X(t)\}$$

2nd law $$\dot{\Sigma}(t) = d_t S_X(t) - \beta\dot{Q}(t)$$

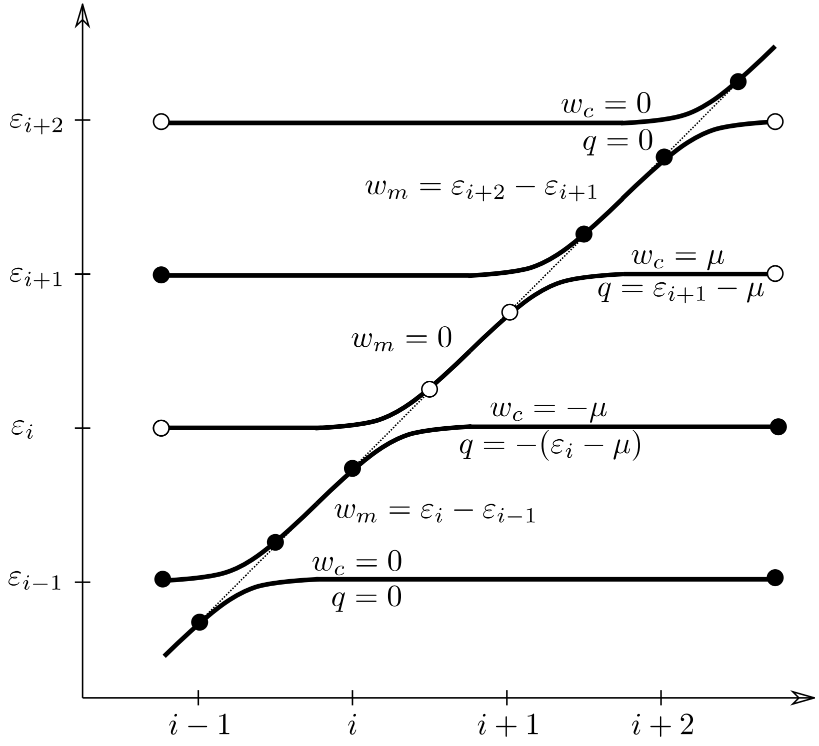
$$= -\mathrm{tr}\{[\mathcal{L}_X(t)\rho_X(t)][\ln\rho_X(t) - \ln\rho_\beta^X(t)]\} \geq 0$$

Summary:

- 0th, 1st, 2nd law, $\dot{\Sigma} \geq 0$, slow trsf. $\dot{\Sigma} \approx 0$
- but weak coupling

4) A Landau-Zener approach

$$H(t) = \epsilon_t c^\dagger c + \sum_{i=1}^{L} \varepsilon_i c_i^\dagger c_i + \gamma \sum_{i=1}^{L} (c^\dagger c_i + c_i^\dagger c)$$



Prob. diabatic transition:

$$R_i = \exp\left\{-\pi\delta_i^2/(2\hbar\dot{\epsilon}_i)\right\}$$

$$t_i^{\mathrm{lz}} = \sqrt{\hbar/\dot{\epsilon}_i}\,\max[1, \sqrt{\delta_i^2/(\hbar\dot{\epsilon}_i)}]$$

Validity: $\Delta t_{i+} > t_i^{\mathrm{lz}}$ $\Delta\varepsilon_{i+} > \delta_i$ → $\Delta\varepsilon_{i+} > \sqrt{\hbar\dot{\epsilon}_i}, \delta_i$

[Barra & Esposito, PRE **93**, 062118 (2016)]

Effective dynamics

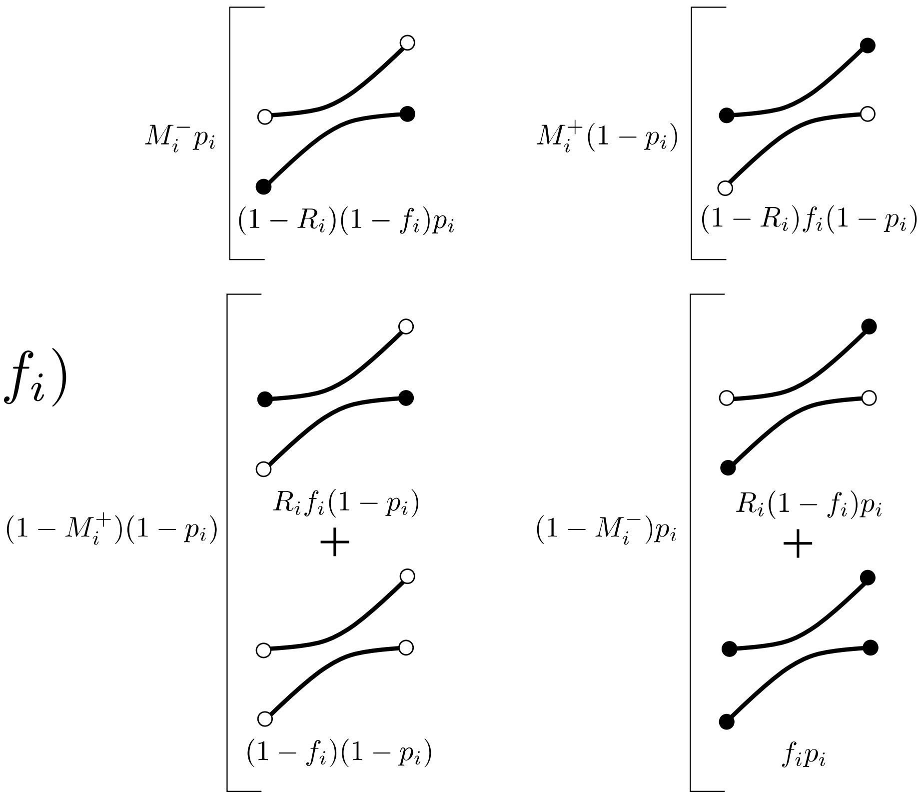
Master equation

$$p_{i+1} = (1 - M_i^-)p_i + M_i^+(1 - p_i)$$

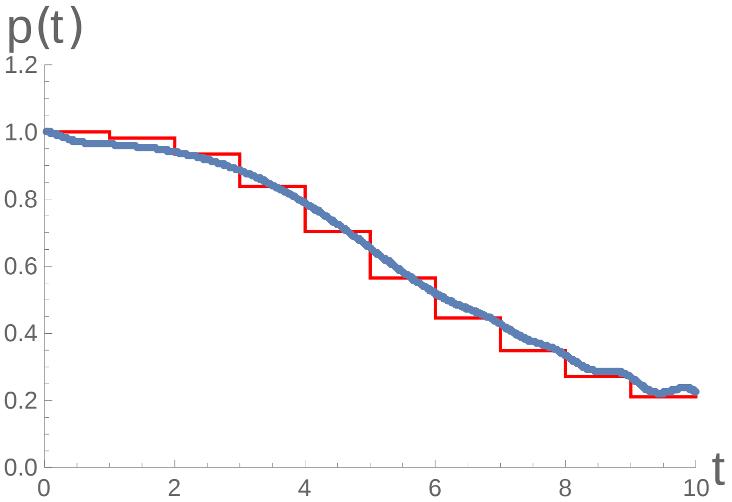
$$M_i^+ = (1 - R_i)f_i \qquad M_i^- = (1 - R_i)(1 - f_i)$$

Local detailed balance

$$M_i^+/M_i^- = e^{-\beta(\varepsilon_i - \mu)}$$

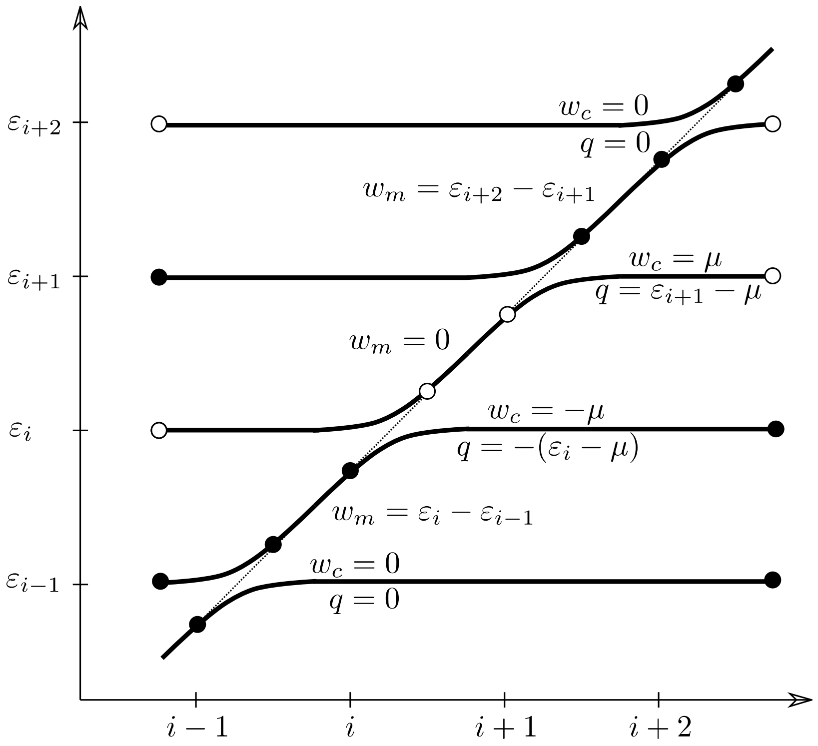


Exact vs stochastic dynamics



[Barra & Esposito, PRE **93**, 062118 (2016)]

Thermodynamics



1st law $\Delta E_{i+} = E_{i+1} - E_i = W_{i+} + Q_{i+}$

$$N_i = p_i \qquad E_i = \varepsilon_i p_i \qquad Q_{i+} = (\varepsilon_i - \mu)(p_{i+1} - p_i)$$

$$W_{i+} = W_{i+}^{\mathrm{m}} + W_{i+}^{\mathrm{c}} \quad \begin{cases} W_{i+}^{\mathrm{m}} = (\varepsilon_{i+1} - \varepsilon_i)p_{i+1} \\ W_{i+}^{\mathrm{c}} = \mu(p_{i+1} - p_i) \end{cases}$$

2nd law $\Delta S_{i+} = S_{i+1} - S_i = \Sigma_{i+} + Q_{i+}/T$

$$S_i = -k_B p_i \ln p_i - k_B (1 - p_i) \ln(1 - p_i)$$

$$\Sigma_{i+} = k_B M_i^+ (1 - p_i) \ln \frac{M_i^+ (1 - p_i)}{M_i^- p_i} + k_B M_i^- p_i \ln \frac{M_i^- p_i}{M_i^+ (1 - p_i)} - k_B D(p_{i+1} | p_i) \geq 0$$

[Barra & Esposito, PRE **93**, 062118 (2016)]

$$\overset{\text{between crossing}}{\underbrace{W^{\mathrm{m}}_{i+} - \Delta\Omega^{\mathrm{eq}}_{i+}}}$$

$$\underbrace{\Sigma_{i+}}_{\text{at crossing}} = \frac{W^{\mathrm{diss}}_{i+}}{T} - k_B D(p_{i+1}|f_{i+1}) + k_B D(p_i|f_i) \geq 0$$

QM adiabatic regime (slow driving)

$$p_i = f_{i-1} \quad p_{i+1} = f_i$$

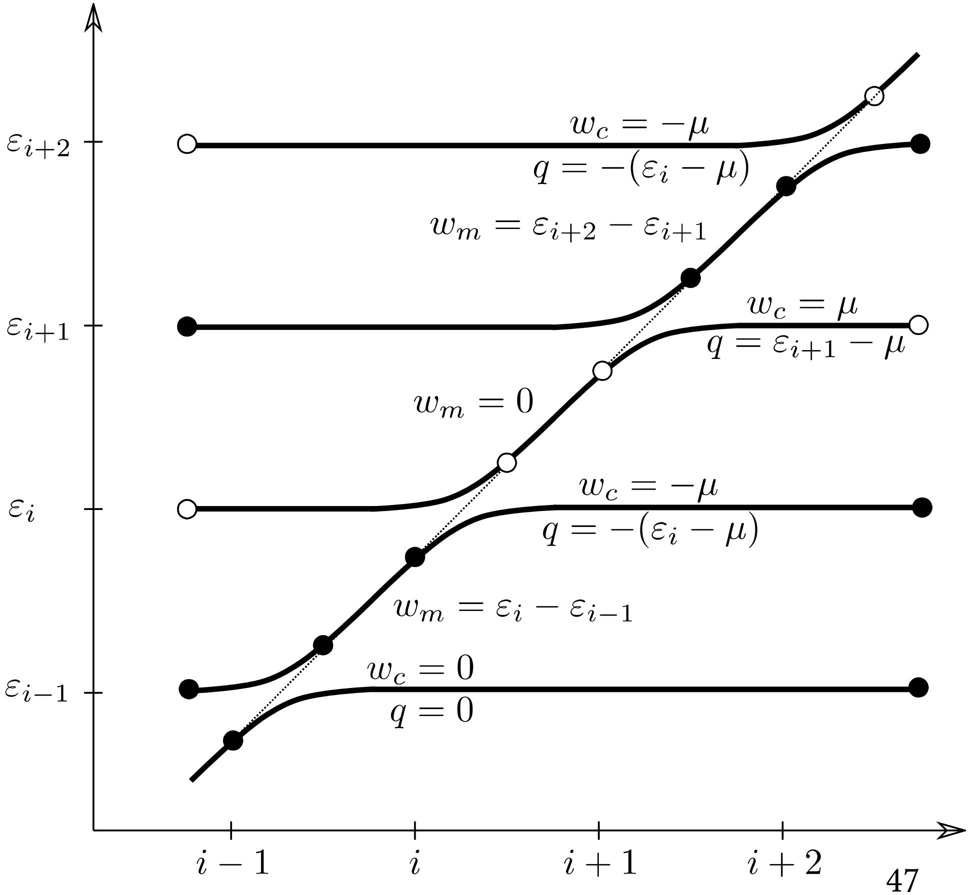
At the crossing: $\Sigma_{i+} = k_B D(f_{i-1}|f_i)$

From $i \to i+1$: $W^{\mathrm{diss}}_{i+} = k_B T D(f_i|f_{i+1})$

Reversibility only occurs if:

$$\Delta\varepsilon, \delta, \dot{\epsilon} \to 0$$

$$\Delta\varepsilon > \delta \gg \sqrt{\hbar\dot{\epsilon}} \qquad \Sigma_{i+}\ ,\ W^{\mathrm{diss}}_{i+} \sim \Delta\varepsilon^2$$



[Barra & Esposito, PRE **93**, 062118 (2016)]

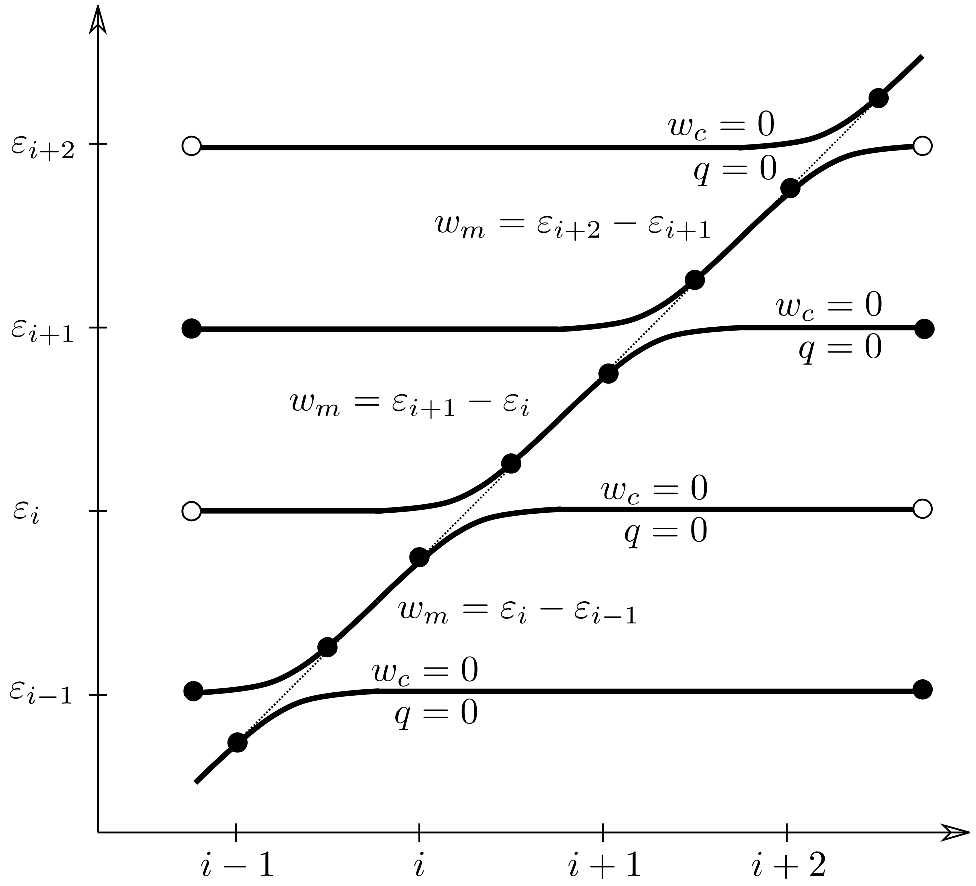
$$\Sigma_{i+} = \frac{W_{i+}^{\text{diss}}}{T} - k_B D(p_{i+1}|f_{i+1}) + k_B D(p_i|f_i) \geq 0$$

QM diabatic regime (fast driving)

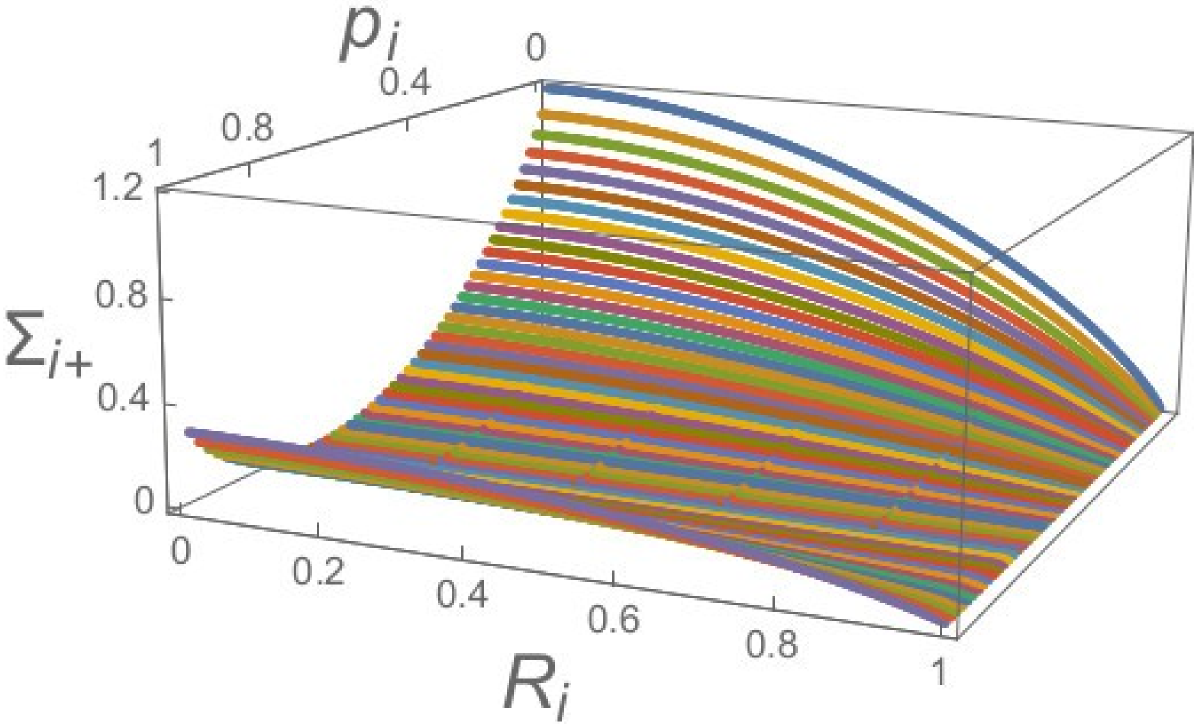
$$p_i = p_1$$

$$\Sigma = \Delta S = Q = 0$$

$$W^{\text{diss}} = k_B T \sum_i \big(D(p_1|f_{i+1}) - D(p_1|f_i)\big)$$



[Barra & Esposito, PRE **93**, 062118 (2016)]



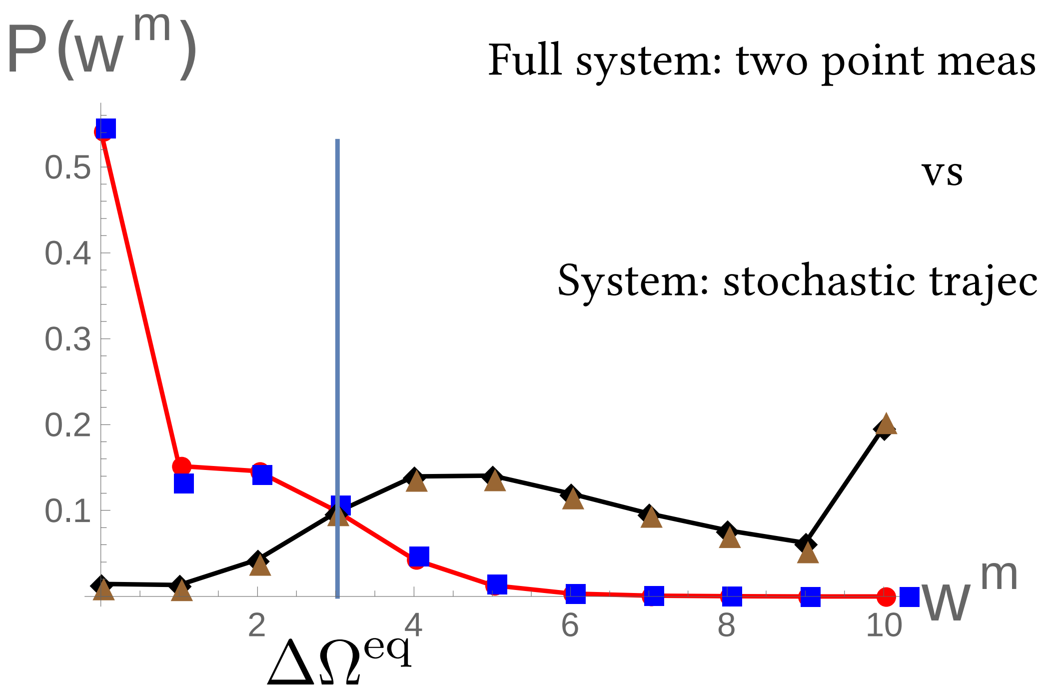
[Barra & Esposito, PRE **93**, 062118 (2016)]

Work fluctuations

Jarzynski and Crooks fluctuation relation

System initially
at equilibrium

$$\frac{P(w^{\mathrm{m}})}{\tilde{P}(-w^{\mathrm{m}})} = \exp\left\{\beta(w^{\mathrm{m}} - \Delta\Omega^{\mathrm{eq}})\right\}$$



Full system: two point measurement approach

vs

System: stochastic trajectory approach

[Barra & Esposito, PRE **93**, 062118 (2016)]

QM diabatic regime: continuous limit

$$\Delta\varepsilon > \sqrt{\hbar\dot{\epsilon}} > \delta \qquad\qquad R \approx 1 - \frac{\delta^2}{\hbar\dot{\epsilon}}\frac{\pi}{2}$$

$$(p_{i+1} - p_i)/\Delta t_{i+} = (f_i - p_i)(1 - R_i)/\Delta t_{i+}$$

$$d_i = 1/\Delta\varepsilon_i \qquad\qquad \Delta t_{i+} = 1/(\dot{\epsilon}_i d_i)$$

$$\Downarrow$$

$$d_t p = w^+(1-p) - w^- p$$

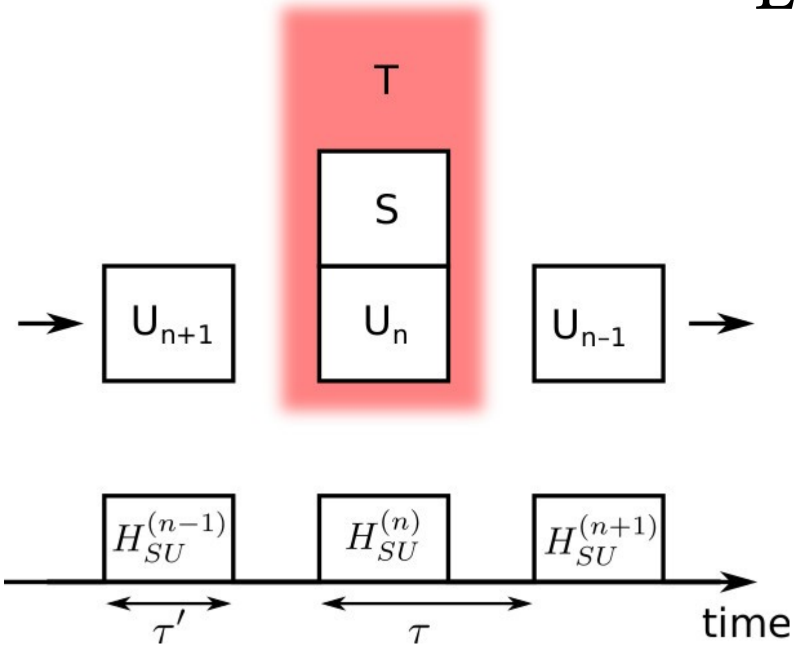
$$w^+ = \frac{\pi\delta^2(\epsilon_t)d(\epsilon_t)}{2\hbar}f(\epsilon_t) \qquad w^- = \frac{\pi\delta^2(\epsilon_t)d(\epsilon_t)}{2\hbar}(1 - f(\epsilon_t))$$

Pauli master equation with Fermi golden rule rates

[Barra & Esposito, PRE **93**, 062118 (2016)]

5) Repeated interactions

Exact identities



$$\Delta E_X = \Delta E_U + \Delta E_S = W + Q$$

$$\Sigma = \underbrace{\Delta S_S + \Delta S_U - I_{S:U}(\tau)}_{\Delta S_X} - \beta Q \geq 0$$

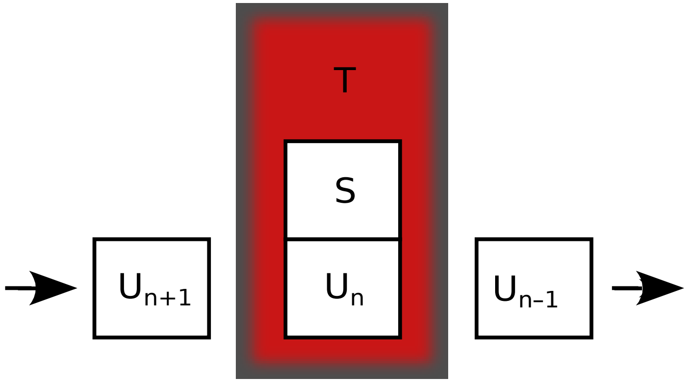
$$\Delta E_{SU} \equiv \lim_{\epsilon \searrow 0} \int_{-\epsilon}^{\tau-\epsilon} dt \frac{dE_X(t)}{dt} = \Delta E_S + \Delta E_U$$

$$W \equiv \lim_{\epsilon \searrow 0} \int_{-\epsilon}^{\tau-\epsilon} dt \dot{W}(t) = W_X + W_{\text{sw}}$$

$$\left\{ \begin{aligned} W_X &= \int_0^{\tau} dt \mathrm{tr}_X \left\{ \rho_X(t) d_t H_S(t) \right\} \\ &\quad + \lim_{\epsilon \searrow 0} \int_{\epsilon}^{\tau'-\epsilon} dt' \mathrm{tr}_X \left\{ \rho_X(t) d_t V_{SU}(t) \right\} \\ W_{\text{sw}} &= \mathrm{tr}_X \{ V_{SU}(0) \rho_X(0) - V_{SU}(\tau') \rho_X(\tau') \} \end{aligned} \right.$$

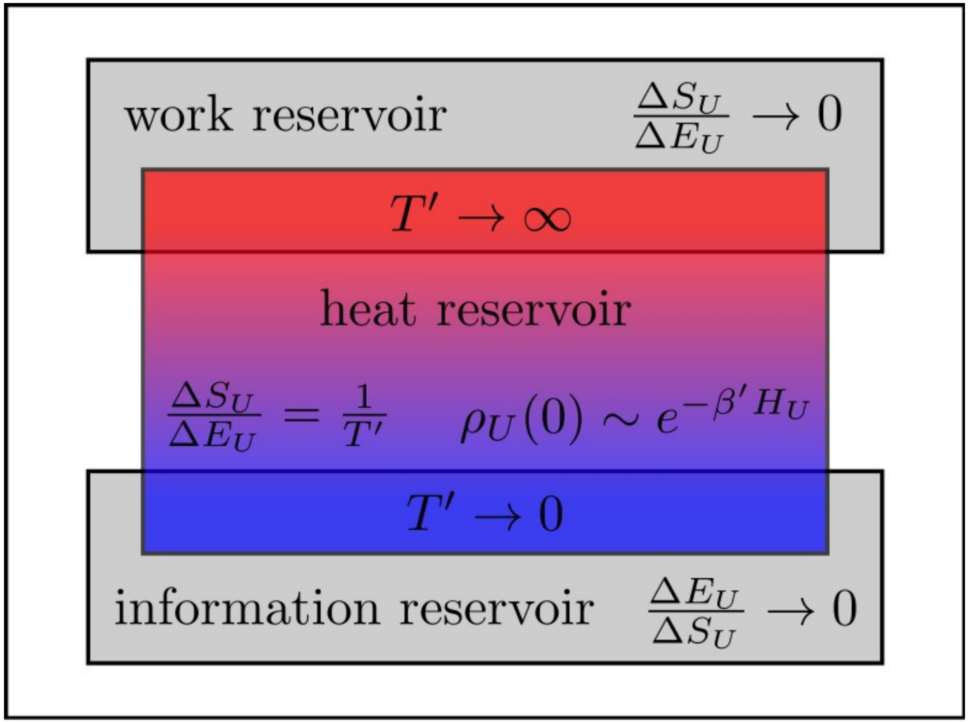
$$Q \equiv \lim_{\epsilon \searrow 0} \int_{-\epsilon}^{\tau-\epsilon} dt \dot{Q}(t)$$

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]



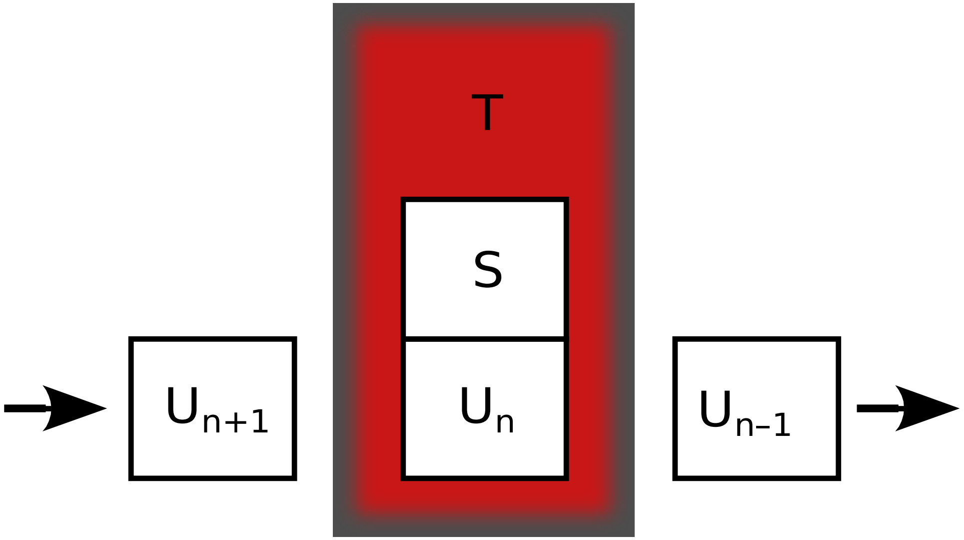
1st law
$$\Delta E_S = W + Q - \Delta E_U$$

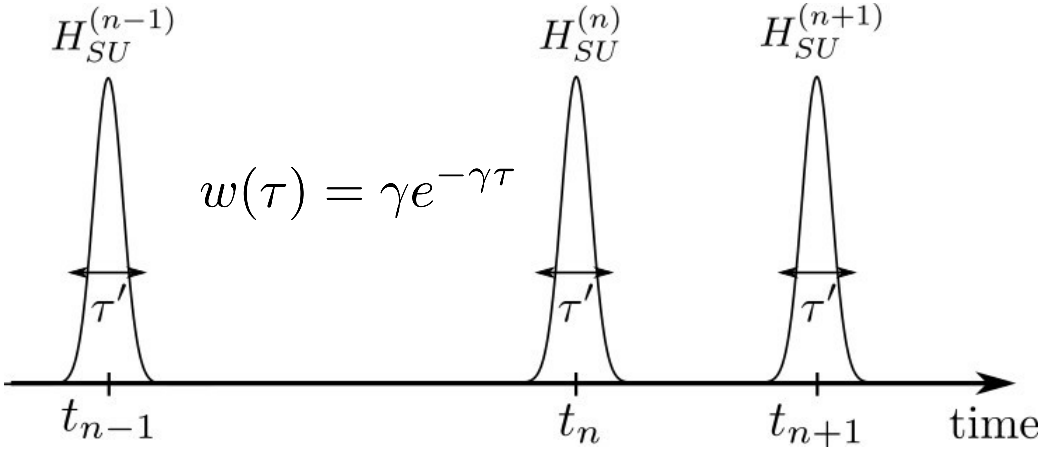
2nd law
$$\Sigma_S \equiv \Delta S_S + \Delta S_U - \beta Q \geq I_{S:U}(\tau) \geq 0$$



[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

Repeated interaction QME





$$H_{SU}(t) = \sum_n \delta(t - t_n) V_{SU}$$

Effective dynamics

Effect of a kick: $U = e^{-iV_{SU}}$

$$\mathcal{J}_S \rho_S(t) \equiv \mathrm{tr}_U\{U \rho_S(t) \otimes \rho_U U^\dagger\}$$

$$\mathcal{J}_U \rho_U \equiv \mathrm{tr}_S\{U \rho_S(t) \otimes \rho_U U^\dagger\}$$

$$d_t \rho_S(t) = -i[H_S(t), \rho_S(t)] + \mathcal{L}_\beta \rho_S(t) + \mathcal{L}_{\mathrm{new}} \rho_S(t)$$

$$\mathcal{L}_{\mathrm{new}} \rho_S(t) \equiv \gamma(\mathcal{J}_S - 1)\rho_S(t)$$

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

Thermodynamics

1st law

$$d_t E_S(t) = \dot{W}_S(t) + \dot{W}_{SU}(t) + \dot{Q}(t) - d_t E_U(t)$$

$$\dot{W}_S = \mathrm{tr}_S\{\rho_S(t) d_t H_S(t)\}$$

$$\begin{aligned}\dot{W}_{SU} &= \gamma \mathrm{tr}_{SU}\{[H_S(t) + H_U][U\rho_S(t)\rho_U U^\dagger - \rho_S(t)\rho_U]\} \\ &= \gamma \mathrm{tr}_S\{H_S(t)(\mathcal{J}_S - 1)\rho_S(t)\} + \gamma \mathrm{tr}_U\{H_U(\mathcal{J}_U - 1)\rho_U\}\end{aligned}$$

$$\dot{Q}(t) = \mathrm{tr}_S\{H_S(t)\mathcal{L}_\beta \rho_S(t)\}$$

$$d_t E_U(t) = \gamma \mathrm{tr}_U\{H_U(\mathcal{J}_U - 1)\rho_U\}$$

2nd law

$$\dot{\Sigma}_S(t) = d_t S_S(t) + d_t S_U(t) - \beta\dot{Q} \geq 0$$

$$\neq -\mathrm{tr}\{[\mathcal{L}_0 \rho_S(t)][\ln \rho_S(t) - \ln \rho_\beta^S(t)]\} - \mathrm{tr}\{[\mathcal{L}_{\mathrm{new}}\rho_S(t)][\ln \rho_S(t) - \ln \bar{\rho}_{\mathrm{new}}]\}$$

thermodynamics cannot always be deduced from dynamics alone

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

Units entropy changes

Approach 1: $\rho_U(t) = \mathcal{J}_U \rho_U$

$$d_t S_U(t) = \gamma(-\mathrm{tr}_U\{(\mathcal{J}_U \rho_U)\ln(\mathcal{J}_U \rho_U)\} + \mathrm{tr}_U\{\rho_U \ln \rho_U\})$$

Approach 2:

Fraction of units which interacted $d_t n_t = \gamma N$

$$\bar{\rho}_U(t) = \frac{n_t}{N}\mathcal{J}_U \rho_U + \frac{N - n_t}{N}\rho_U$$

$$d_t \bar{S}_U(t) = -d_t \mathrm{tr}_U\{\bar{\rho}_U(t) \ln \bar{\rho}_U(t)\} = -\gamma \mathrm{tr}_U\{[(\mathcal{J}_U - 1)\rho_U] \ln \rho_U\}$$

Different by $d_t \bar{S}_U(t) - d_t S_U(t) = \gamma D(\mathcal{J}_U \rho_U \| \rho_U)$ "mixing" contribution

Thermal units $\rho_U = \rho^U_{\beta'}$

$$\begin{cases} d_t S_U(t) = \beta' d_t E_U(t) - \gamma D(\mathcal{J}_U \rho^U_{\beta'} \| \rho^U_{\beta'}) \\ d_t \bar{S}_U(t) = \beta' d_t \bar{E}_U(t) \quad \text{ideal reservoir} \end{cases}$$

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

More...

Quantum master equation including degenerate states:

[Bulnes-Cuetara, Esposito & Schaller, Entropy **18**, 447 (2016)]

Fast periodic driving using master equation and Floquet theory:

[Bulnes-Cuetara, Engel & Esposito, NJP **18**, 447 (2016)]

Strong coupling using polaron transformation and quantum master equation:

[Krause, Brandes, Esposito & Schaller, JCP **142**, 134106 (2015)]
[Schaller, Krause, Brandes & Esposito, NJP **15**, 033032 (2013)]

Strong coupling using Nonequilibrium Green's functions:

[Esposito, Ochoa & Galperin, PRL **114**, 080602 (2015)]
[Esposito, Ochoa & Galperin, PRB **92**, 235440 (2015)]

Strong coupling (classical) using time scale separation:

[Strasberg & Esposito, arxiv:1703.05098]
[Esposito, Phys. Rev. E **85**, 041125 (2012)]

Perspectives

- Stochastic thermodynamics in the thermodynamic limit

Interplays between $N \longrightarrow \infty$ and $t \longrightarrow \infty$

- Chemical Reaction Networks (Stochastic & Deterministic)

Toward energy and information processing in biology

Thank you