Finite VS infinite weights

or...

How to fight fluctuations by saving old wit

Joint work:

Patryk Lipka-Bartosik

Michał Horodecki

Jonathan Oppenheim

Let S be a qubit in a thermal state with trivial Hamiltonian:

$$H_S = 0$$

$$\tau_S = \frac{1}{2}\mathbb{I}_S$$

Consider the following process...

$$\Phi\left(\frac{1}{2}\mathbb{I}_S\right) = (1-\epsilon)|0\rangle\langle 0|_S + \epsilon|1\rangle\langle 1|_S$$

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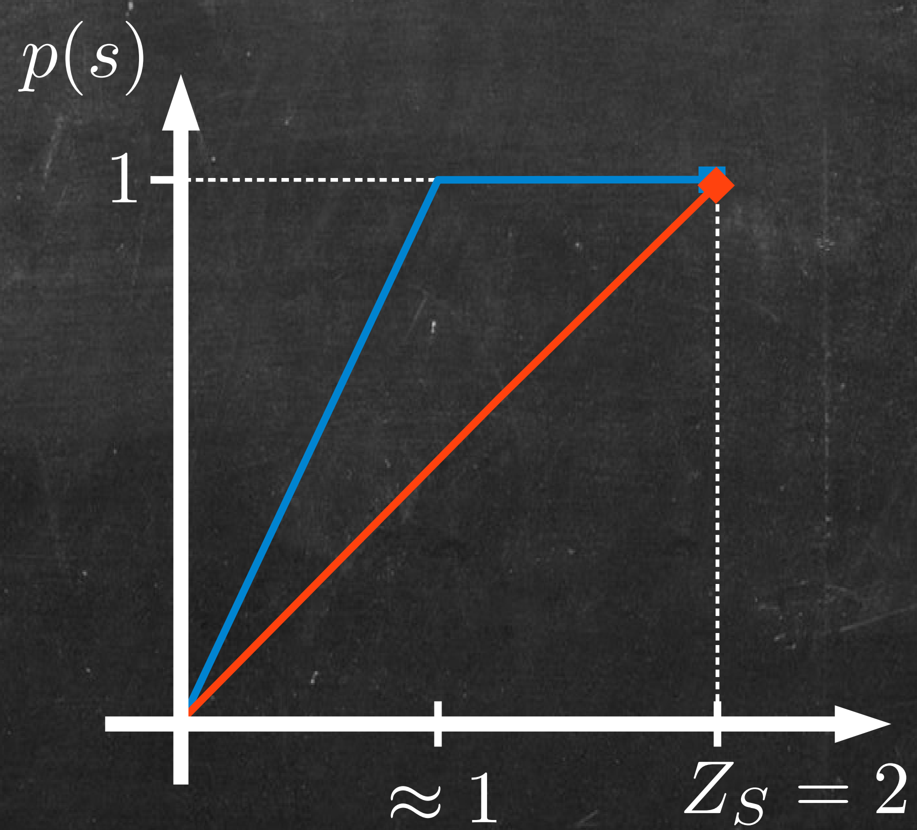
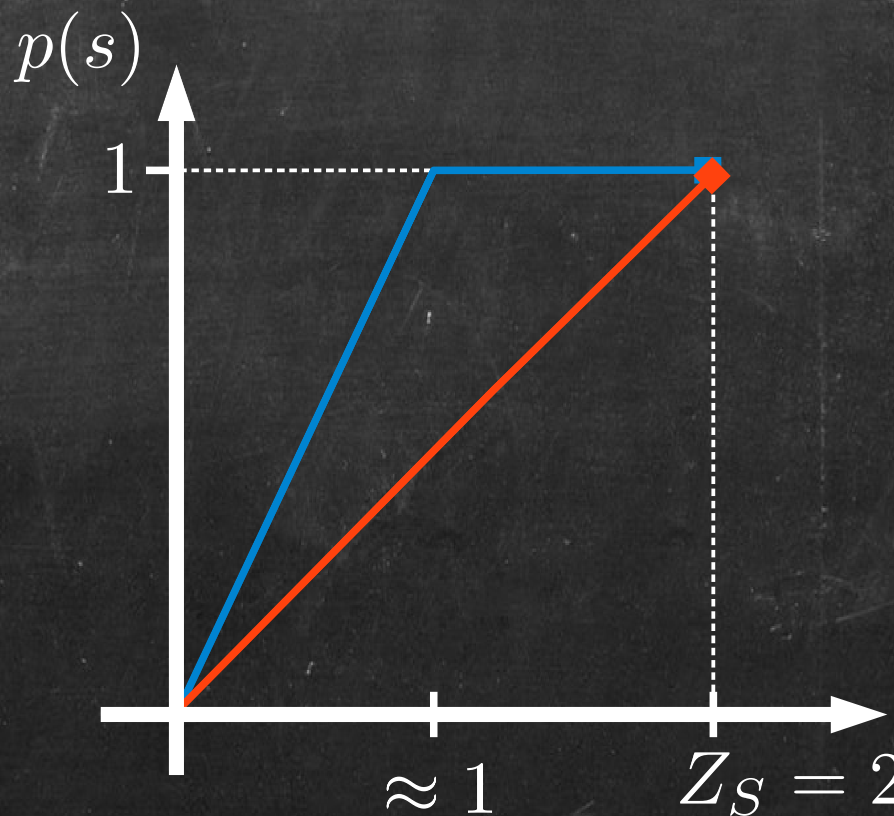


$$\Phi\left(\frac{1}{2}\mathbb{I}_S\right) = (1-\epsilon)|0\rangle\langle 0|_S + \epsilon|1\rangle\langle 1|_S$$

$$\frac{1}{2}\mathbb{I}_S \not\succ_T (1-\epsilon)|0\rangle\langle 0|_S + \epsilon|1\rangle\langle 1_S$$



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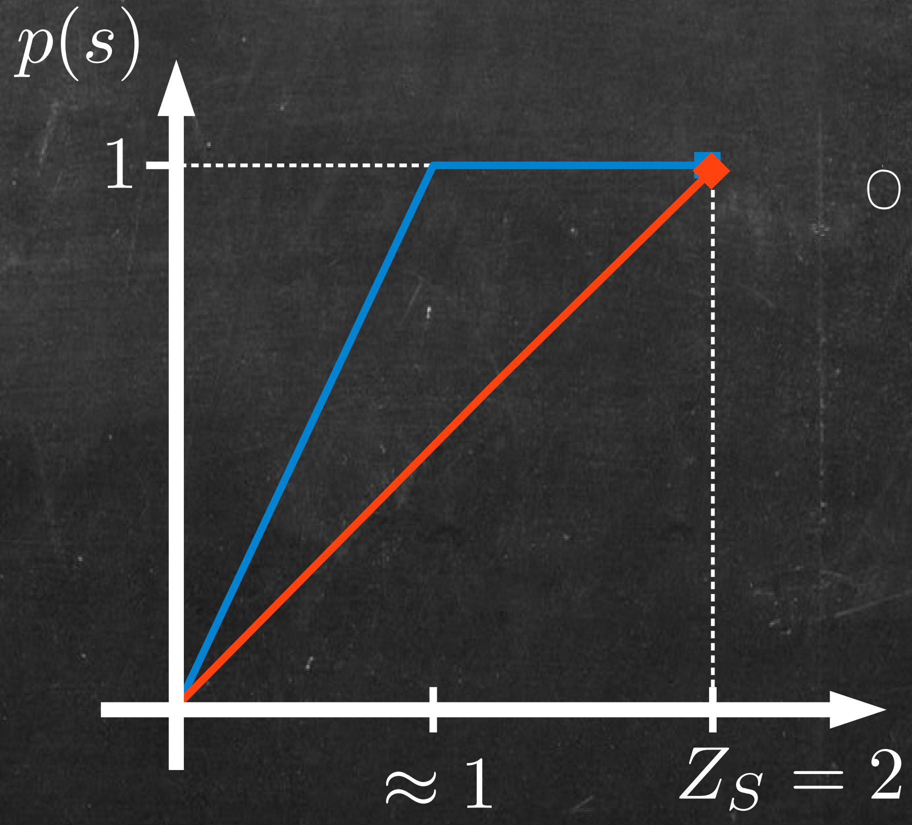

No thermal operation can do this

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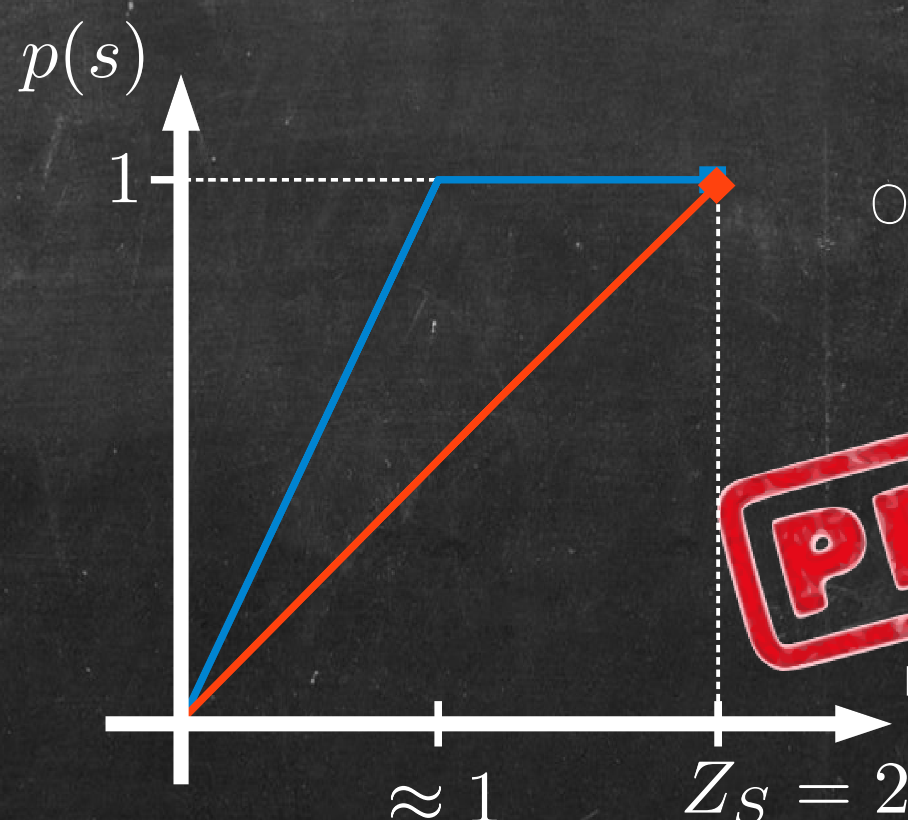


No thermal operation can do this

PROVEN

Horodecki, Oppenheim (2013)
Janzig et. al. (2000)
E. Ruch, A. Mead (1976)

To use this channel one has to perform work!

How?...

Add an explicit work-storage system!

Horodecki, Oppenheim (2013)

Option 1: Wit

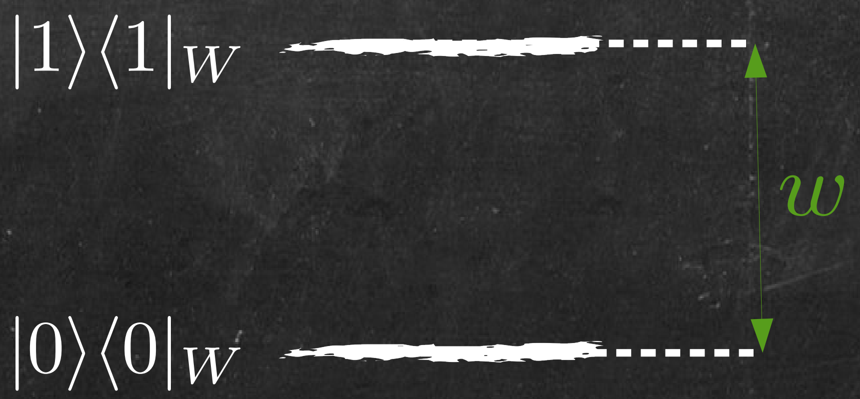
Two level ancilla with Hamiltonian

$$H_W = \mathrm{diag}(0, w)$$

Horodecki, Oppenheim (2013)

Option 1: Wit

Drawback 1: Must be tuned to the transition.



Horodecki, Oppenheim (2013)

Option 1: Wit

Drawback 1: Must be tuned to the transition.

Drawback 2: Must start in a specific state.

Linden, Popescu (2010)

Alhambra, Masanes et. al (2016)

Option 2: Infinite weight

Weight with continous energy spectrum and Hamiltonian:

$$H_W = \int_{\mathbb{R}} \mathrm{d}x \, x |x\rangle\langle x|_W$$

Linden, Popescu (2010)

Alhambra, Masanes et. al (2016)

Option 2: Infinite weight

Weight with continous energy spectrum and Hamiltonian:

$$H_W = \int_{\mathbb{R}} \mathrm{d}x \, x|x\rangle\langle x|_W$$

...and additional assumption:

Linden, Popescu (2010)

Alhambra, Masanes et. al (2016)

Option 2: Infinite weight

Translational invariance:

$$[U, \Delta_W] = 0$$

Linden, Popescu (2010)

Alhambra, Masanes et. al (2016)

Option 2: Infinite weight

Translational invariance:

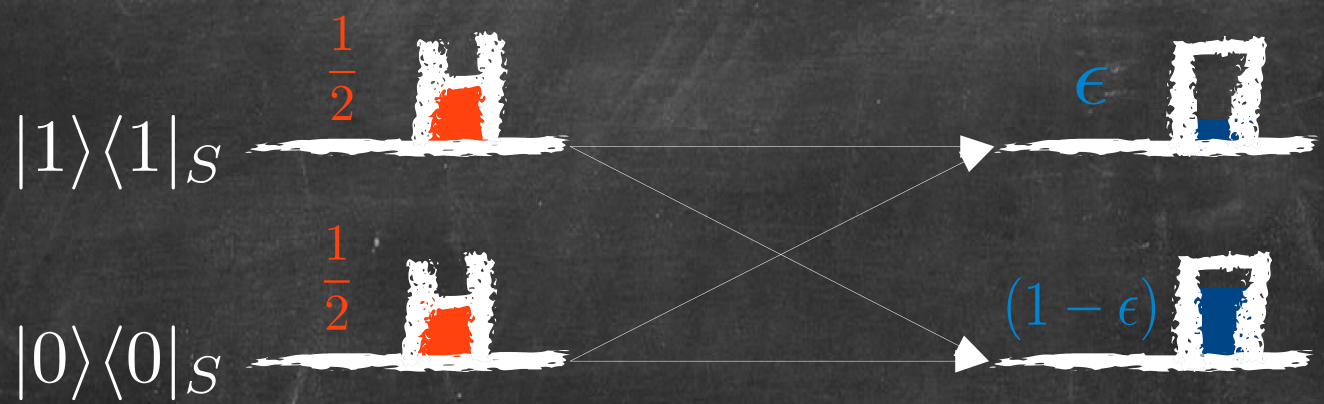
$$[U, \Delta_W] = 0$$

$$[H_W \,, \Delta_W] = i$$

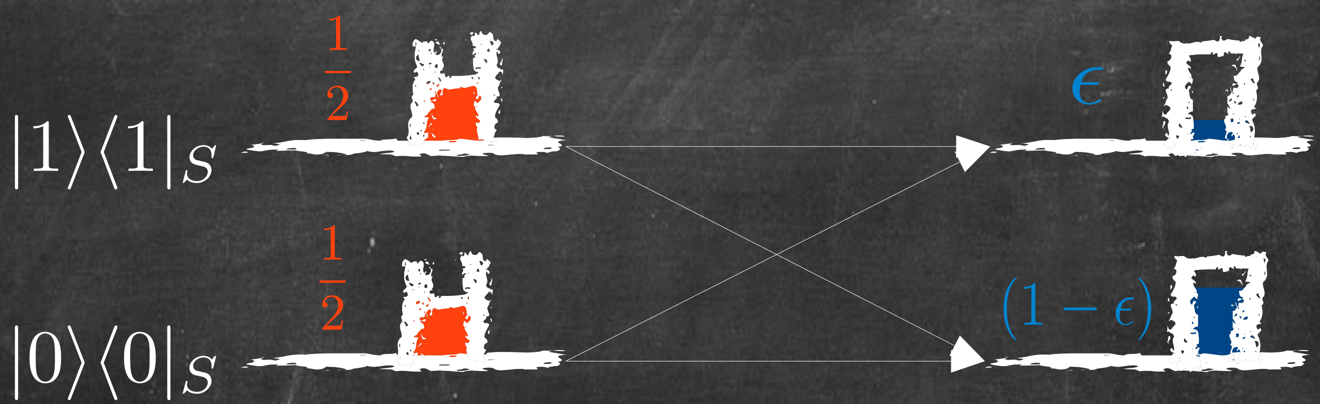
(shift operator)

Infinite weight

Infinite weight

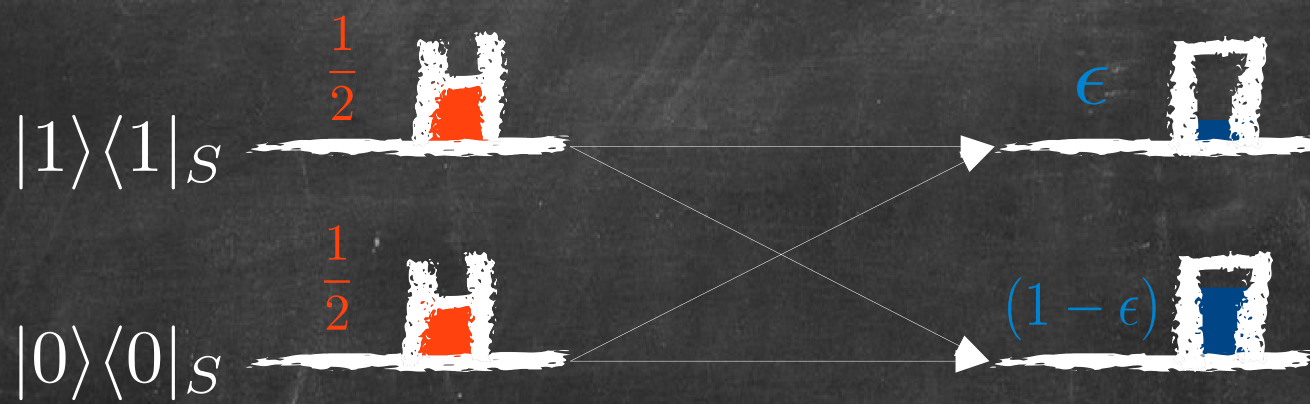


Infinite weight



$\vert s\rangle_S \to \vert s'\rangle_S$	$p(s'\vert s)$	*work*
$\vert 0\rangle_S \to \vert 0\rangle_S$	$1-\epsilon$	w_{00}
$\vert 1\rangle_S \to \vert 0\rangle_S$	$1-\epsilon$	w_{10}
$\vert 0\rangle_S \to \vert 1\rangle_S$	ϵ	w_{01}
$\vert 1\rangle_S \to \vert 1\rangle_S$	ϵ	w_{11}

Infinite weight



$\|s\rangle_S \to \|s'\rangle_S$	$p(s'\|s)$	$work$
$\|0\rangle_S \to \|0\rangle_S$	$1-\epsilon$	w_{00}
$\|1\rangle_S \to \|0\rangle_S$	$1-\epsilon$	w_{10}
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$\|1\rangle_S \to \|1\rangle_S$	ϵ	w_{11}

Deterministic work

$$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes |0\rangle\langle 0|_W\right) = (1-\epsilon)\, |0\rangle\langle 0|_S \otimes |w_{00}\rangle\langle w_{00}|_W$$
$$+ \; (1-\epsilon)\, |0\rangle\langle 0|_S \otimes |w_{10}\rangle\langle w_{10}|_W$$
$$+ \; \epsilon \; |1\rangle\langle 1|_S \otimes |w_{01}\rangle\langle w_{01}|_W$$
$$+ \; \epsilon \; |1\rangle\langle 1|_S \otimes |w_{11}\rangle\langle w_{11}|_W$$

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$\|0\rangle_S \to \|0\rangle_S$	$1-\epsilon$	w_{00}
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$$+\ (1-\epsilon)\,|0\rangle\langle 0|_S \otimes |w_{10}\rangle\langle w_{10}|_W$$
$$+\ \epsilon\ |1\rangle\langle 1|_S \otimes |w_{01}\rangle\langle w_{01}|_W$$
$$+\ \epsilon\ |1\rangle\langle 1|_S \otimes |w_{11}\rangle\langle w_{11}|_W$$

translational invariance

$$[U, \Delta_W] = 0$$

$\lvert s\rangle_S \to \lvert s'\rangle_S$	$p(s' \mid s)$	*work*
$\lvert 0\rangle_S \to \lvert 0\rangle_S$	$1-\epsilon$	w_{00}
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$\lvert 0\rangle_S \to \lvert 1\rangle_S$	ϵ	w_{01}
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Alhambra, Masanes et. al (2016)

Second-law equality:

$$\forall_{s'} \qquad \sum_{s,w} p(s', w|s) e^{\beta w} = 1$$

Alhambra, Masanes et. al (2016)

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Deterministic work: $p(s, s'|w) = \delta_{w, w_{ss'}}$

Alhambra, Masanes et. al (2016)

Second-law equality:

$$p(s', w|s) = p(w|s, s') \cdot p(s'|s)$$
$$= \delta_{w,w_{ss'}} p(s'|s)$$

$$\forall_{s'} \quad \sum_{s,w} p(s', w|s) e^{\beta w} = 1$$

Deterministic work: $p(s, s'|w) = \delta_{w,w_{ss'}}$

Second-law equality (deterministic work):

$$\forall_{s'} \qquad \sum_{s} p(s'|s) e^{\beta w_{ss'}} = 1$$

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$$\forall_{s'} \qquad \sum_s p(s'|s) e^{\beta w_{ss'}} = 1$$

$\lvert s\rangle_S \to \lvert s'\rangle_S$	$p(s'\lvert s)$	$work$
$\lvert 1\rangle_S \to \lvert 0\rangle_S$	$1-\epsilon$	w_{00}
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$$e^{-\beta w_{00}} + e^{-\beta w_{10}} = \frac{1}{1-\epsilon}$$

$$e^{-\beta w_{01}} + e^{-\beta w_{11}} = \frac{1}{\epsilon}$$

Conditional average work

$$\langle w(\epsilon) \rangle_{s'} = \sum_{work} p(work|s') \cdot work$$

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$$= \sum_{s,w} p(w|s,s') \cdot p(s) \cdot w$$

$$= \sum_{s} p(s) w_{ss'}$$

Conditional average work

$$\langle w(\epsilon)\rangle_{s'} = \sum_{work} p(work|s') \cdot work$$

$$e^{-\beta w_{00}} + e^{-\beta w_{10}} = \frac{1}{1-\epsilon}$$

$$e^{-\beta w_{01}} + e^{-\beta w_{11}} = \frac{1}{\epsilon}$$

$$\langle w(\epsilon)\rangle_0 = \frac{1}{2}(w_{00} + w_{01})$$

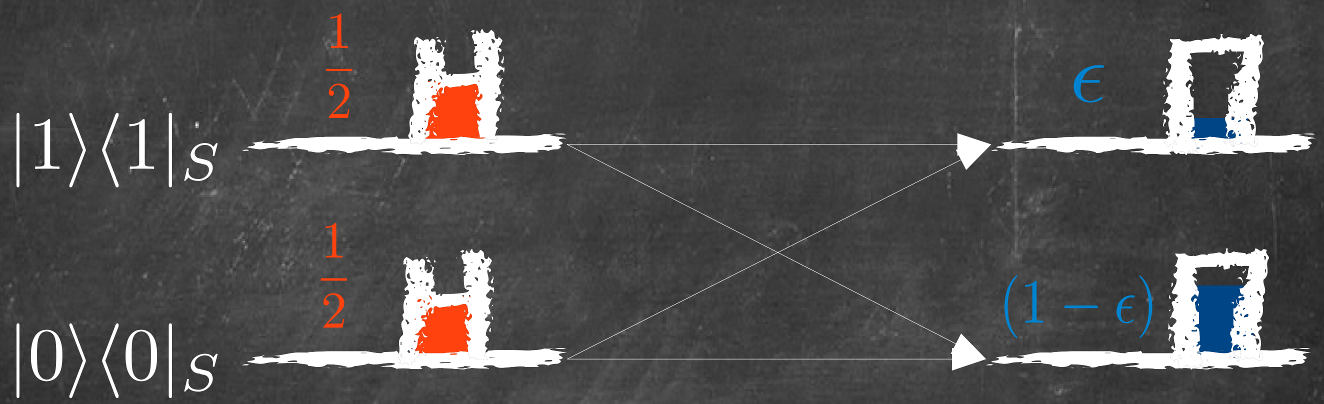
$$= \frac{1}{2}\left[w_{00} + \log\left(\frac{1}{1-\epsilon} - e^{\beta w_{00}}\right)\right]$$

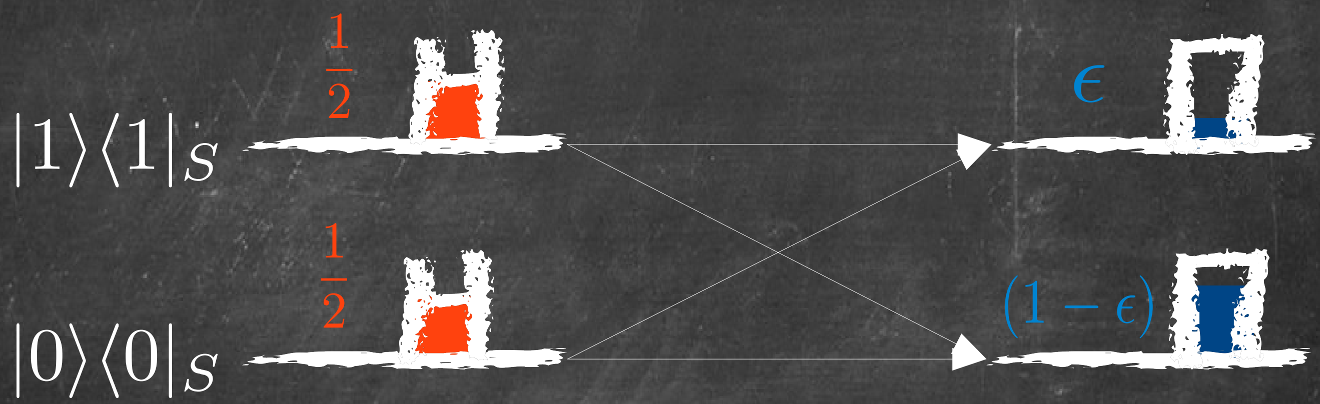
$$\langle w(\epsilon)\rangle_1 = \frac{1}{2}(w_{01} + w_{11})$$

$$= \frac{1}{2}\left[w_{01} + \log\left(\frac{1}{\epsilon} - e^{\beta w_{01}}\right)\right]$$

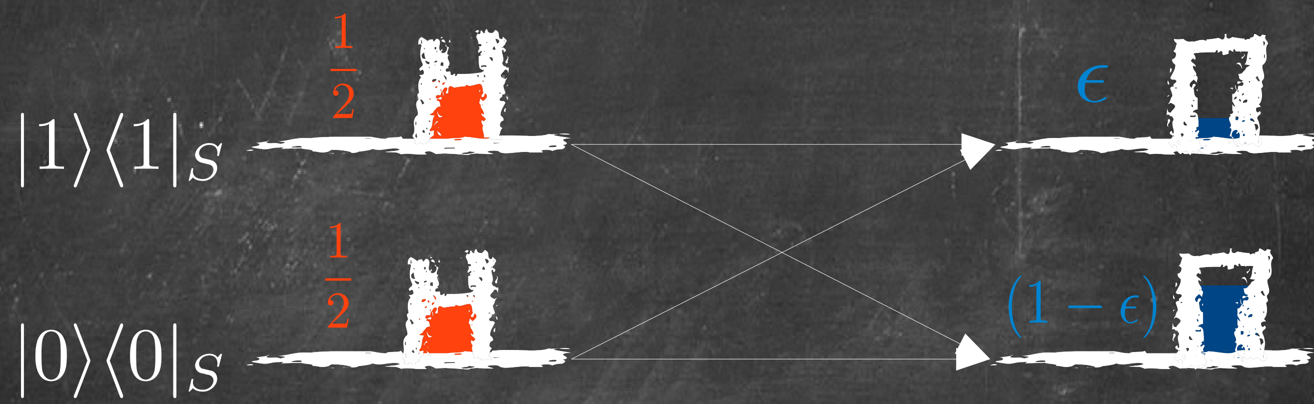
What happens in the limit of perfect erasure?

$$\epsilon \to 0$$


$|1\rangle\langle 1|_S$
$|0\rangle\langle 0|_S$
$\frac{1}{2}$
$\frac{1}{2}$
ϵ
$(1-\epsilon)$



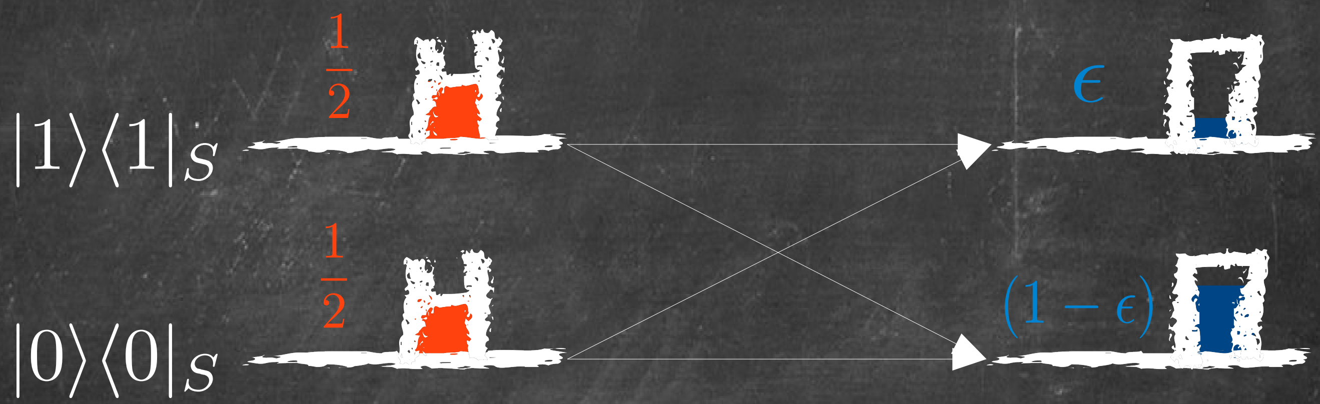
$$\begin{aligned}\langle w(\epsilon)\rangle_0 &= \frac{1}{2}\left(w_{00} + w_{10}\right) \\ &= \frac{1}{2}\left[w_{00} + \log\left(\frac{1}{1-\epsilon} - e^{\beta w_{00}}\right)\right]\end{aligned}$$



$$\langle w(\epsilon)\rangle_0 = \frac{1}{2}\left(w_{00} + w_{10}\right)$$
$$= \frac{1}{2}\left[w_{00} + \log\left(\frac{1}{1-\epsilon} - e^{\beta w_{00}}\right)\right]$$

$\epsilon \to 0$

$$\frac{1}{2}\left[w_{00} + \log\left(1 - e^{\beta w_{00}}\right)\right]$$

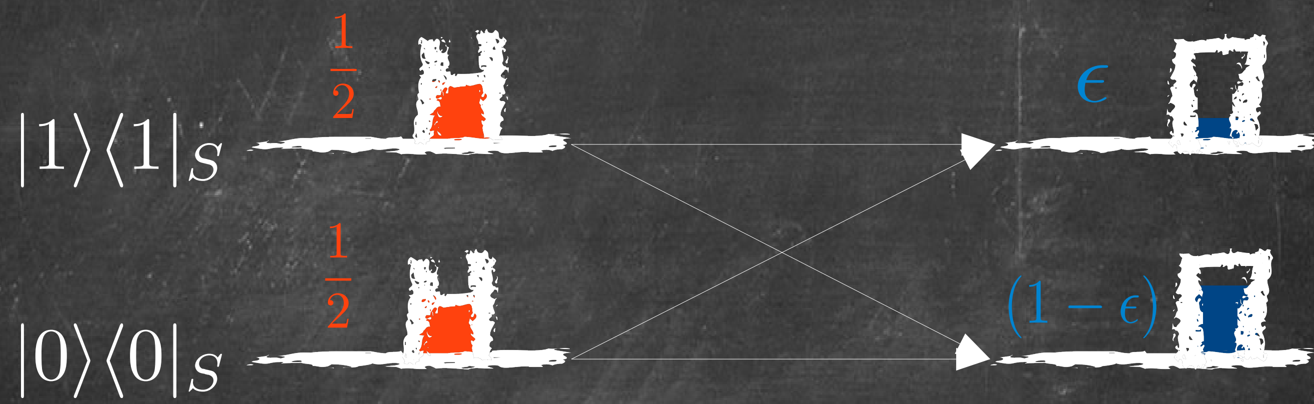


$$
\begin{aligned}
\langle w(\epsilon)\rangle_0 &= \frac{1}{2}\left(w_{00} + w_{10}\right) \\
&= \frac{1}{2}\left[w_{00} + \log\left(\frac{1}{1-\epsilon} - e^{\beta w_{00}}\right)\right]
\end{aligned}
$$

$\epsilon \to 0$

$$
\frac{1}{2}\left[w_{00} + \log\left(1 - e^{\beta w_{00}}\right)\right]
$$

LIMITED



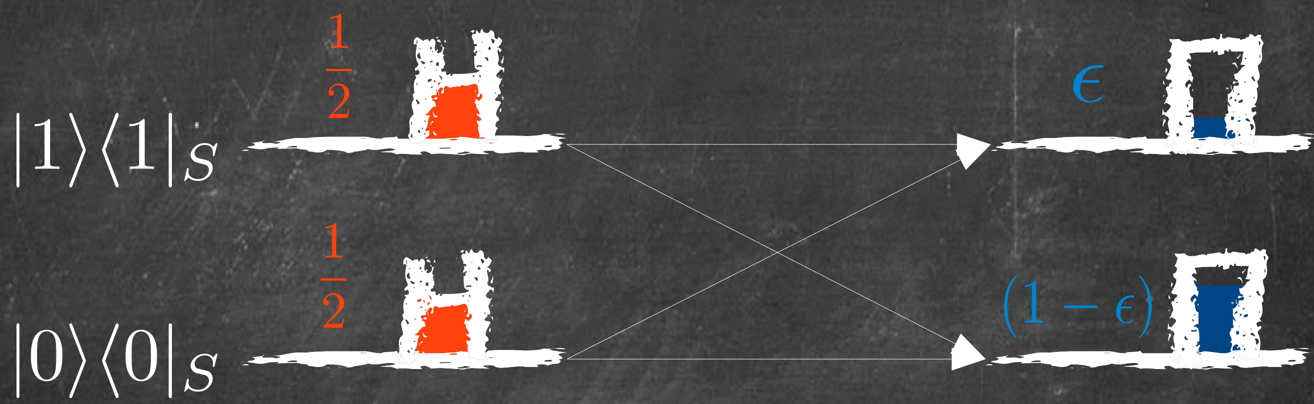
$$\langle w(\epsilon)\rangle_0 = \frac{1}{2}\left(w_{00} + w_{10}\right) = \frac{1}{2}\left[w_{00} + \log\left(\frac{1}{1-\epsilon} - e^{\beta w_{00}}\right)\right]$$

$$\langle w(\epsilon)\rangle_1 = \frac{1}{2}\left(w_{01} + w_{11}\right) = \frac{1}{2}\left[w_{01} + \log\left(\frac{1}{\epsilon} - e^{\beta w_{01}}\right)\right]$$

$\epsilon \to 0$

$$\frac{1}{2}\left[w_{00} + \log\left(1 - e^{\beta w_{00}}\right)\right]$$

LIMITED



$$\langle w(\epsilon)\rangle_0 = \frac{1}{2}\left(w_{00} + w_{10}\right) = \frac{1}{2}\left[w_{00} + \log\left(\frac{1}{1-\epsilon} - e^{\beta w_{00}}\right)\right]$$

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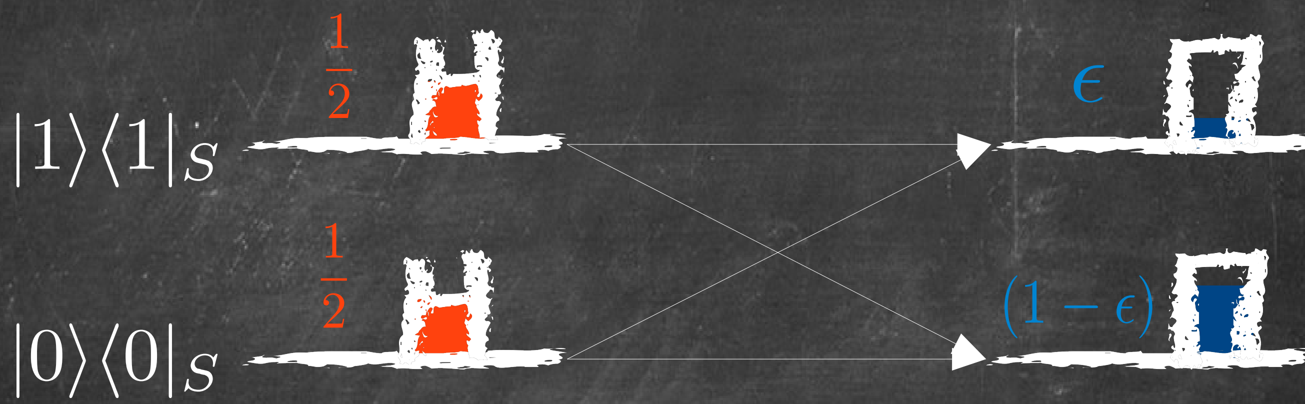
$\epsilon \to 0$

$$\frac{1}{2}\left[w_{00} + \log\left(1 - e^{\beta w_{00}}\right)\right]$$

LIMITED

$\epsilon \to 0$

$$\infty!$$



$$\langle w(\epsilon)\rangle_0 = \frac{1}{2}\left(w_{00} + w_{10}\right) = \frac{1}{2}\left[w_{00} + \log\left(\frac{1}{1-\epsilon} - e^{\beta w_{00}}\right)\right]$$

$\epsilon \to 0$

$$\frac{1}{2}\left[w_{00} + \log\left(1 - e^{\beta w_{00}}\right)\right]$$

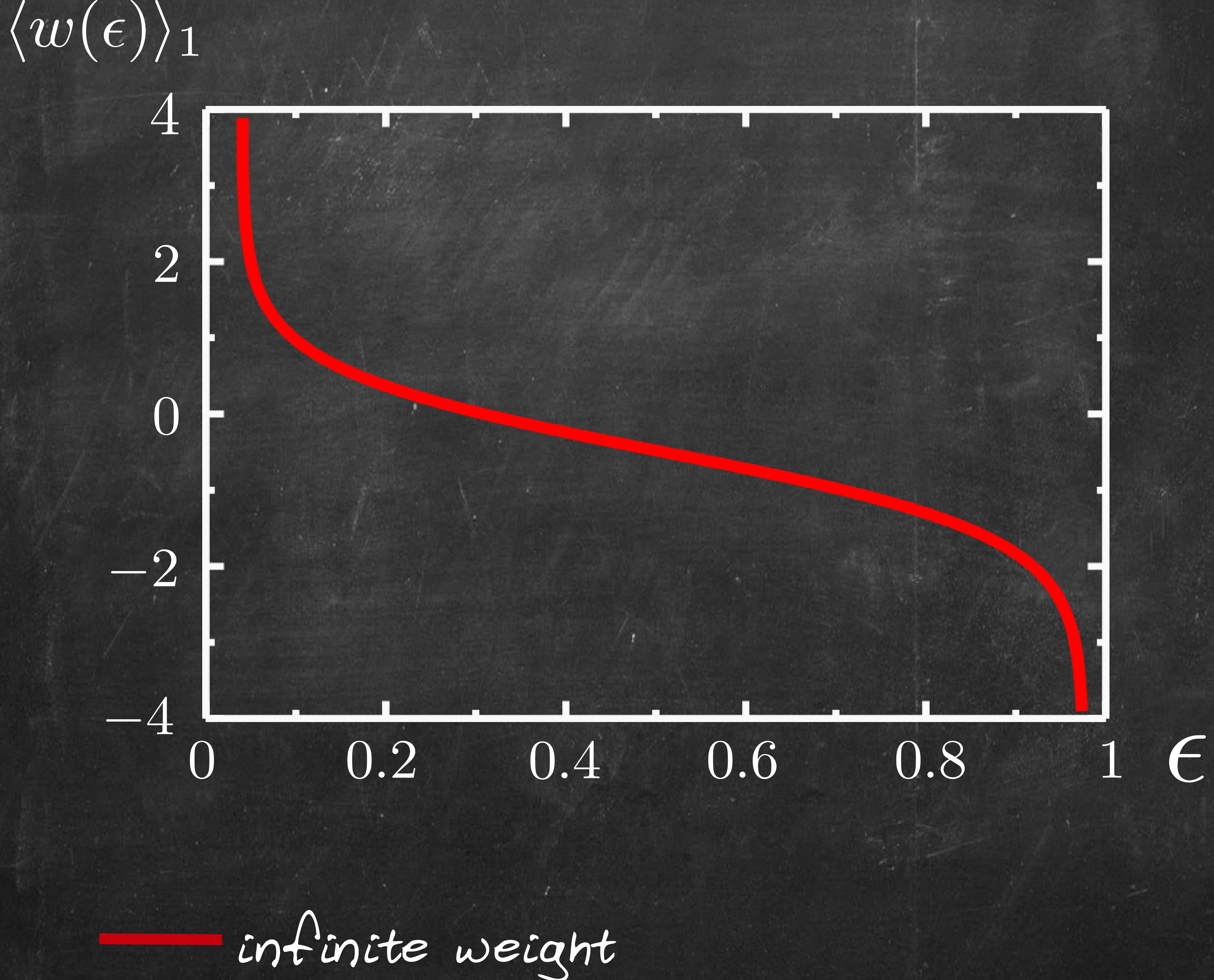
LIMITED

$$\langle w(\epsilon)\rangle_1 = \frac{1}{2}\left(w_{01} + w_{11}\right) = \frac{1}{2}\left[w_{01} + \log\left(\frac{1}{\epsilon} - e^{\beta w_{01}}\right)\right]$$

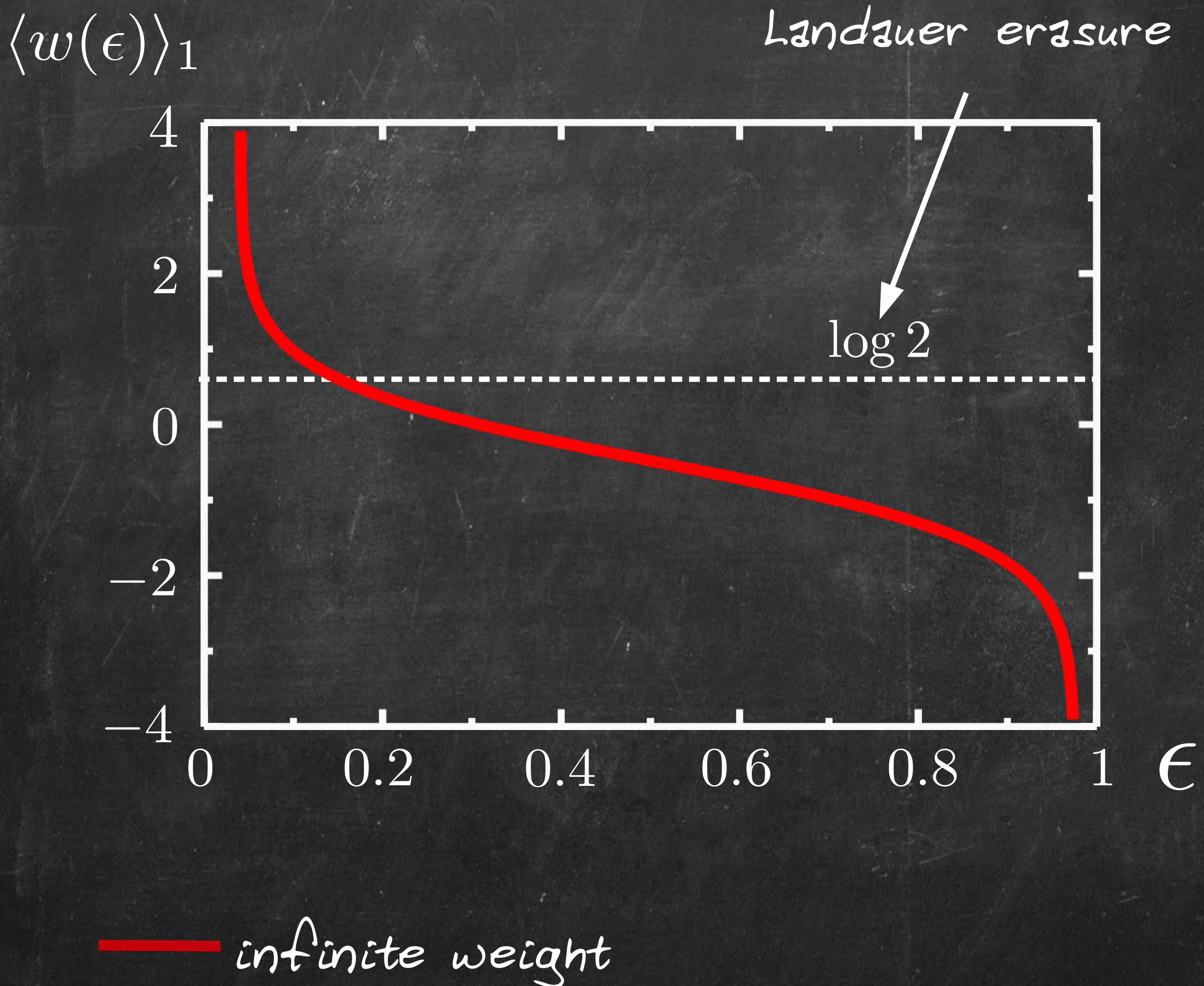
$\epsilon \to 0$

∞!

UNLIMITED



$\langle w(\epsilon) \rangle_1$
4
2
0
−2
−4
0
0.2
0.4
0.6
0.8
1
ϵ
infinite weight



$\langle w(\epsilon)\rangle_1$
Landauer erasure
log 2
4
2
0
−2
−4
0
0.2
0.4
0.6
0.8
1
ϵ
infinite weight

Alternative: Wit

Wit

$$\Phi\left(\frac{1}{2}\mathbb{I}_S\right) = \underbrace{(1-\epsilon)|0\rangle\langle 0|_S + \epsilon|1\rangle\langle 1|_S}_{:= \rho_S(\epsilon)}$$

Wit

$$\Phi\left(\frac{1}{2}\mathbb{I}_S\right) = \rho_S(\epsilon)$$

Wit

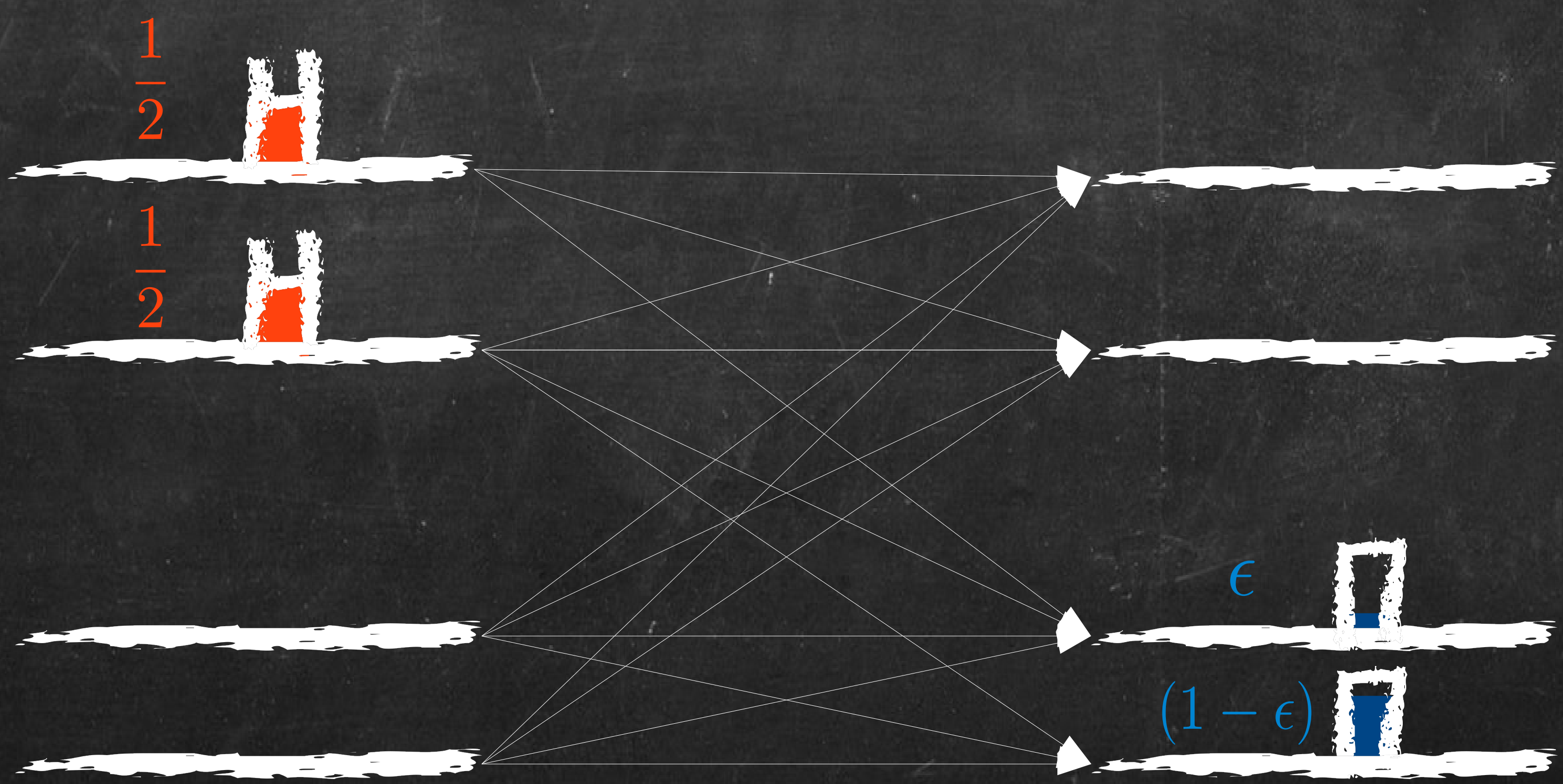
$$\Phi\left(\frac{1}{2}\mathbb{I}_S\right) = \rho_S(\epsilon)$$

Wit

$$\Phi\left(\frac{1}{2}\mathbb{I}_S\right) \neq \rho_S(\epsilon)$$

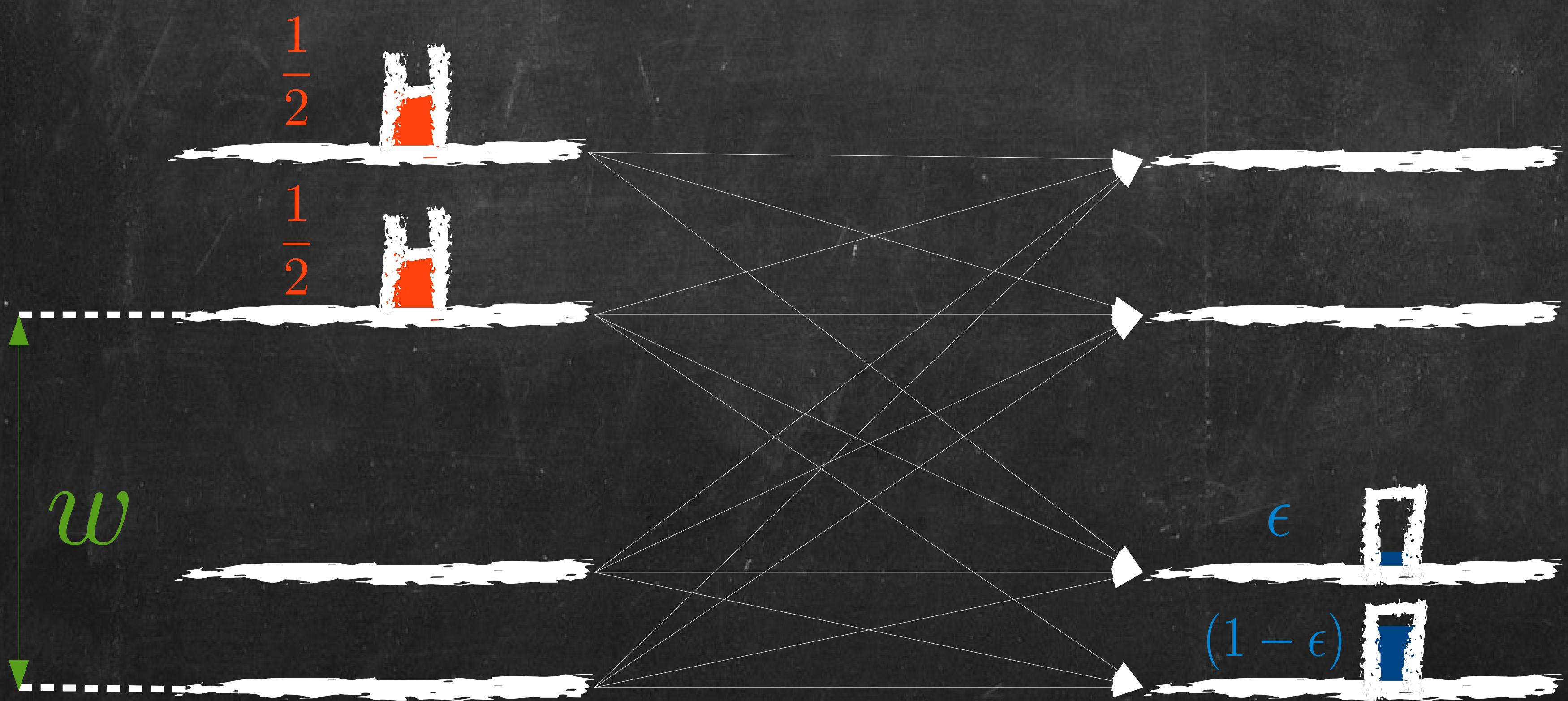
Wit

$$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes |1\rangle\langle 1|_W\right) = \rho_S(\epsilon) \otimes |0\rangle\langle 0|_W$$

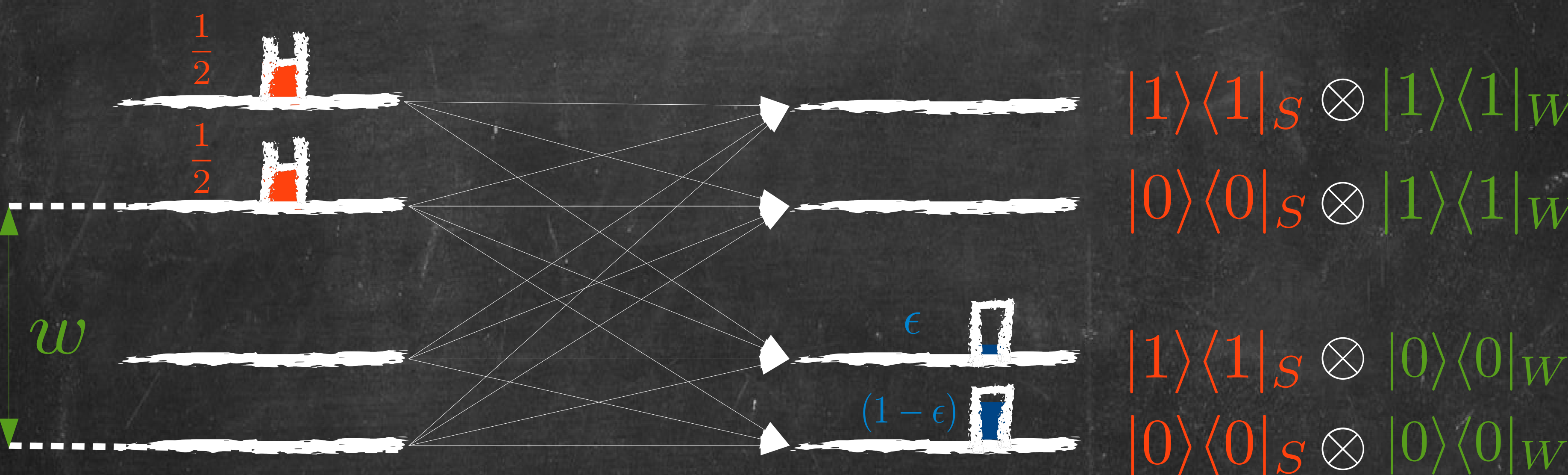


Wit

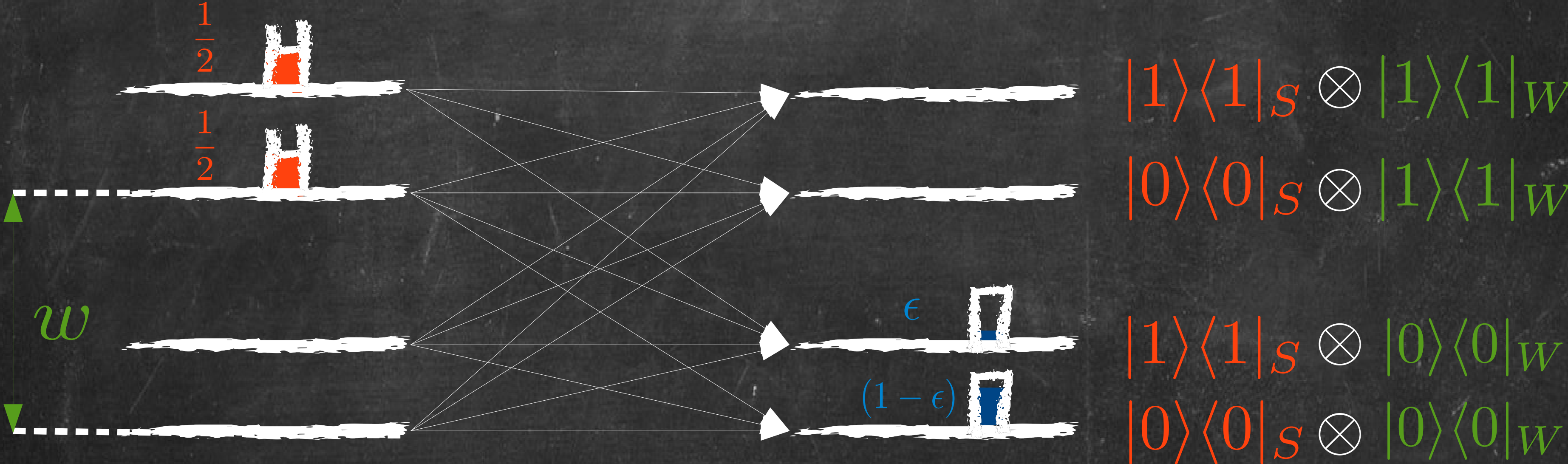
$$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes |1\rangle\langle 1|_W\right) = \rho_S(\epsilon) \otimes |0\rangle\langle 0|_W$$



$$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes |1\rangle\langle 1|_W\right) \overset{?}{=} \rho_S(\epsilon) \otimes |0\rangle\langle 0|_W$$



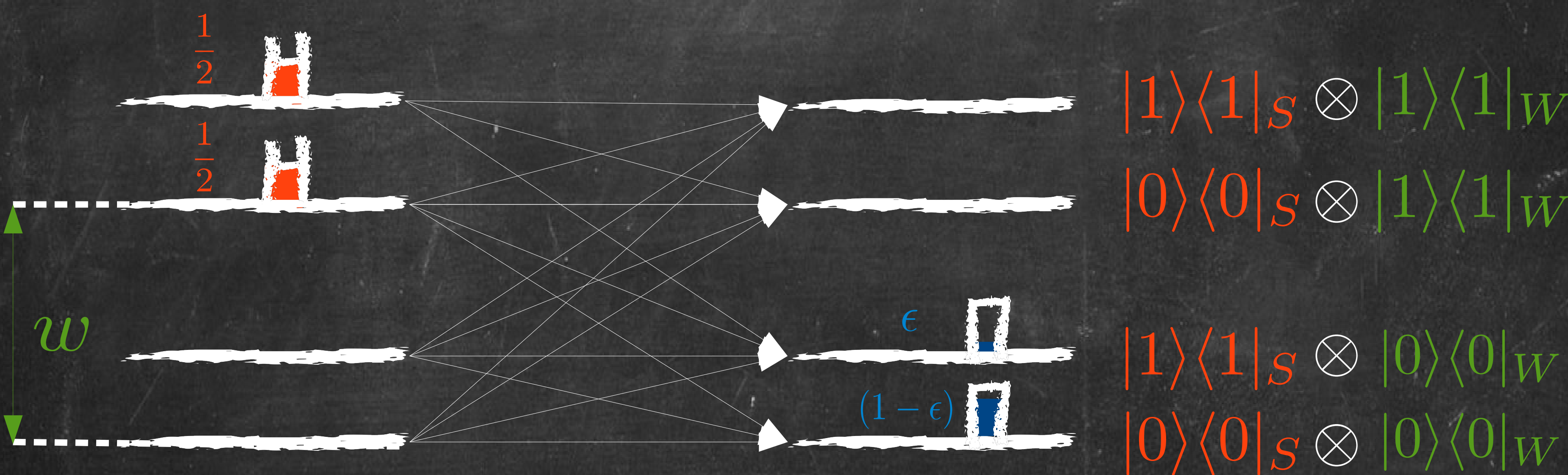
$$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes |1\rangle\langle 1|_W\right) = \rho_S(\epsilon) \otimes |0\rangle\langle 0|_W$$



Thermal operation exists

$$\iff w = F_{max}\left(\rho(\epsilon)_S\right) - F_{max}\left(\frac{1}{2}\mathbb{I}_S\right)$$

$$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes |1\rangle\langle 1|_W\right) = \rho_S(\epsilon) \otimes |0\rangle\langle 0|_W$$



Horodecki, Oppenheim (2013)

PROVEN

Thermal operation exists

$$\iff w = F_{max}\left(\rho(\epsilon)_S\right) - F_{max}\left(\frac{1}{2}\mathbb{I}_S\right)$$

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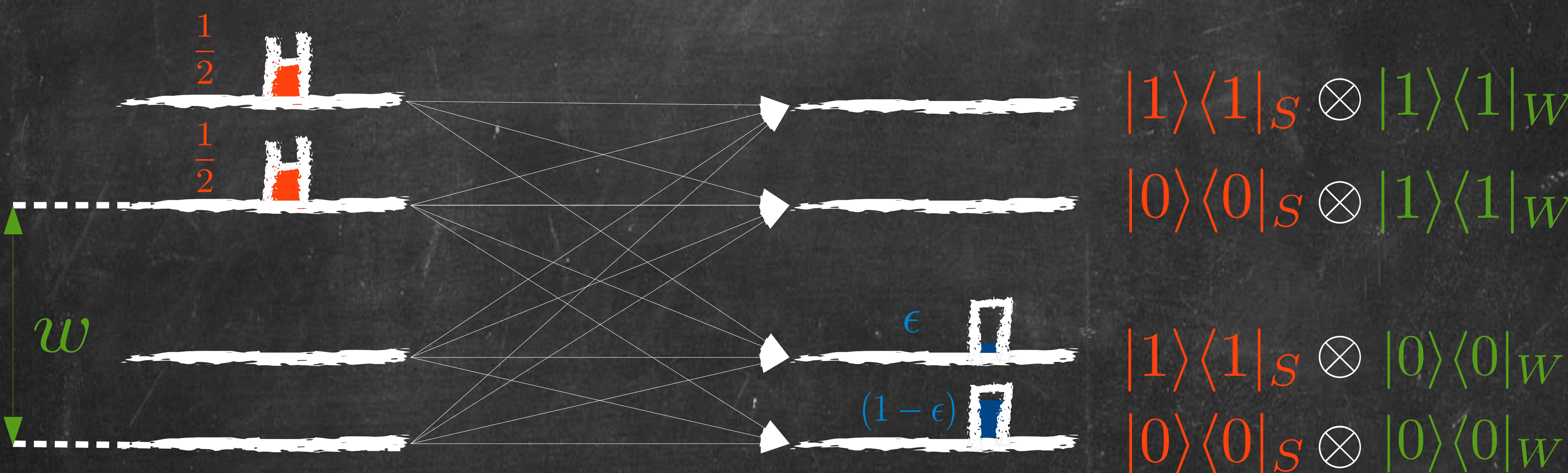
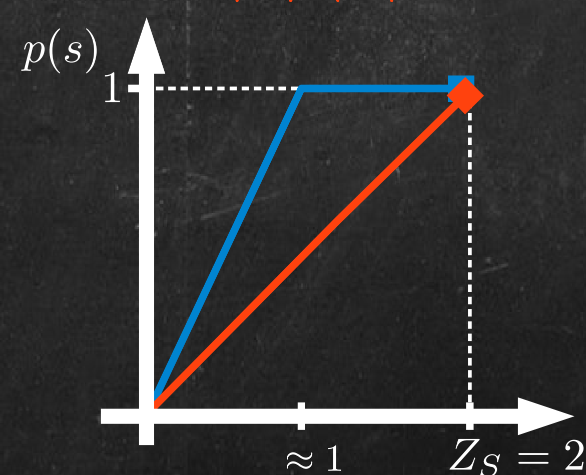
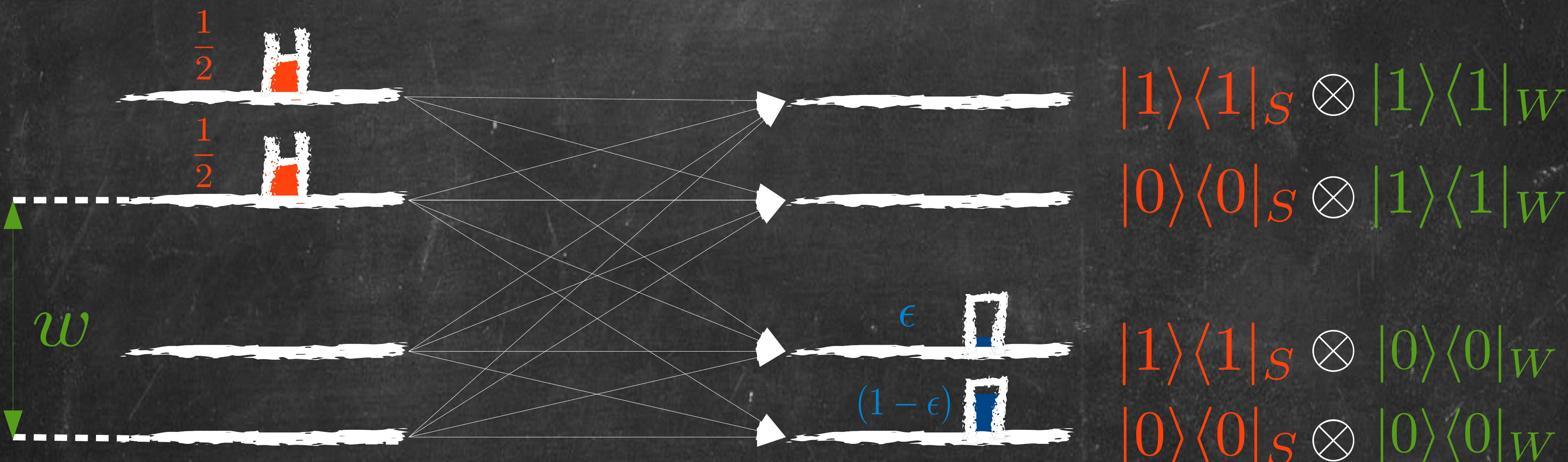


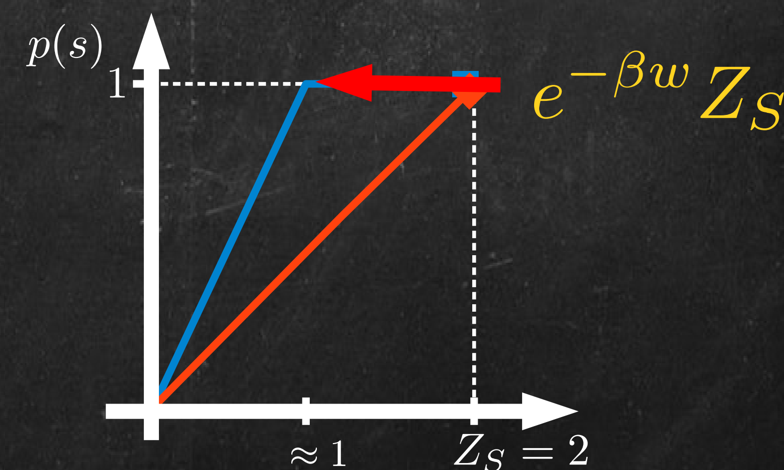


Thermal operation exists

$$\Longleftrightarrow \quad w = kT \log 2$$

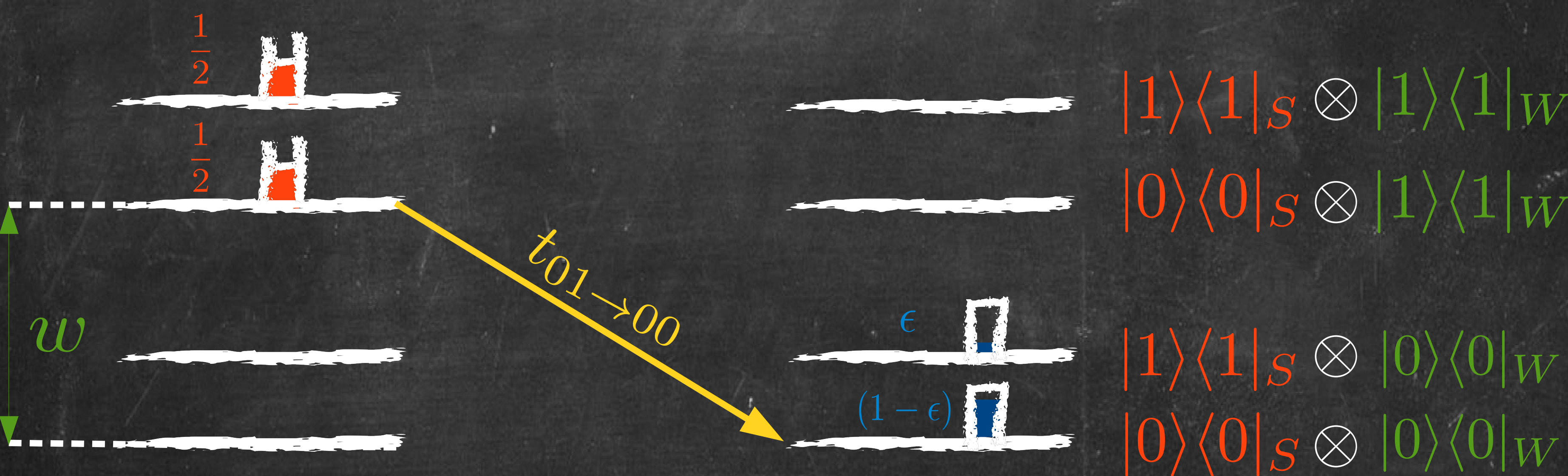
$$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes |1\rangle\langle 1|_W\right) = \rho_S(\epsilon) \otimes |0\rangle\langle 0|_W$$





Thermal operation exists

$$\iff w = kT\log 2$$

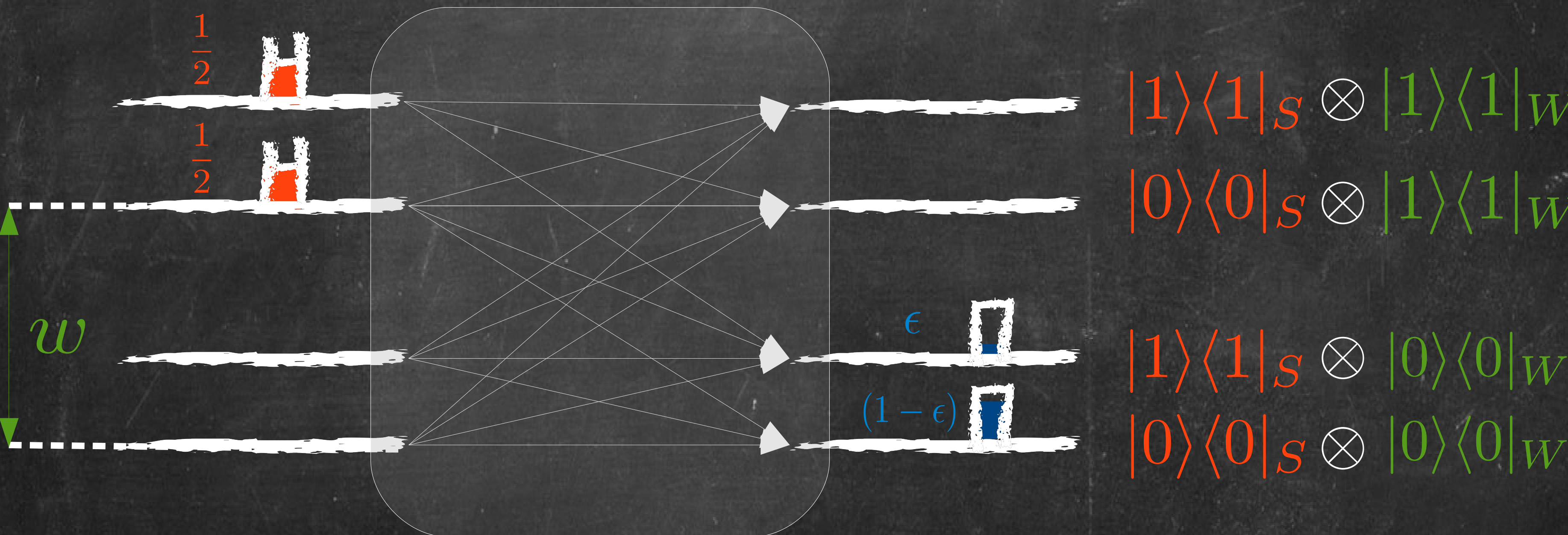


$$A_{sk\to s'k'} := \sqrt{t_{sk\to s'k'}}\; |s'\rangle\langle s|_S \otimes |k'\rangle\langle k|_W$$

$$0 \leq t_{sk \to s'k'} \leq 1$$

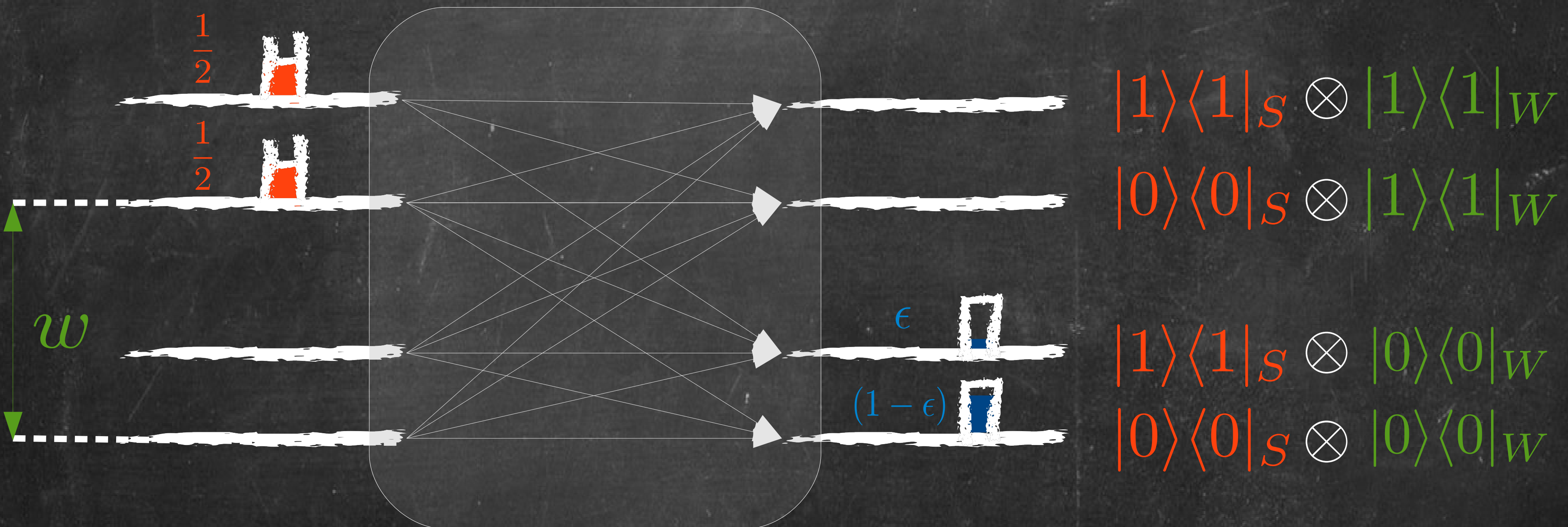
$$\forall_{s,k} \sum_{s',k'} t_{sk \to s'k'} = 1$$

$$A_{sk \to s'k'} := \sqrt{t_{sk \to s'k'}} \; |s'\rangle\langle s|_S \otimes |k'\rangle\langle k|_W$$



$$\Phi\left(\cdot\right) = \sum_{s,s'} \sum_{k,k'} A_{sk \to s'k'} \left(\cdot\right) A^{\dagger}_{sk \to s'k'}$$

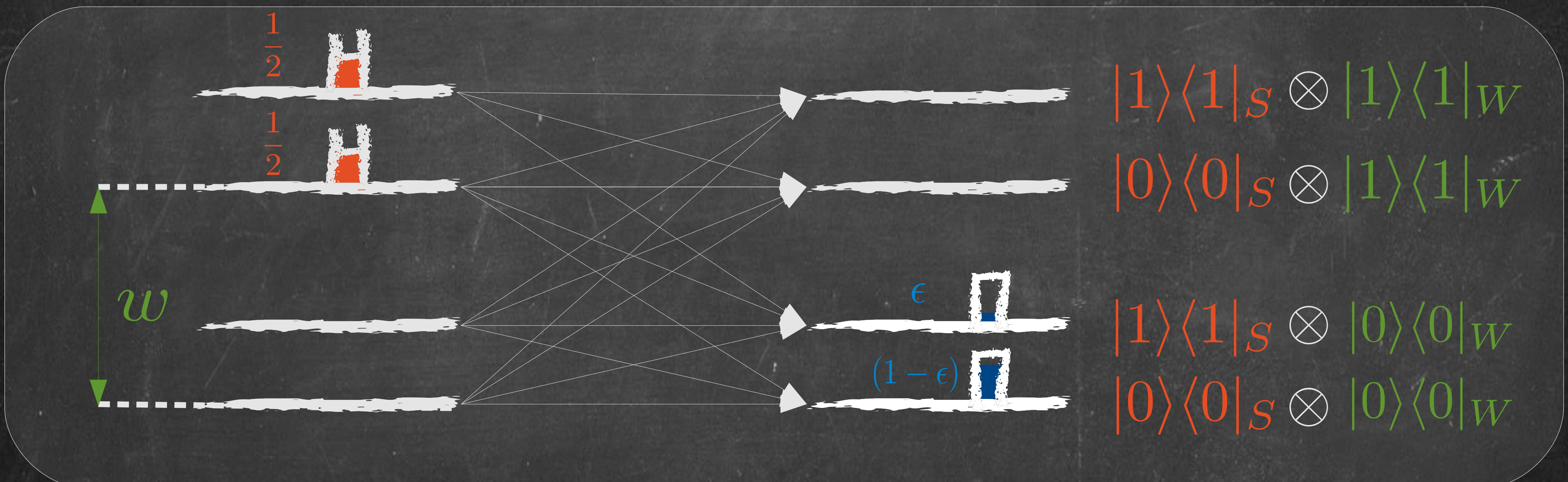
$$\mathcal{R}_{kk'}(\rho_S) := \mathrm{tr}_W \left(\sum_{s,s'} A_{sk\to s'k'} \left(\rho_S \otimes |k\rangle\langle k|_W\right) A^\dagger_{sk\to s'k'} \right)$$



$$\Phi\left(\rho_S \otimes \rho_W\right) = \sum_{s,s'} \sum_{k,k'} A_{sk\to s'k'} \left(\rho_S \otimes \rho_W\right) A^\dagger_{sk\to s'k'}$$

$$= \sum_{k,k'} p(k) \mathcal{R}_{kk'}(\rho_S) \otimes |k'\rangle\langle k'|_W$$

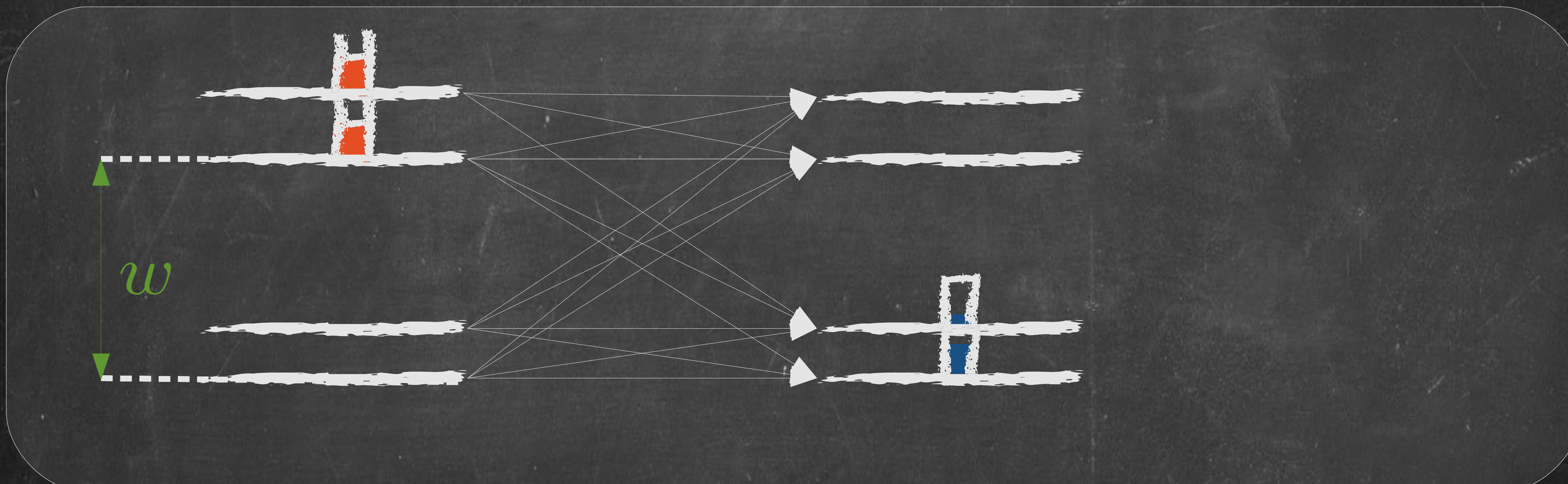
$$\mathcal{R}_{kk'}(\rho_S) := \mathrm{tr}_W\left(\sum_{s,s'} A_{sk\to s'k'}\left(\rho_S \otimes |k\rangle\langle k|_W\right) A^\dagger_{sk\to s'k'}\right)$$



$$\Phi\left(\rho_S \otimes \rho_W\right) = \sum_{s,s'}\sum_{k,k'} A_{sk\to s'k'}\left(\rho_S \otimes \rho_W\right) A^\dagger_{sk\to s'k'}$$

$$= \sum_{k,k'} p(k)\mathcal{R}_{kk'}(\rho_S) \otimes |k'\rangle\langle k'|_W$$

$$\mathcal{R}_{kk'}(\rho_S) := \mathrm{tr}_W \left(\sum_{s,s'} A_{sk\to s'k'} \left(\rho_S \otimes |k\rangle\langle k|_W\right) A^\dagger_{sk\to s'k'} \right)$$



$$\Phi\left(\rho_S \otimes \rho_W\right) = \sum_{s,s'} \sum_{k,k'} A_{sk\to s'k'} \left(\rho_S \otimes \rho_W\right) A^\dagger_{sk\to s'k'}$$

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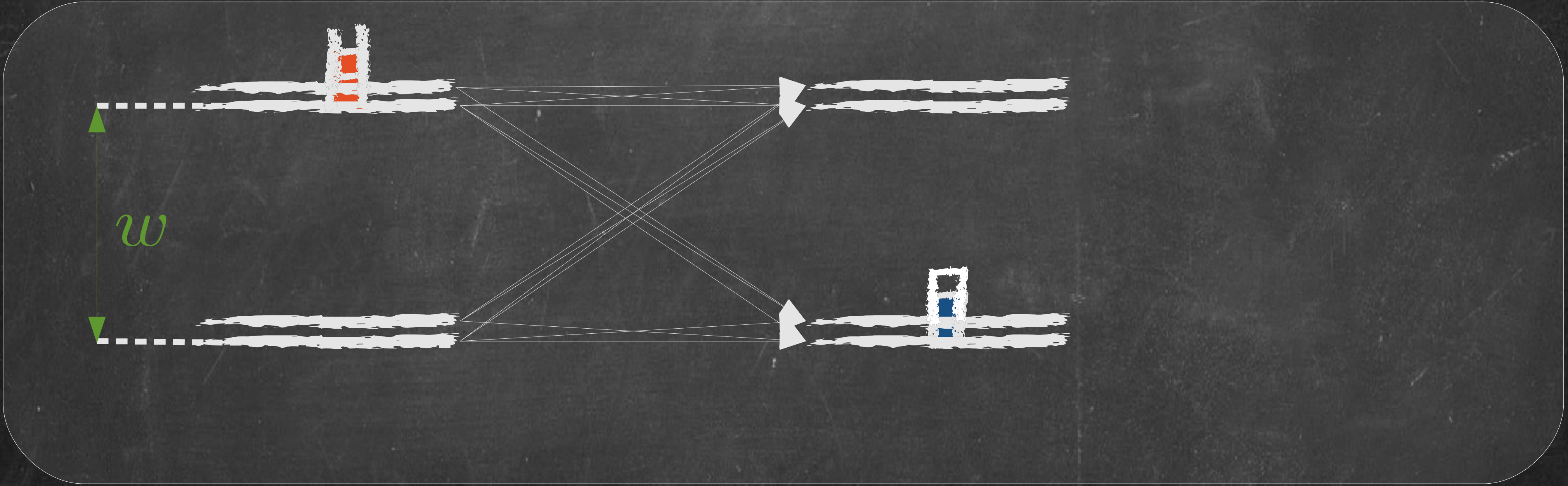

$$\Phi\left(\rho_S \otimes \rho_W\right) = \sum_{s,s'} \sum_{k,k'} A_{sk \to s'k'} \left(\rho_S \otimes \rho_W\right) A^{\dagger}_{sk \to s'k'}$$

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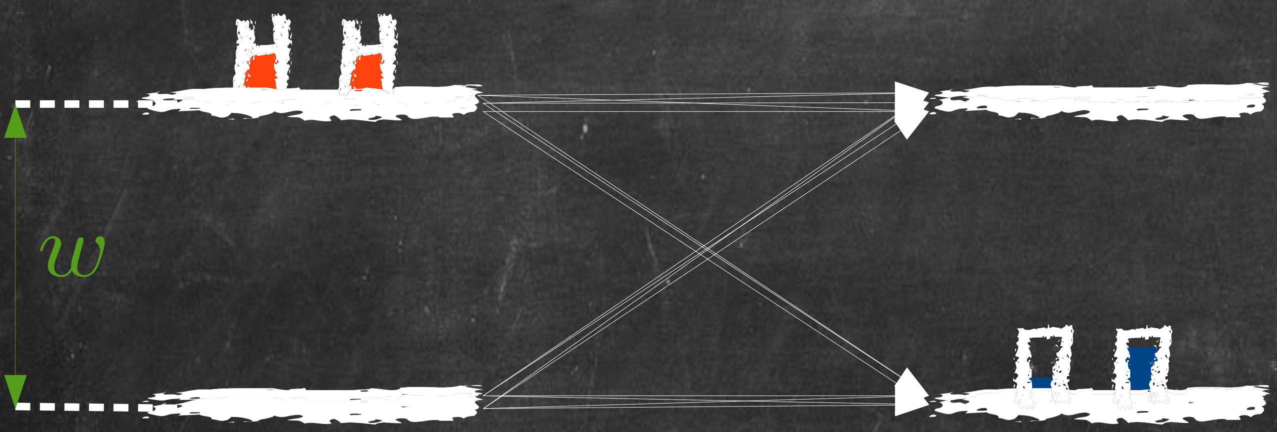
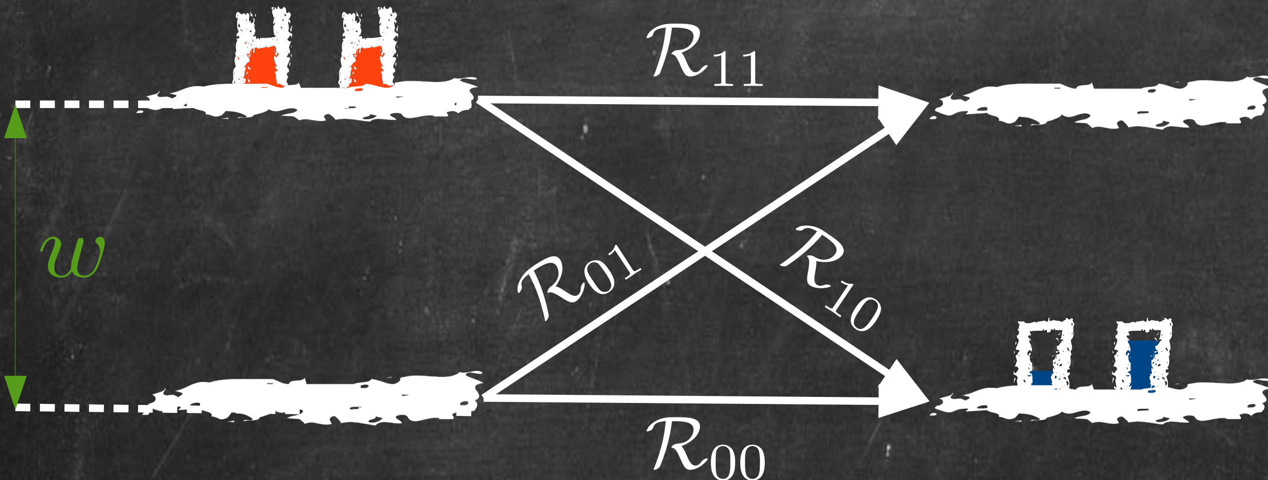


$$\Phi\left(\rho_S \otimes \rho_W\right) = \sum_{s,s'} \sum_{k,k'} A_{sk\to s'k'} \left(\rho_S \otimes \rho_W\right) A^\dagger_{sk\to s'k'}$$

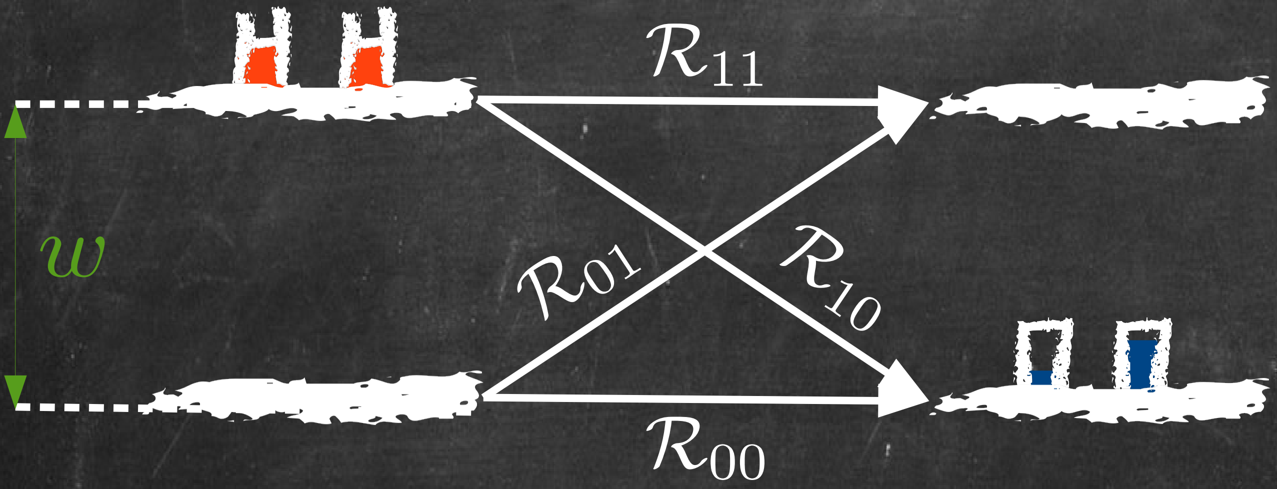
$$= \sum_{k,k'} p(k) \mathcal{R}_{kk'}(\rho_S) \otimes |k'\rangle\langle k'|_W$$

$$\mathcal{R}_{kk'}(\rho_S) := \mathrm{tr}_W \left(\sum_{s,s'} A_{sk\to s'k'} \left(\rho_S \otimes |k\rangle\langle k|_W\right) A^\dagger_{sk\to s'k'} \right)$$

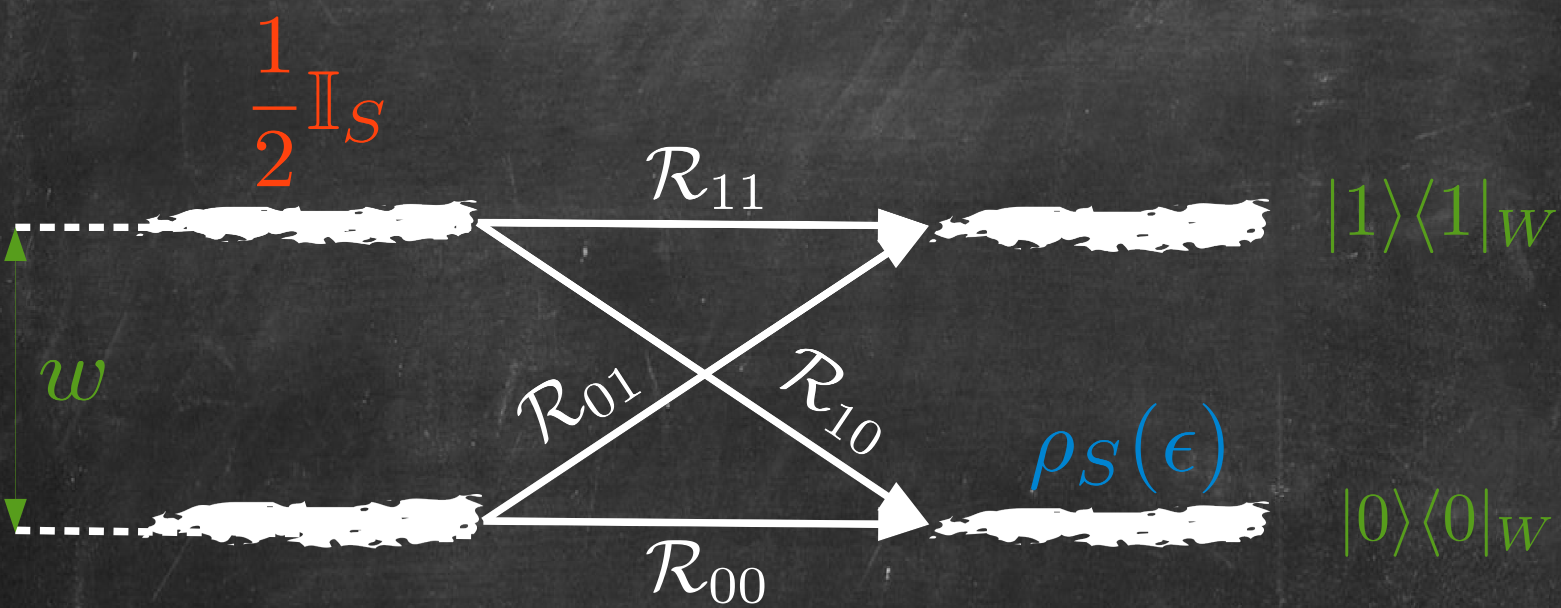


$$\Phi\left(\rho_S \otimes \rho_W\right) = \sum_{s,s'} \sum_{k,k'} A_{sk\to s'k'} \left(\rho_S \otimes \rho_W\right) A^\dagger_{sk\to s'k'}$$

$$= \sum_{k,k'} p(k) \mathcal{R}_{kk'}(\rho_S) \otimes |k'\rangle\langle k'|_W$$

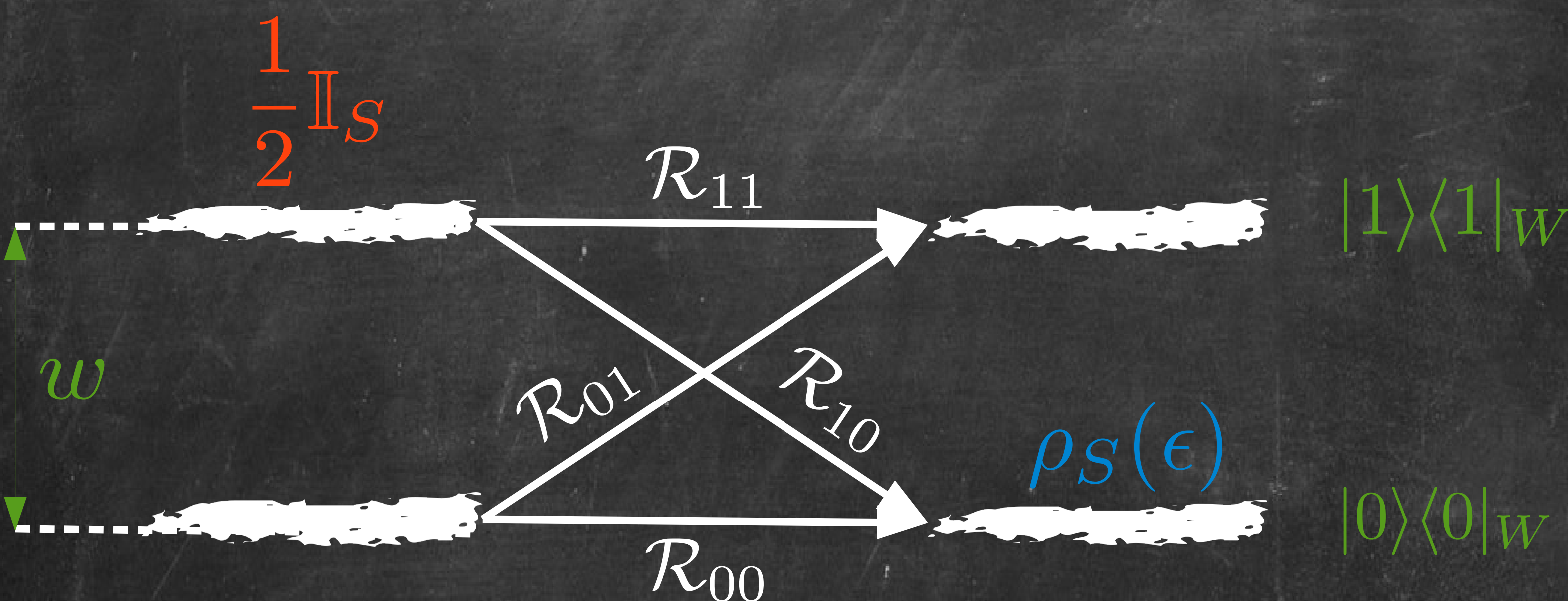

$\mathcal{R}_{11}$
w
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$

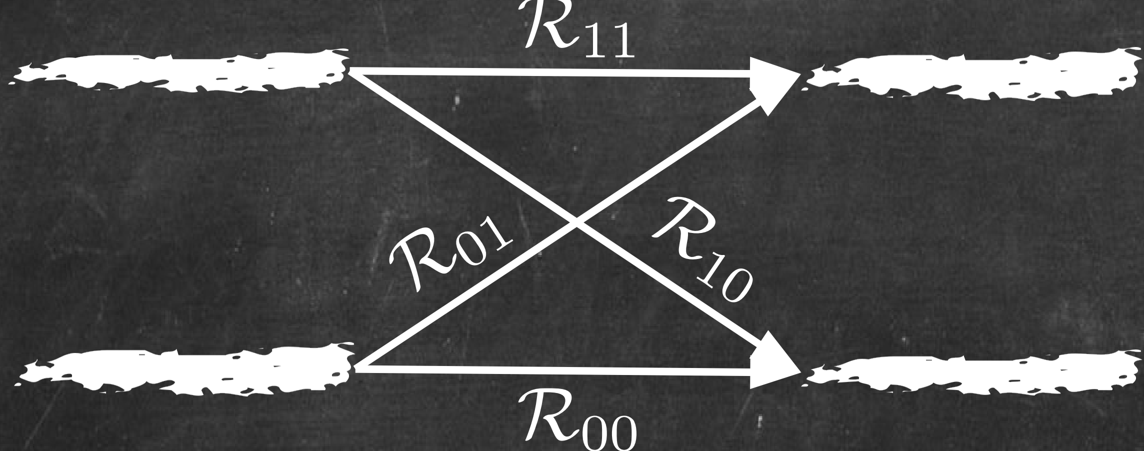
$$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes |1\rangle\langle 1|_W\right) = \rho_S(\epsilon) \otimes |0\rangle\langle 0|_W$$

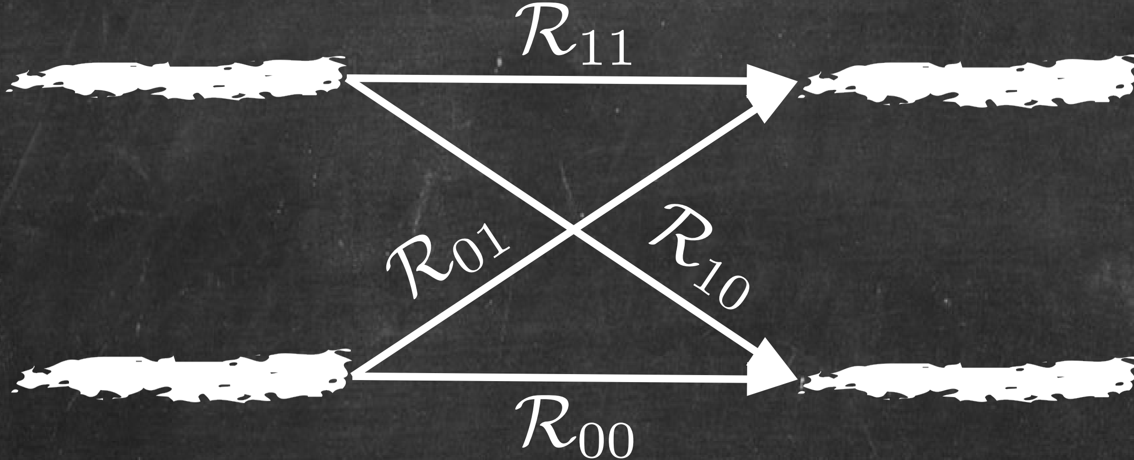


$$\Phi\left(\rho_S \otimes \rho_W\right) = \sum_{k,k'} p(k)\mathcal{R}_{kk'}(\rho_S) \otimes |k'\rangle\langle k'|_W$$

How to extend this to an arbitrary N-level weight?


$\frac{1}{2}\mathbb{I}_S$
$\mathcal{R}_{11}$
$|1\rangle\langle 1|_W$
w
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\rho_S(\epsilon)$
$|0\rangle\langle 0|_W$
$\mathcal{R}_{00}$


$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$


$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
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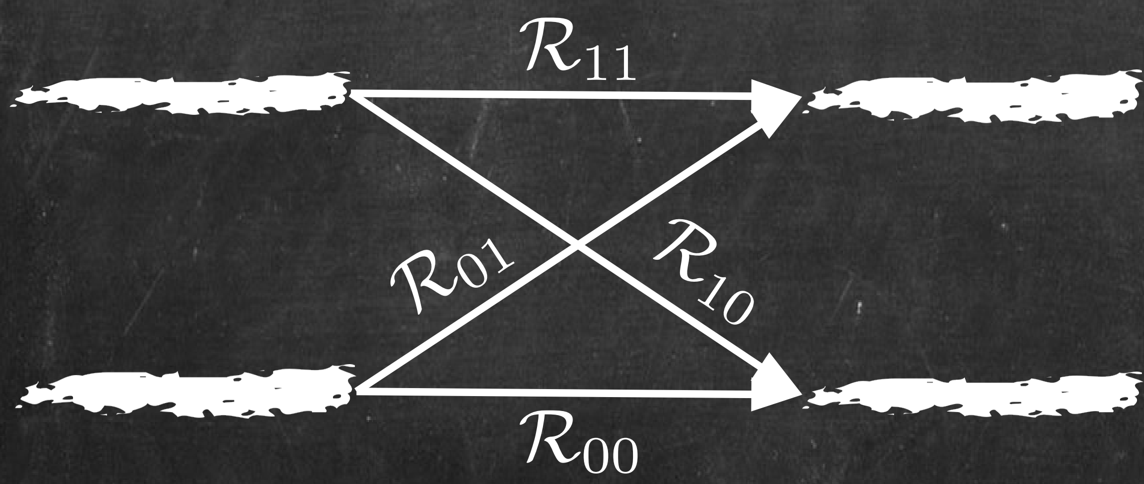
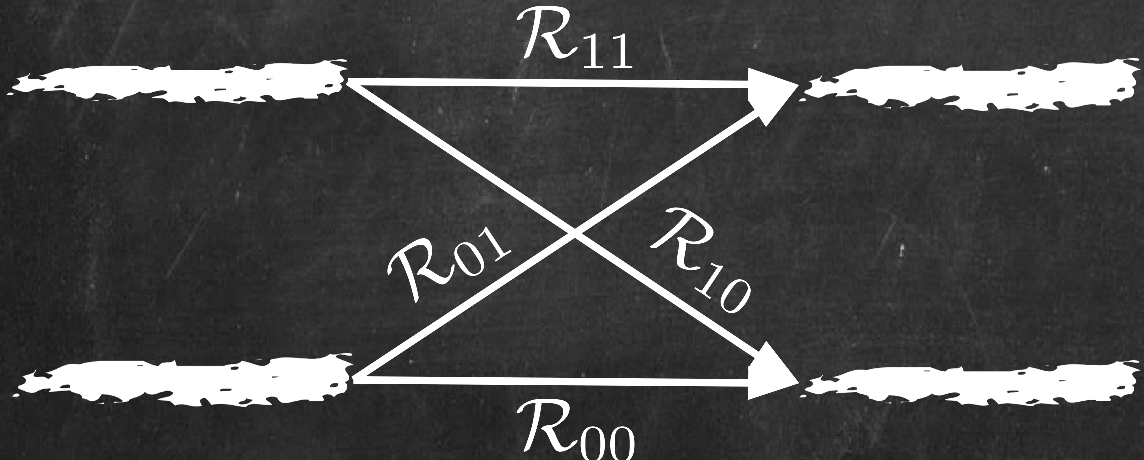


$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$



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$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
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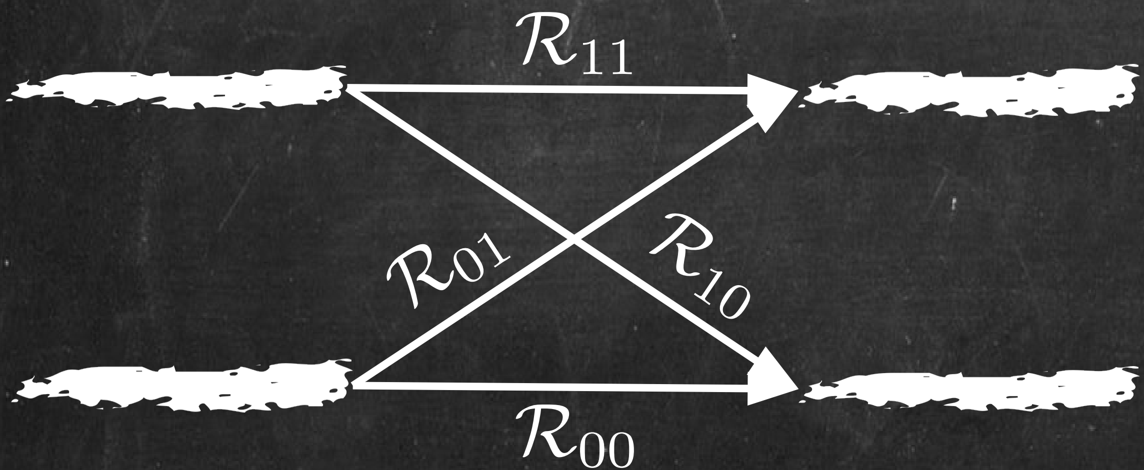
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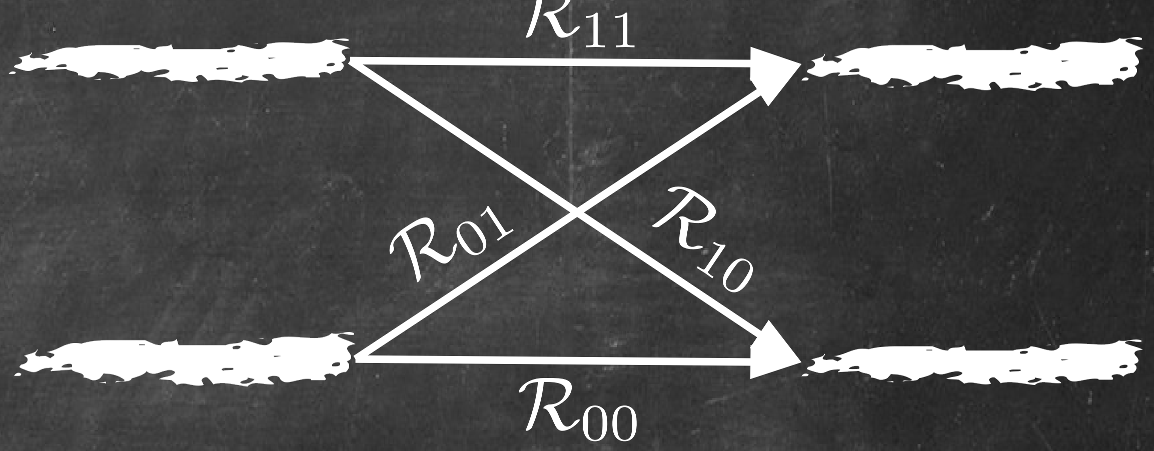
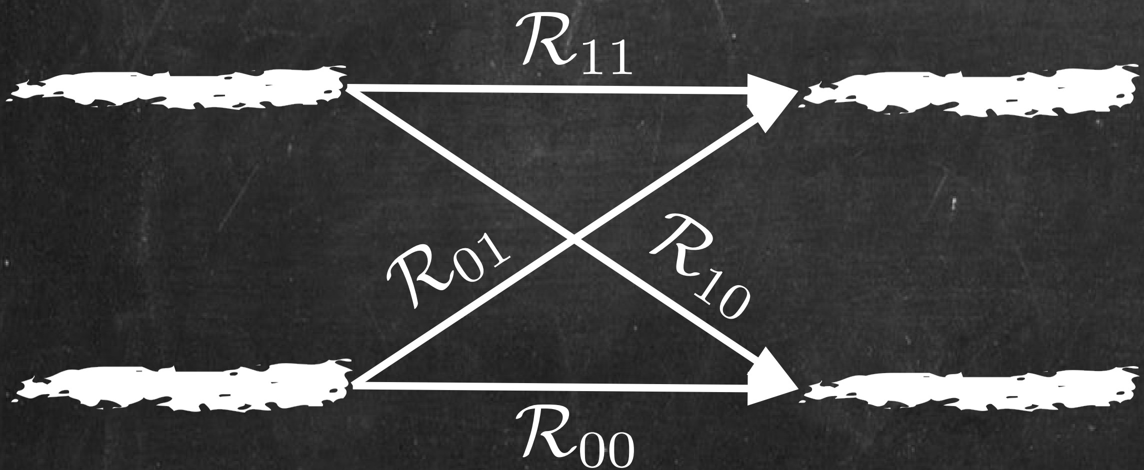
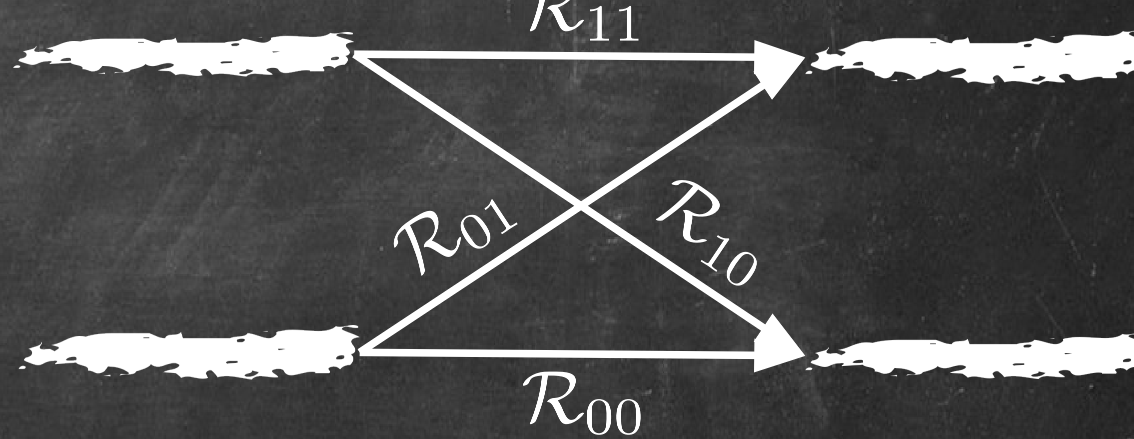
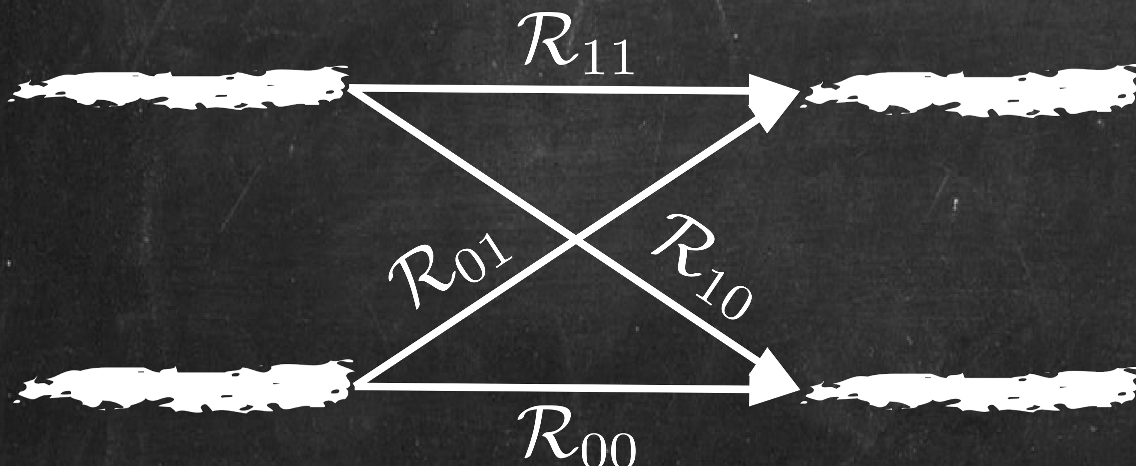
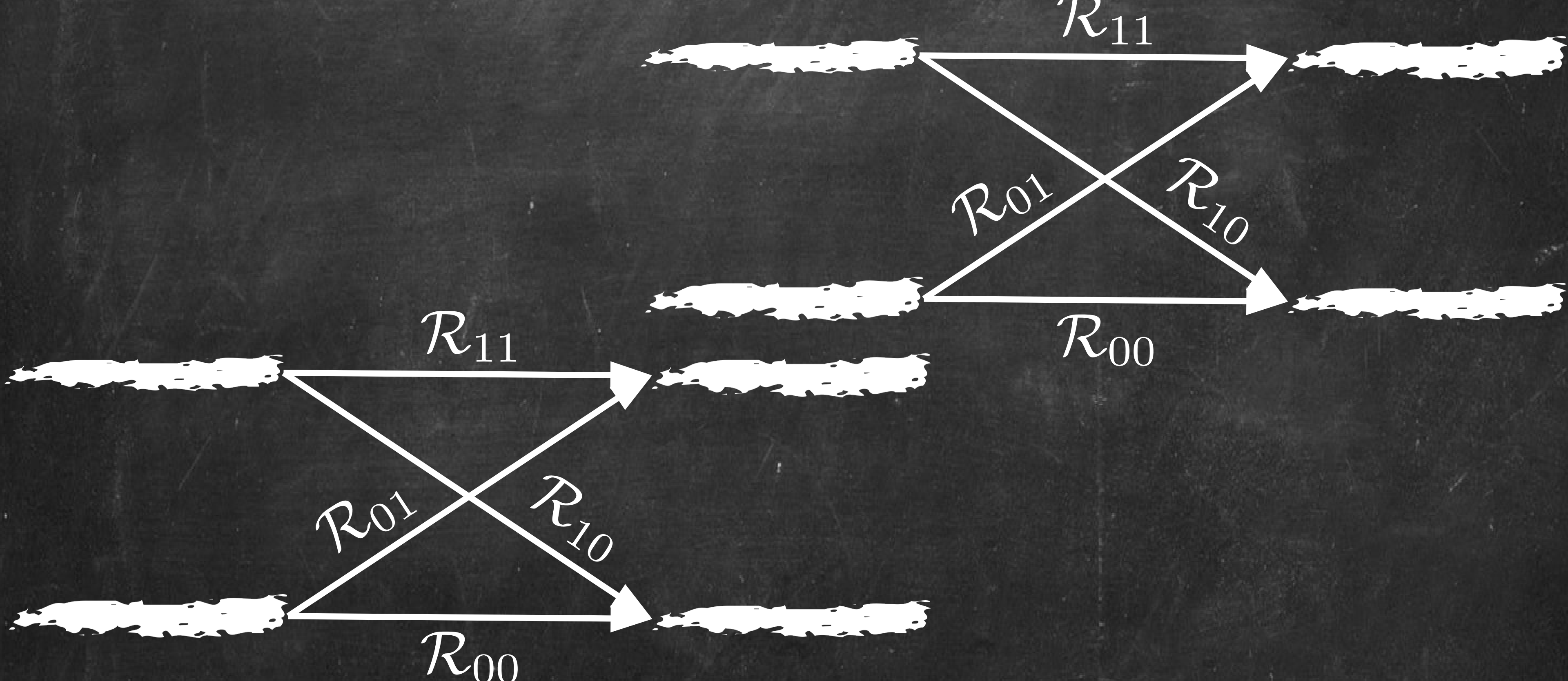
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$


$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$

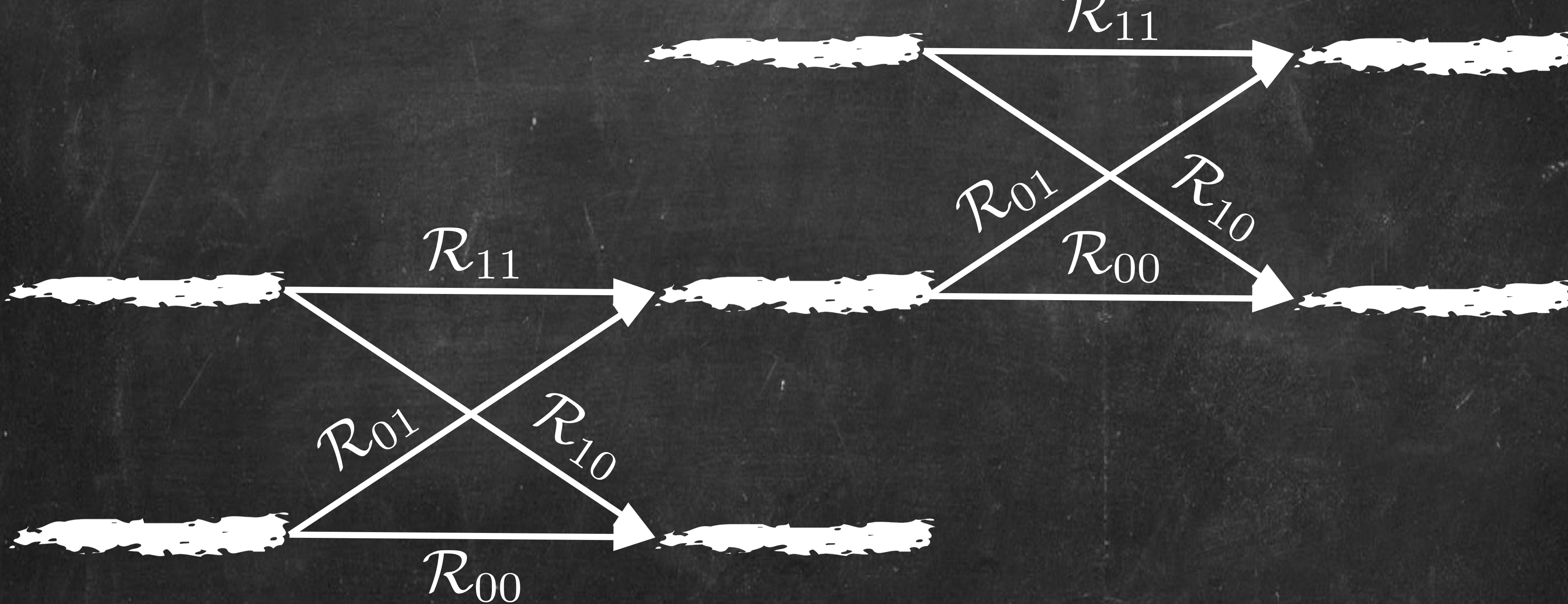

$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$


$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$


$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$



$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$
$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$


$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$
$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
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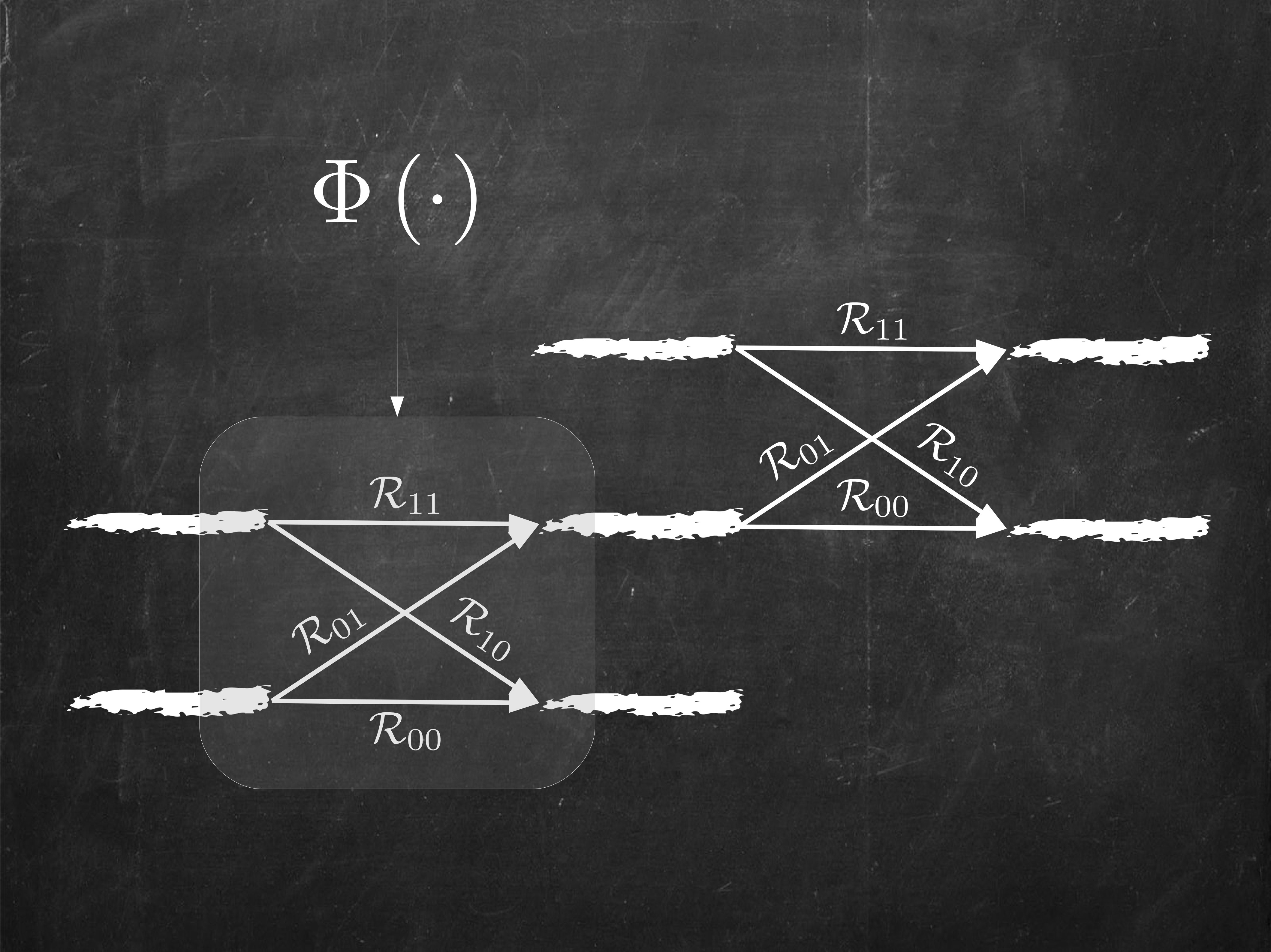
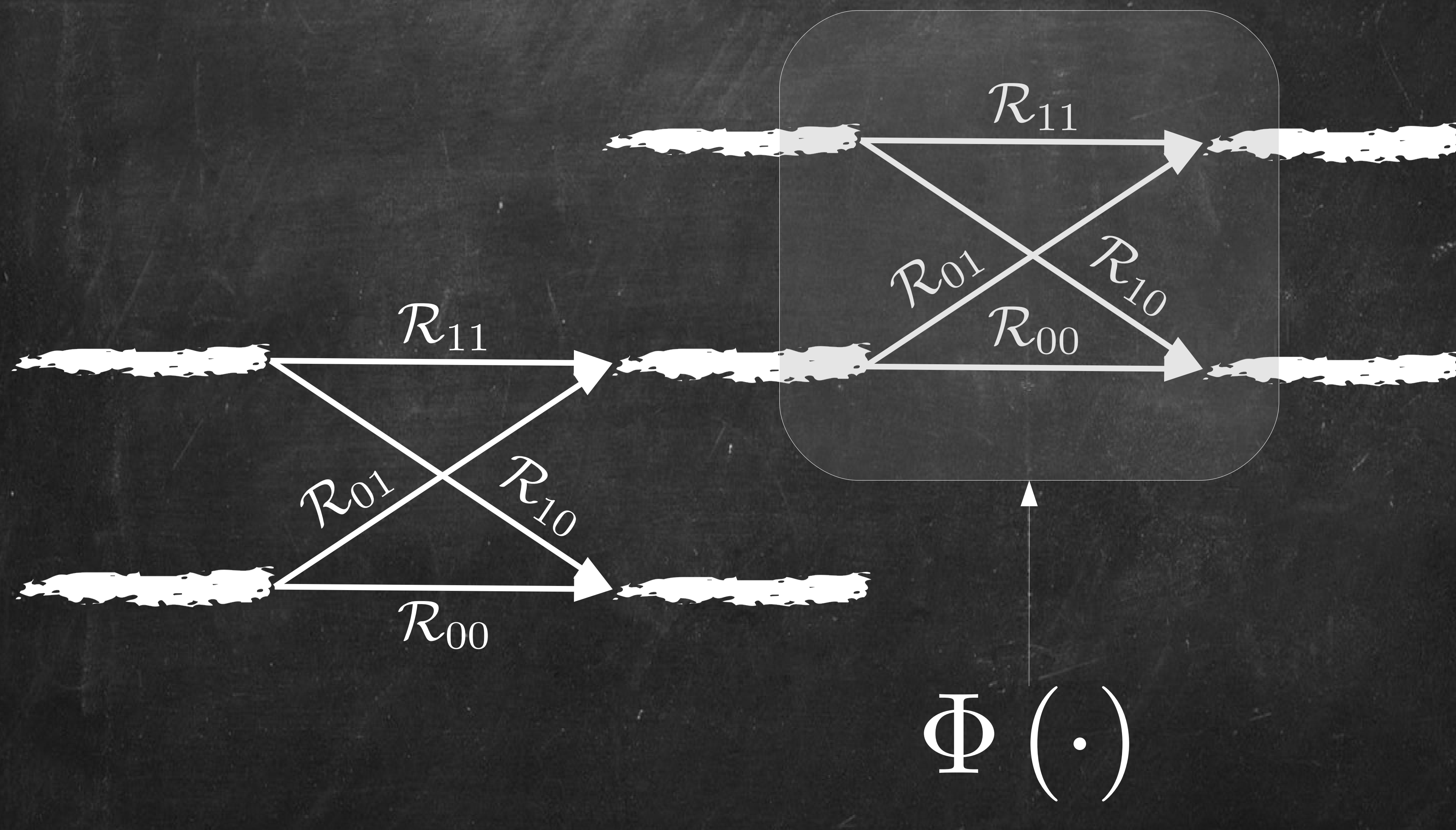
$\mathcal{R}_{00}$

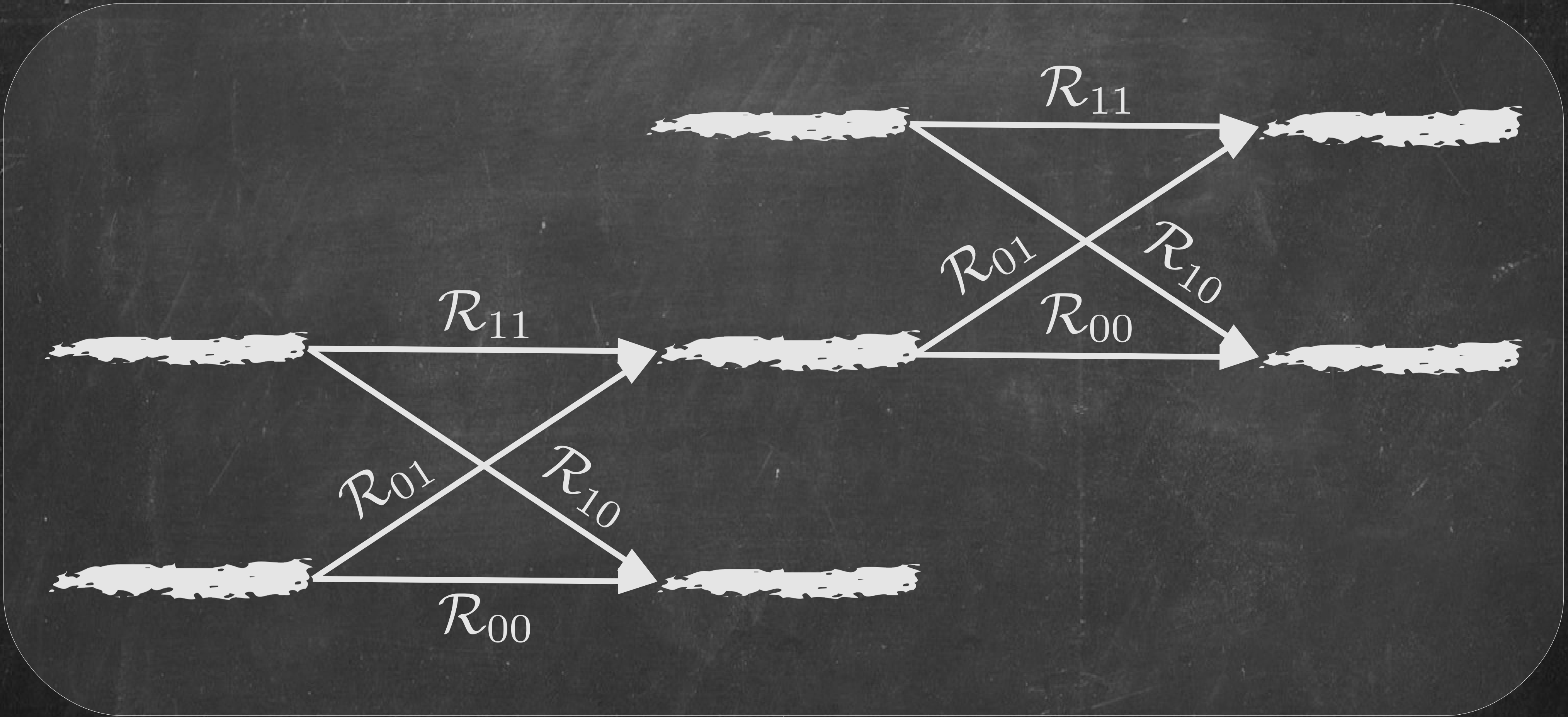


$\Phi(\cdot)$
$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$
$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$



$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$
$\mathcal{R}_{11}$
$\mathcal{R}_{01}$
$\mathcal{R}_{10}$
$\mathcal{R}_{00}$
$\Phi(\cdot)$

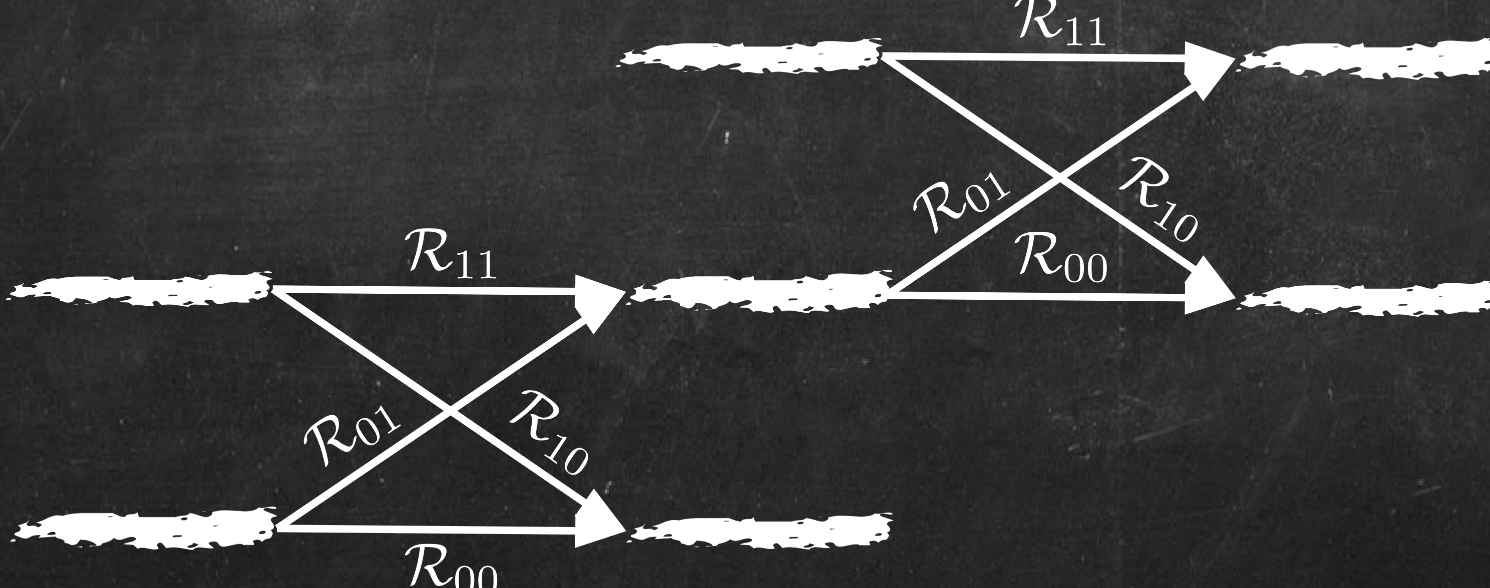
$$\Phi_2 : \mathcal{H}_S \otimes \mathcal{H}_{W_2} \to \mathcal{H}_S \otimes \mathcal{H}_{W_2}$$



$$\Phi_2(\cdot)$$

Is Φ_2 valid?

- Gibbs-preserving?
- Performs the desired transformation?



Is Φ_2 valid?

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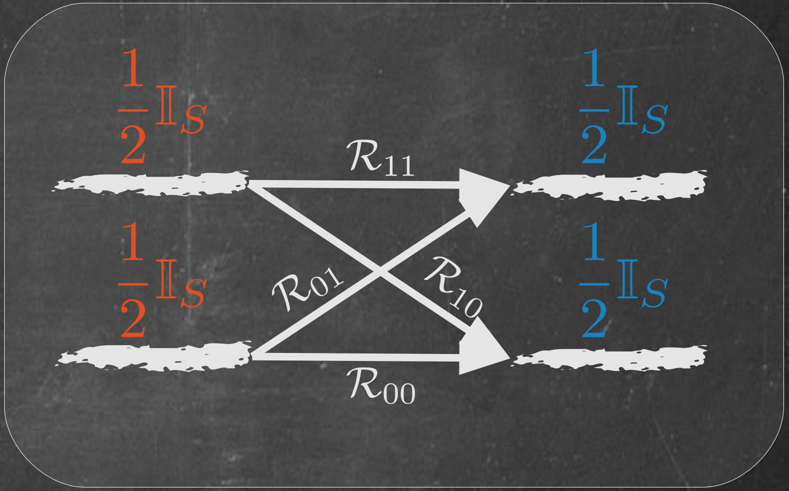
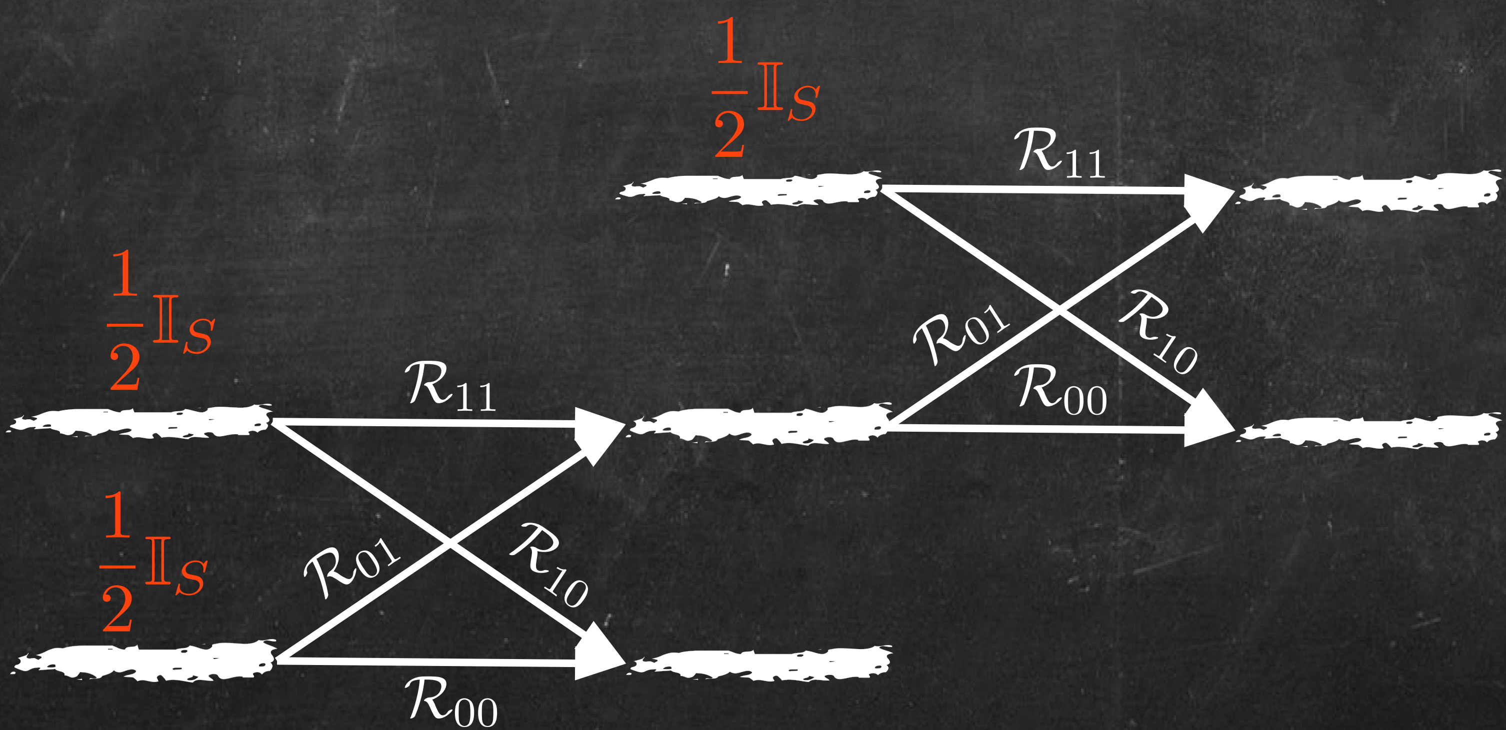
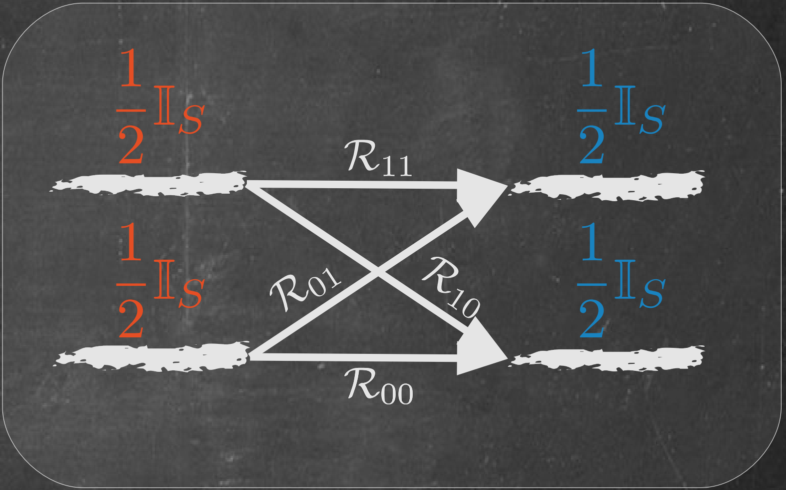
- [ ] Performs the desired transformation?







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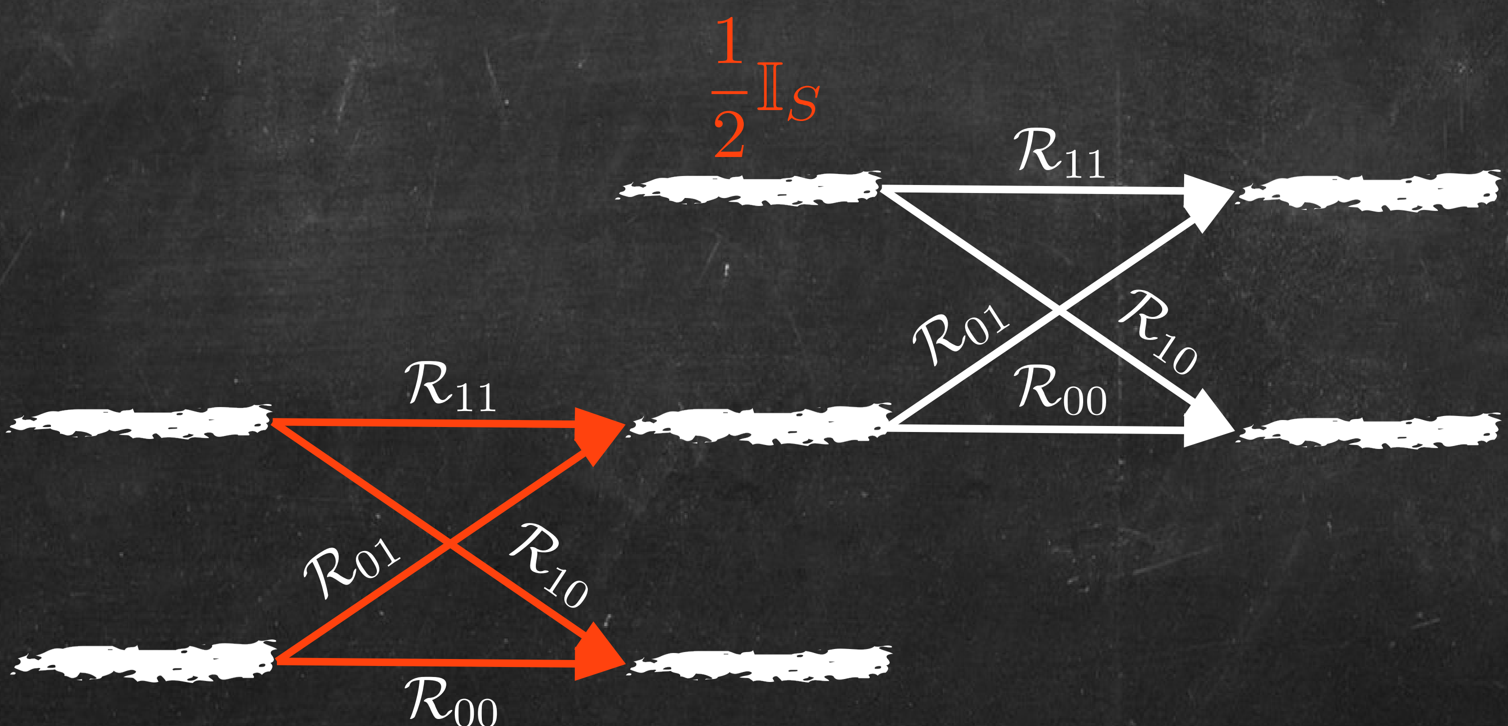
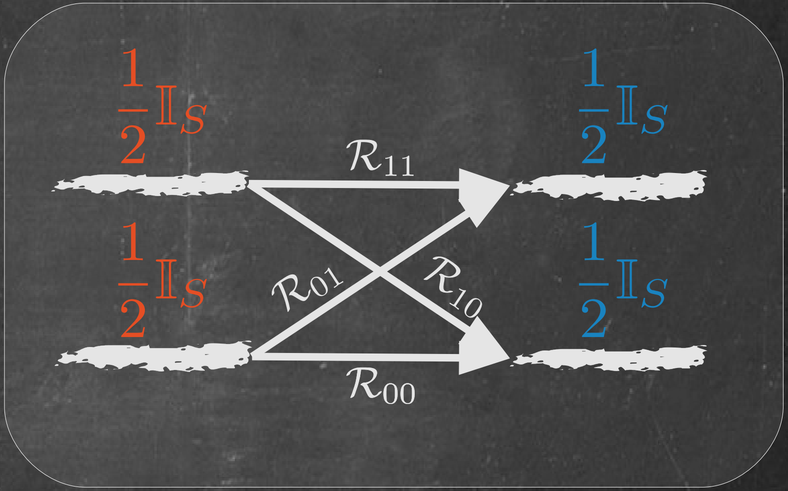


Is Φ_2 valid?



- Gibbs-preserving?
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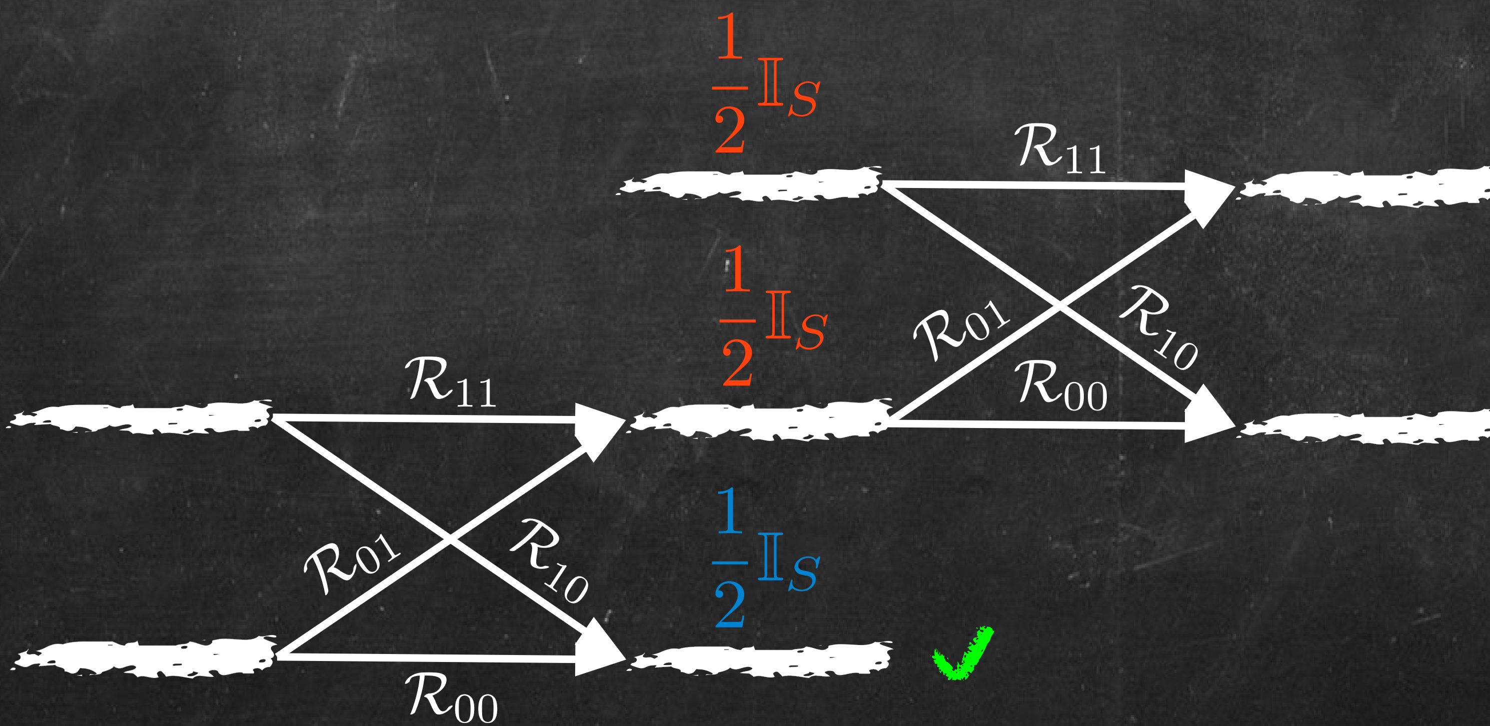
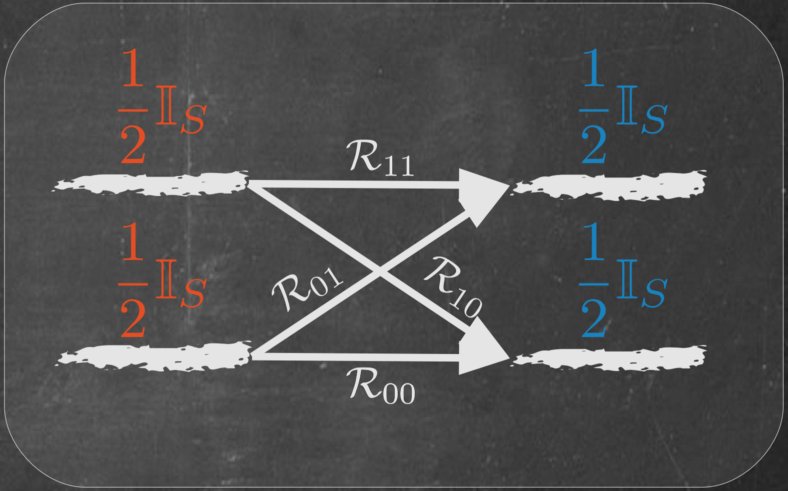
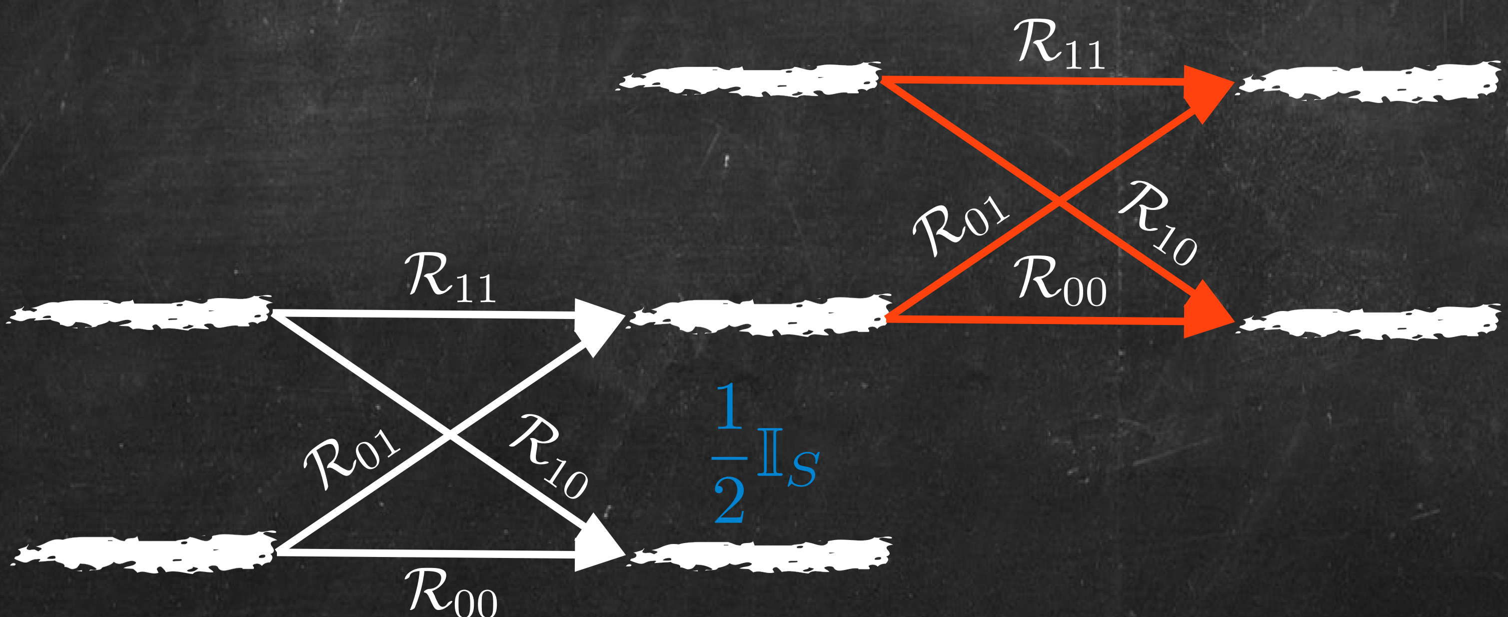


Is Φ_2 valid?



- Gibbs-preserving?
- Performs the desired transformation?



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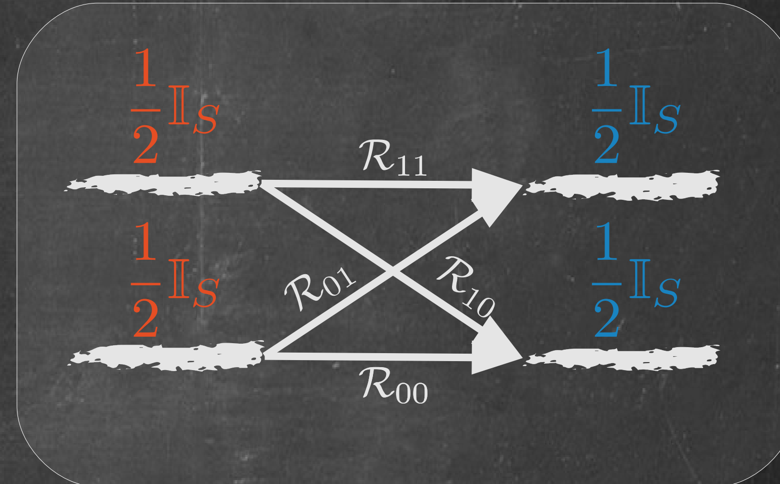
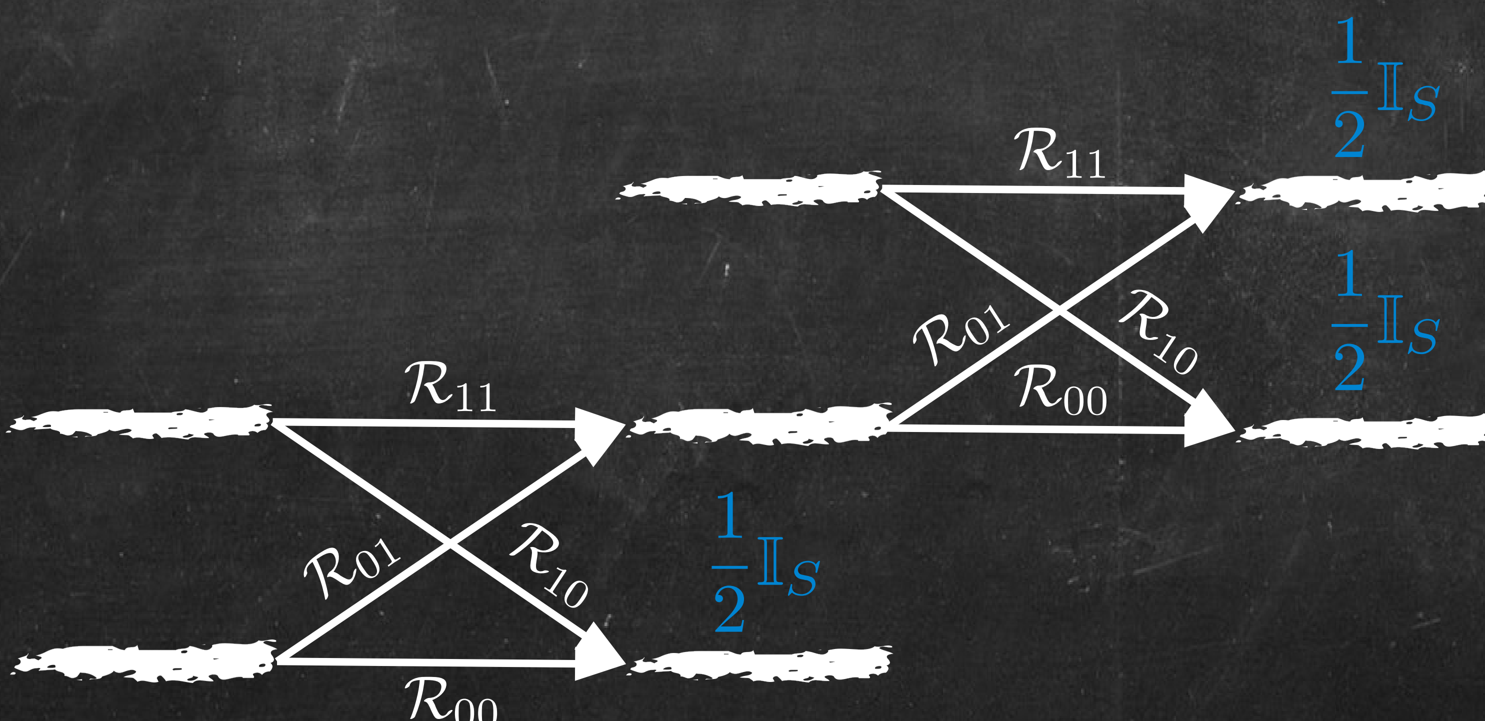
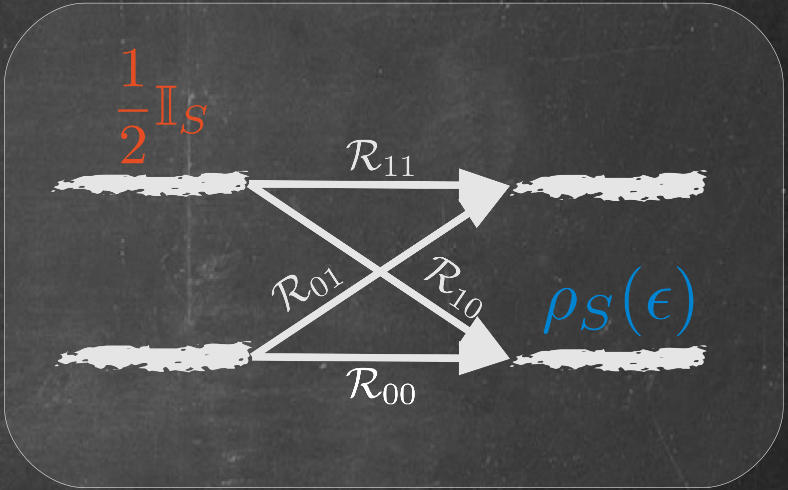
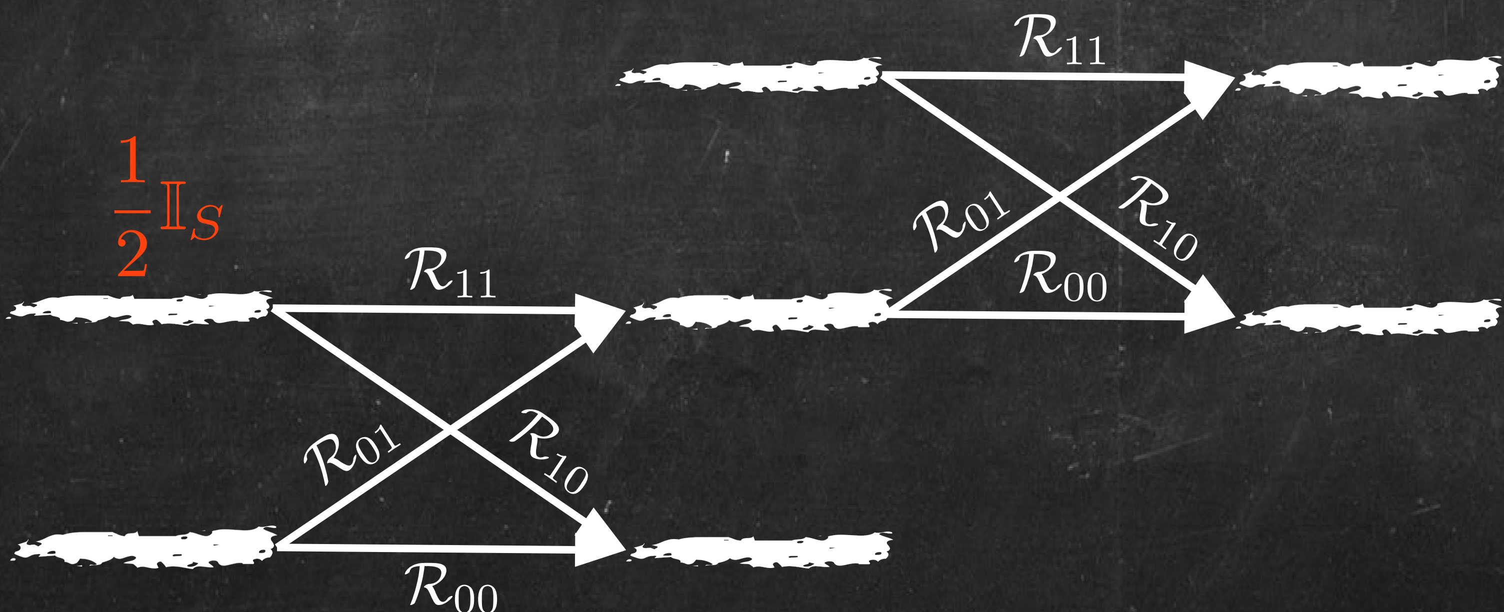




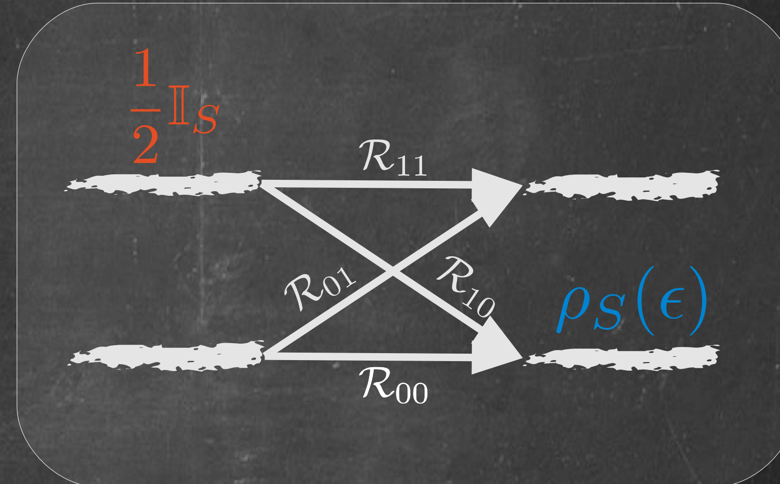
Is Φ_2 valid?

- [x] Gibbs-preserving?
- [ ] Performs the desired transformation?





Is Φ_2 valid?



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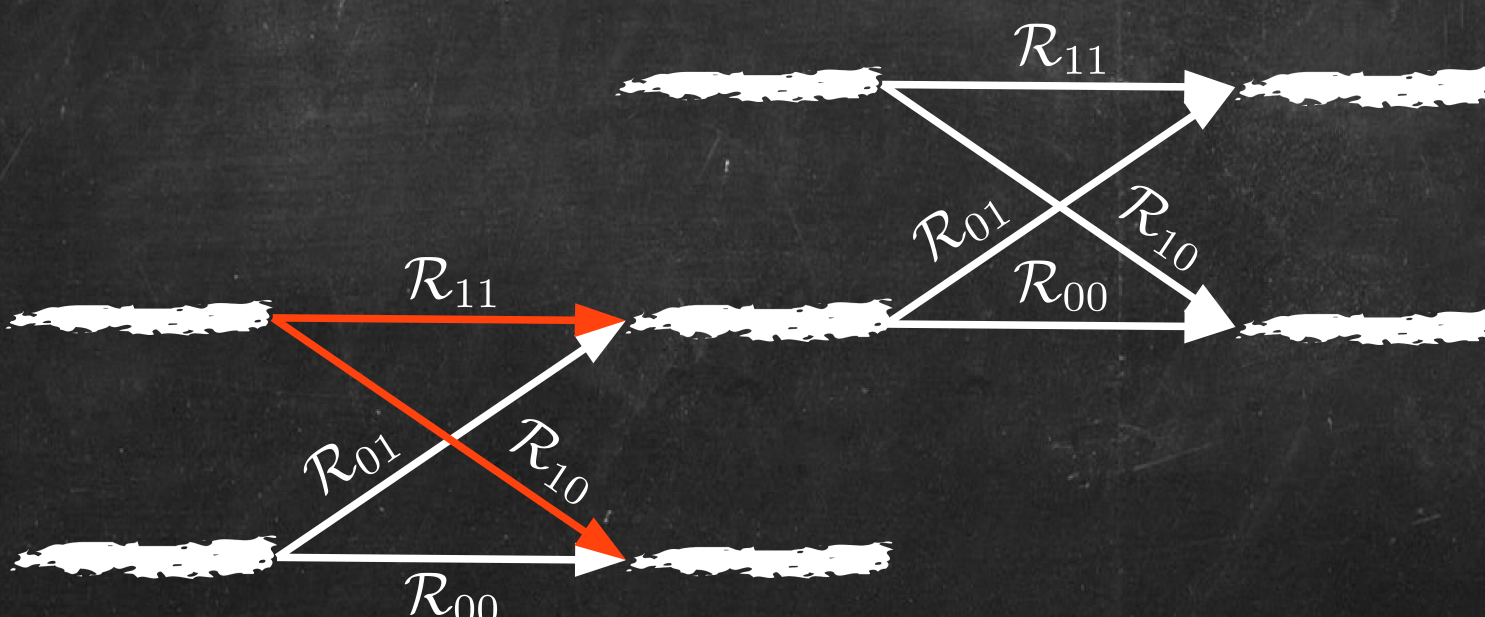
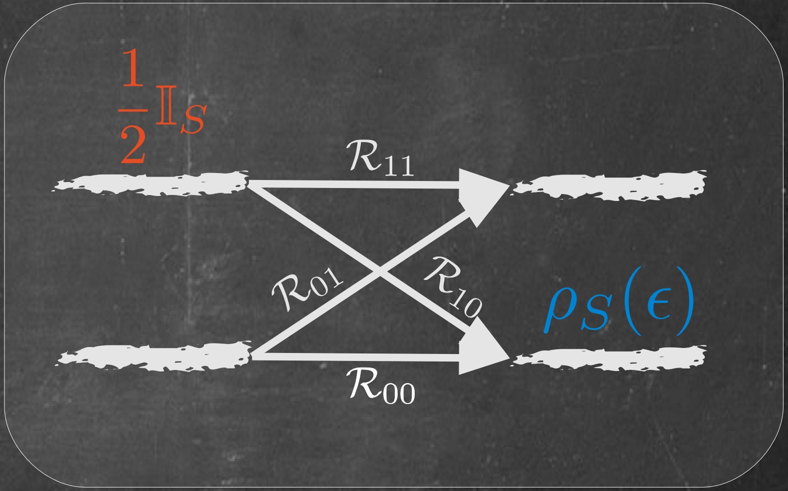
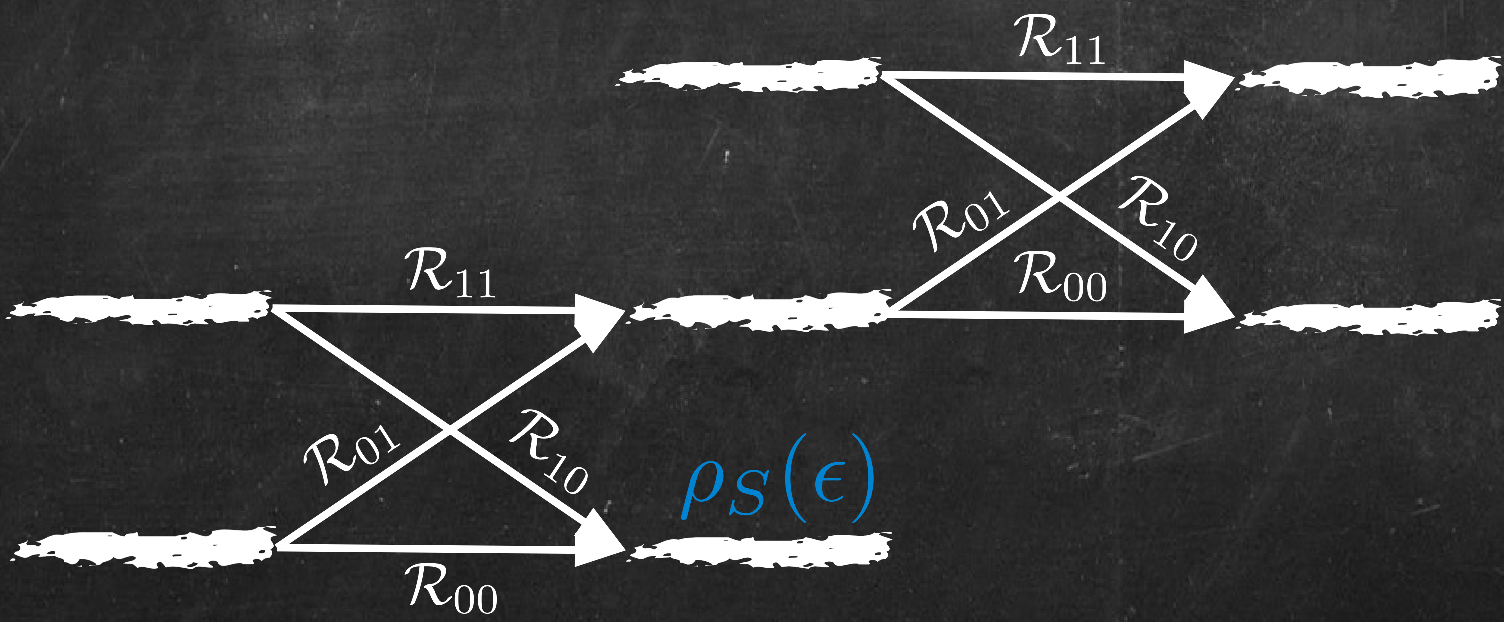




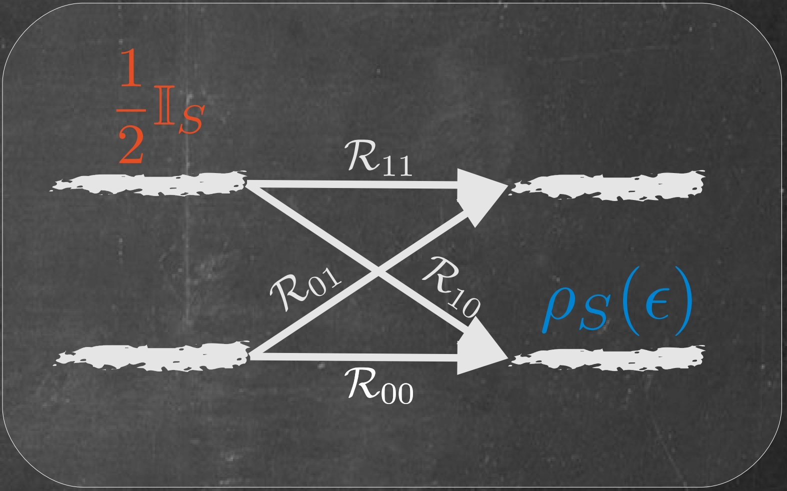
Is Φ_2 valid?

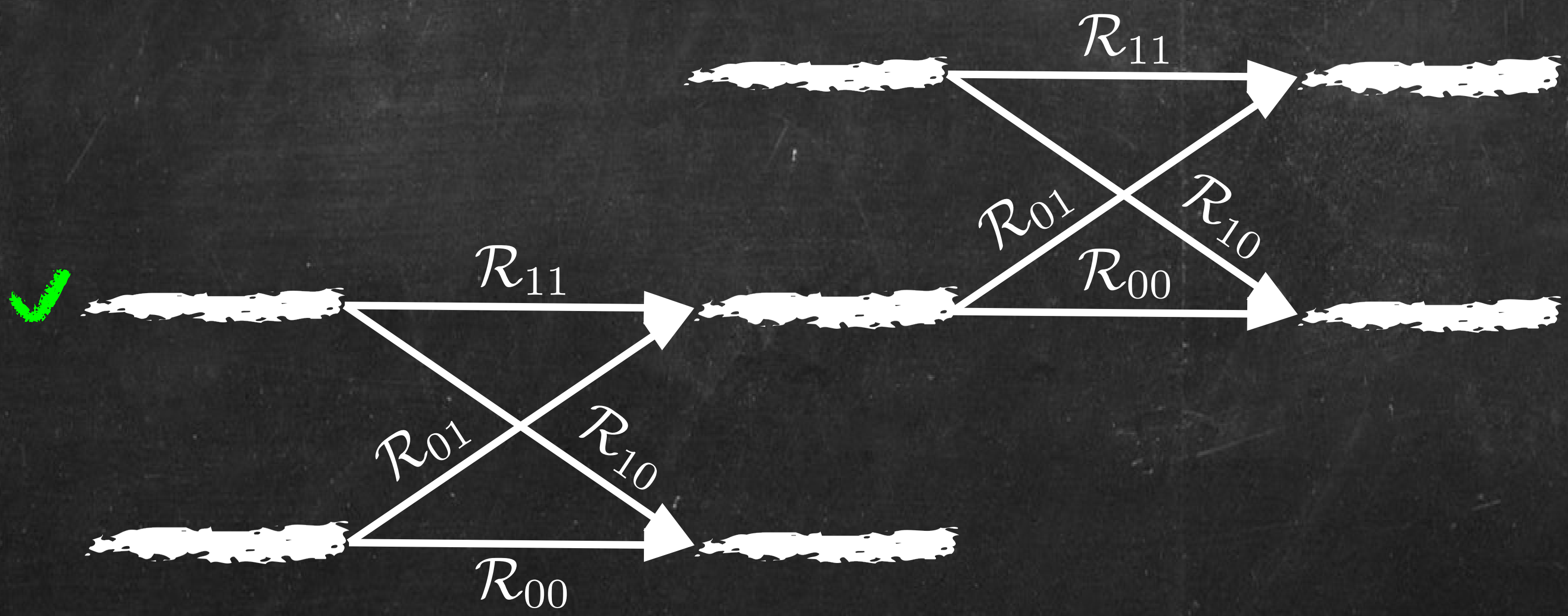
- [x] Gibbs-preserving?
- [ ] Performs the desired transformation?



Is Φ_2 valid?

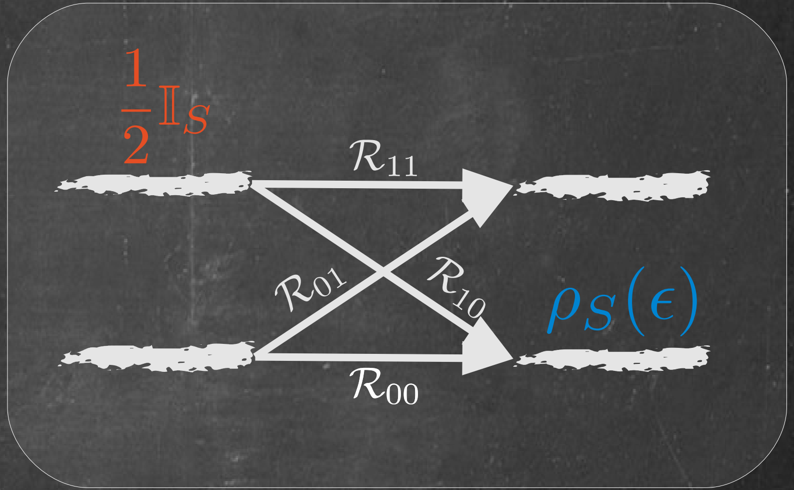
- ✓ Gibbs-preserving?
- ☐ Performs the desired transformation?

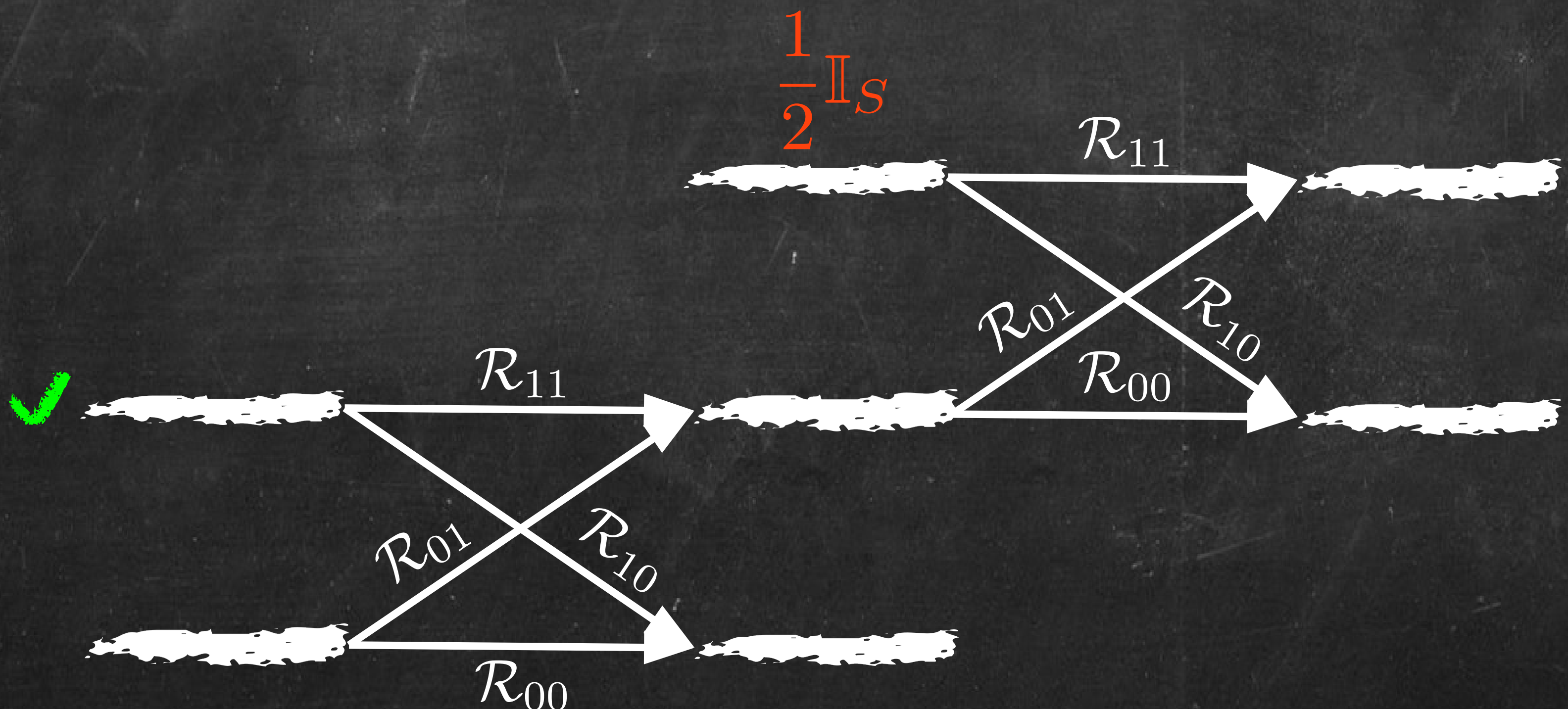




Is Φ_2 valid?

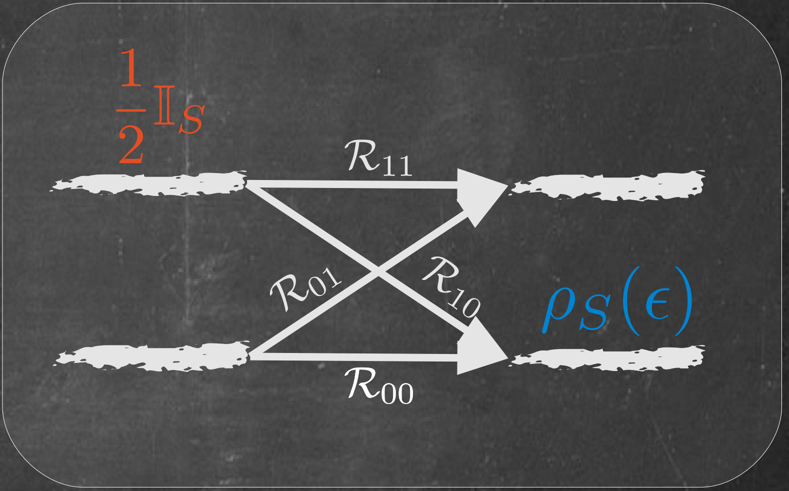
- [x] Gibbs-preserving?
- [ ] Performs the desired transformation?

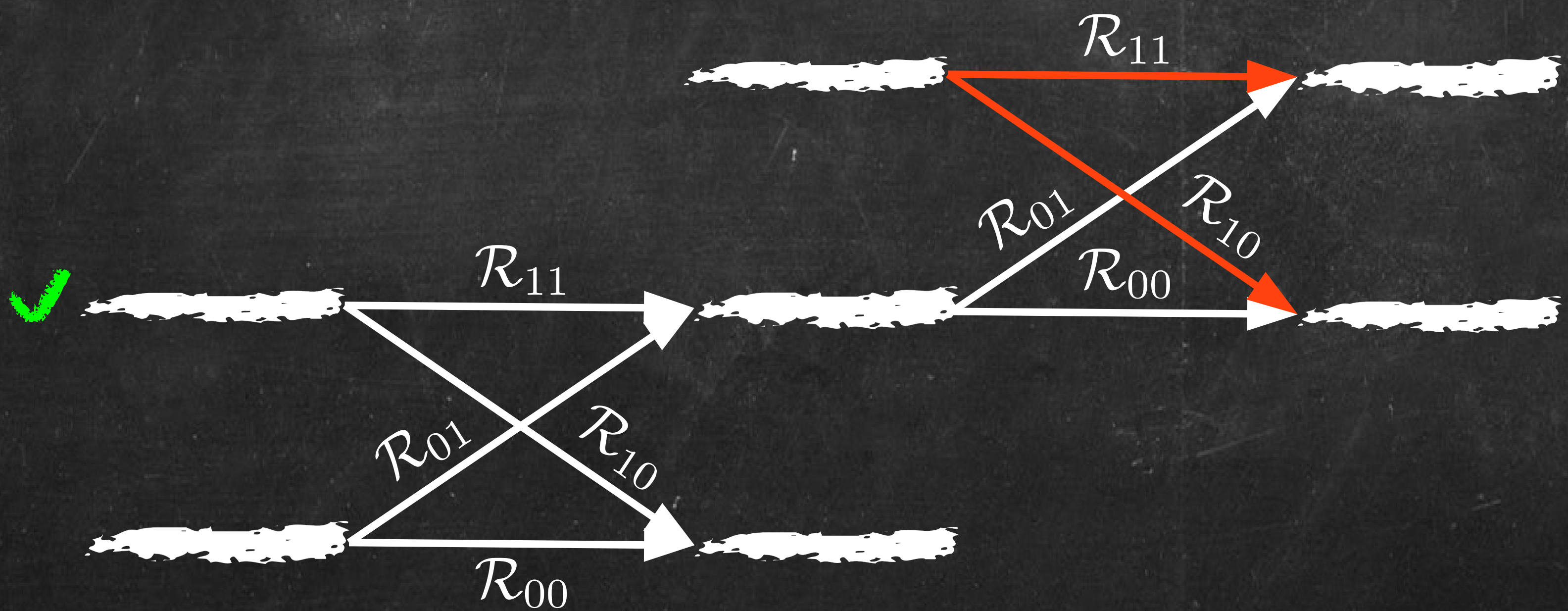




Is Φ_2 valid?

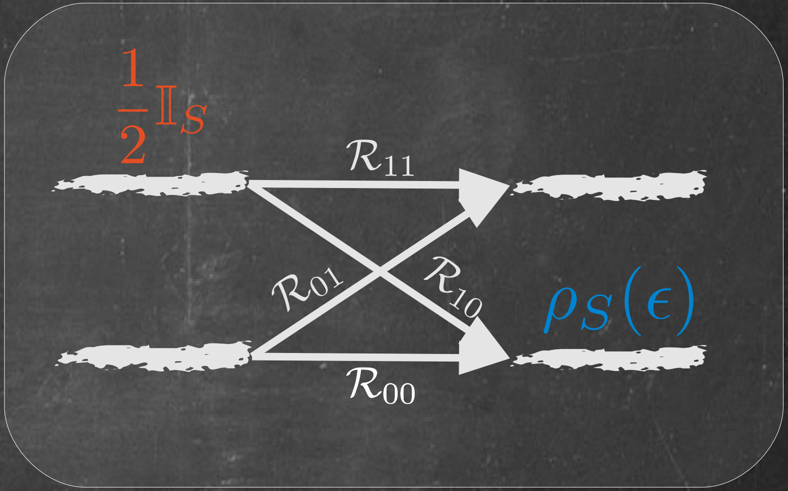
- ✓ Gibbs-preserving?
- ☐ Performs the desired transformation?

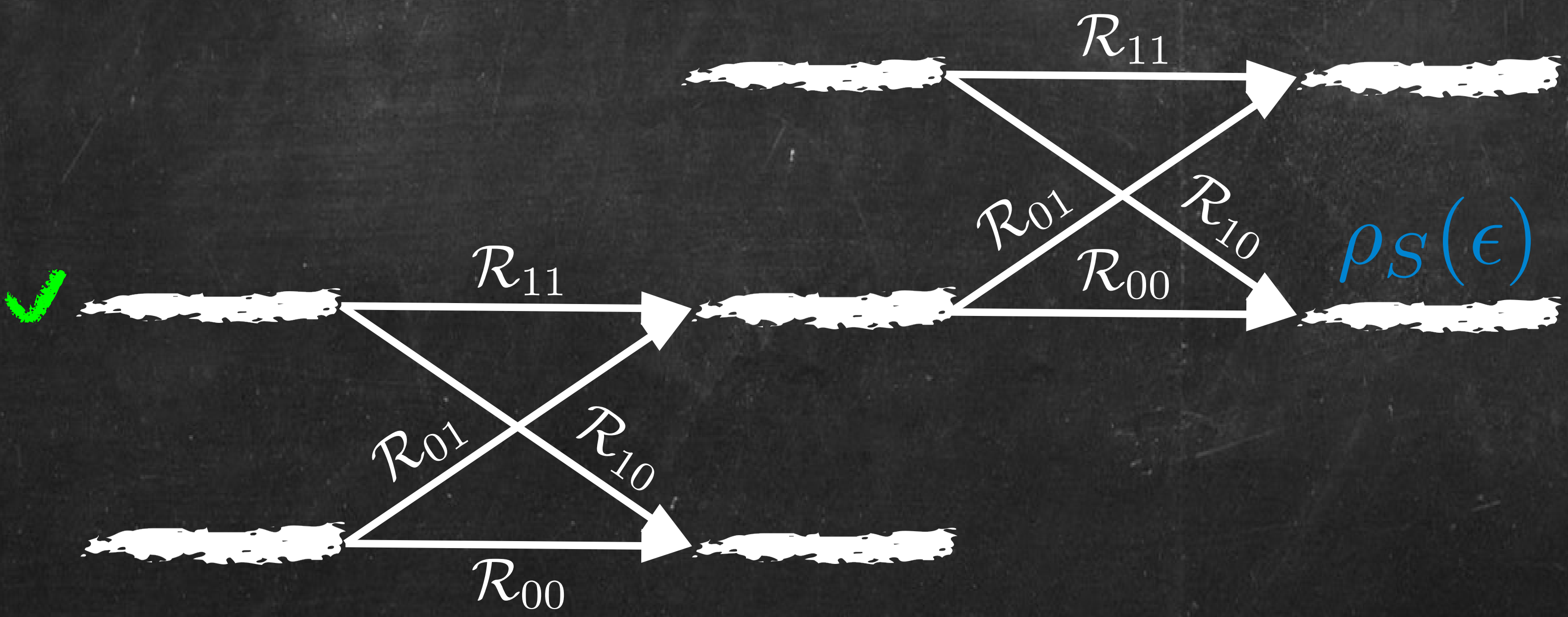




Is Φ_2 valid?

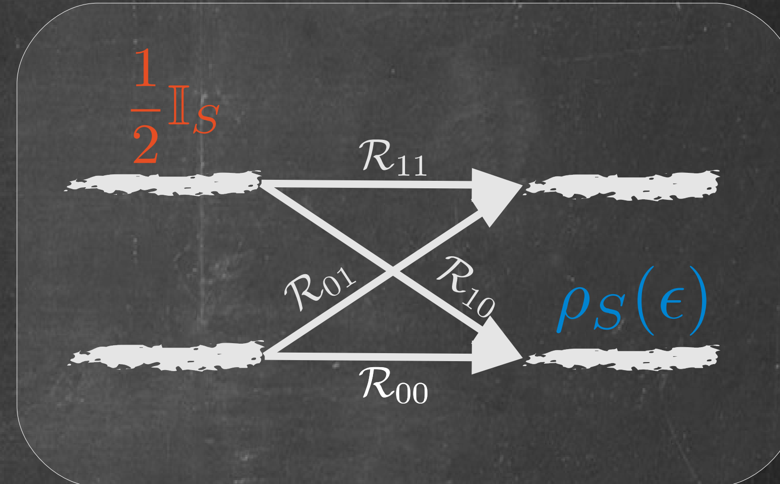
- [x] Gibbs-preserving?
- [ ] Performs the desired transformation?

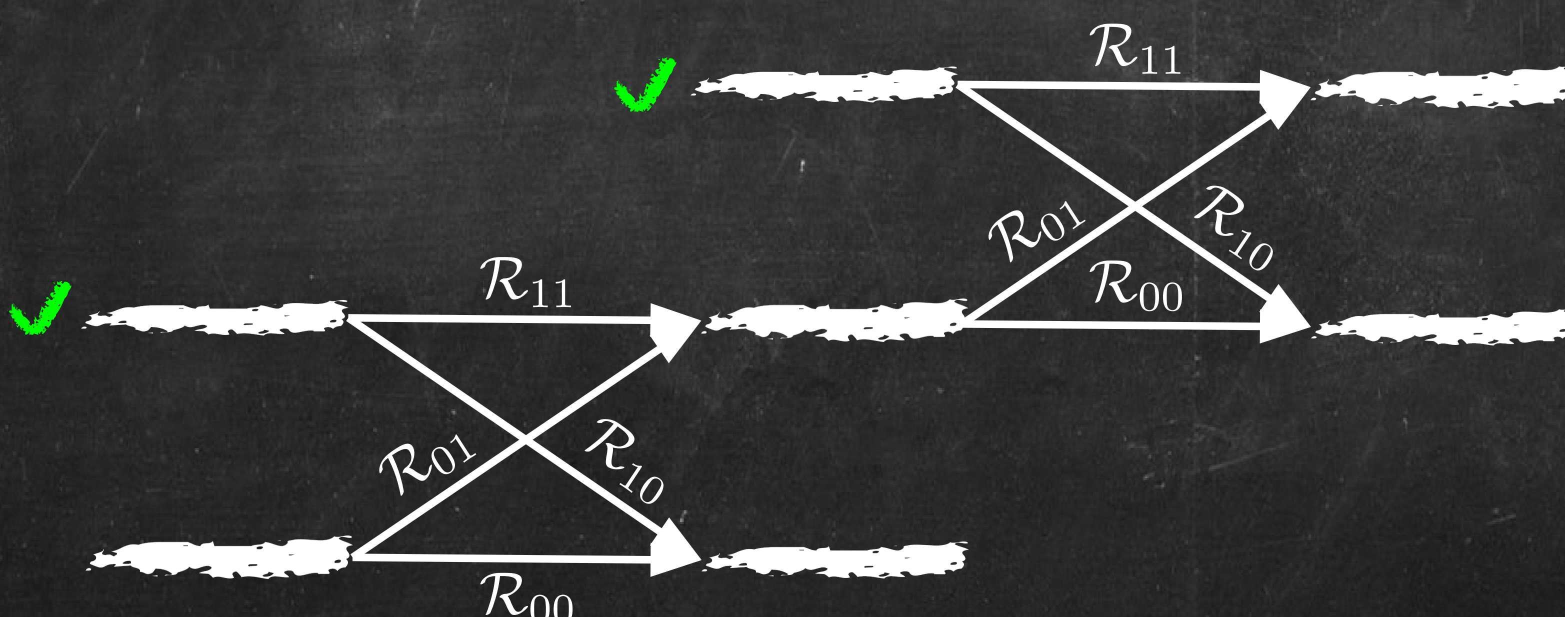




Is Φ_2 valid?

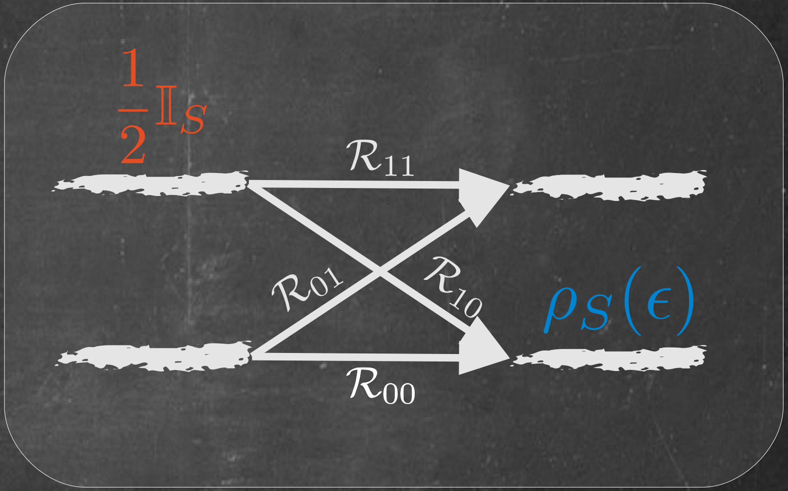
- [x] Gibbs-preserving?
- [ ] Performs the desired transformation?

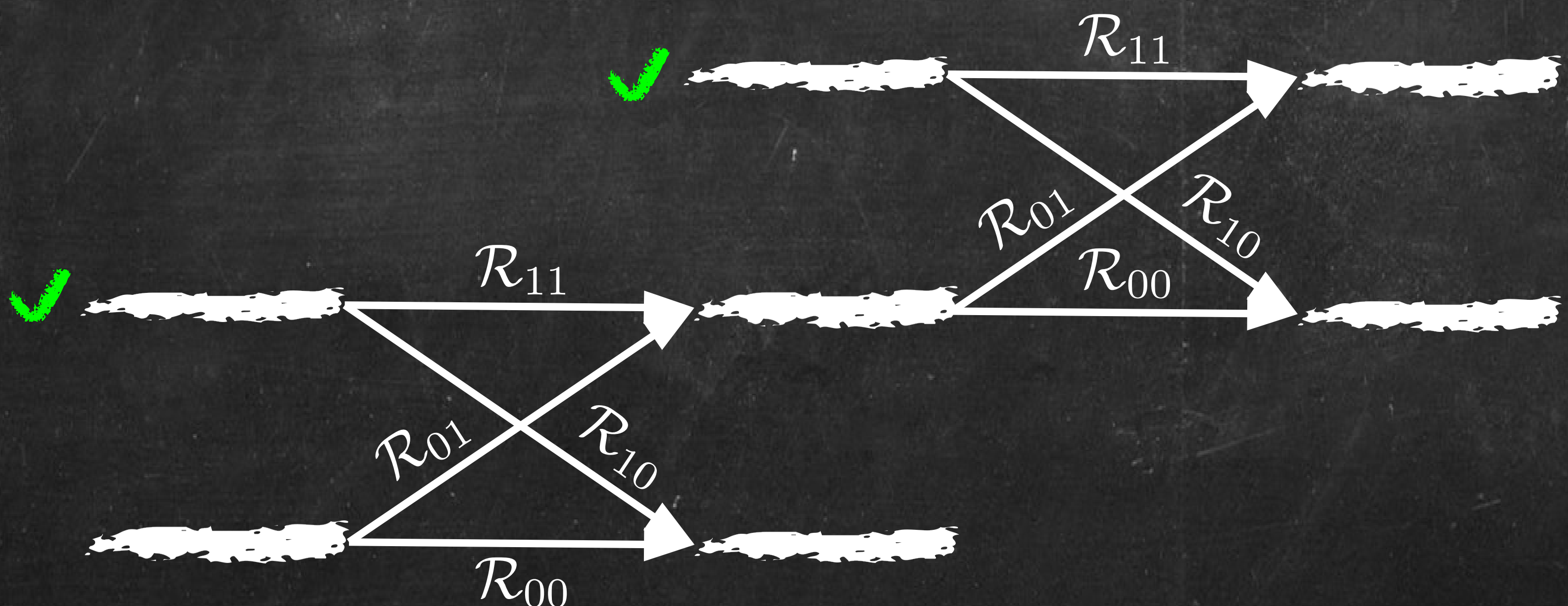




Is Φ_2 valid?

- [x] Gibbs-preserving?
- [x] Performs the desired transformation?

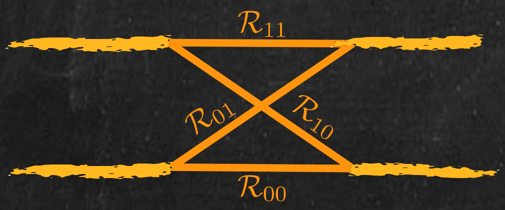




One can proceed iteratively to arbitrary N:

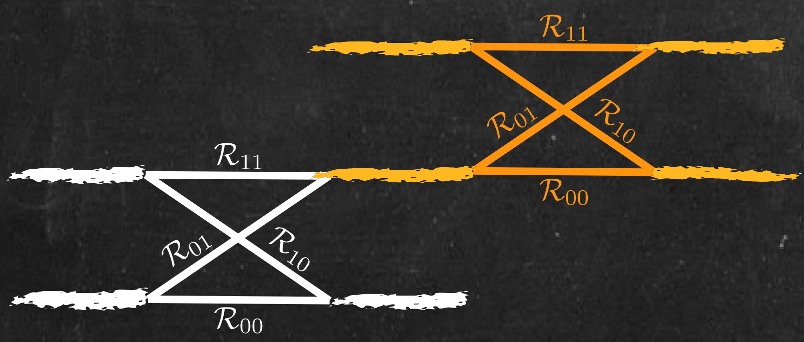
One can proceed iteratively to arbitrary N:

$\Phi_1 \to$



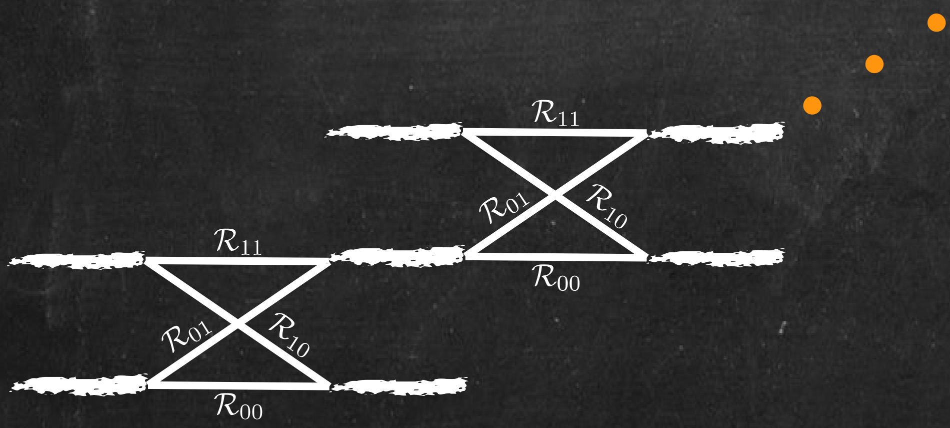
One can proceed iteratively to arbitrary N:

$$\Phi_1 \to \Phi_2$$



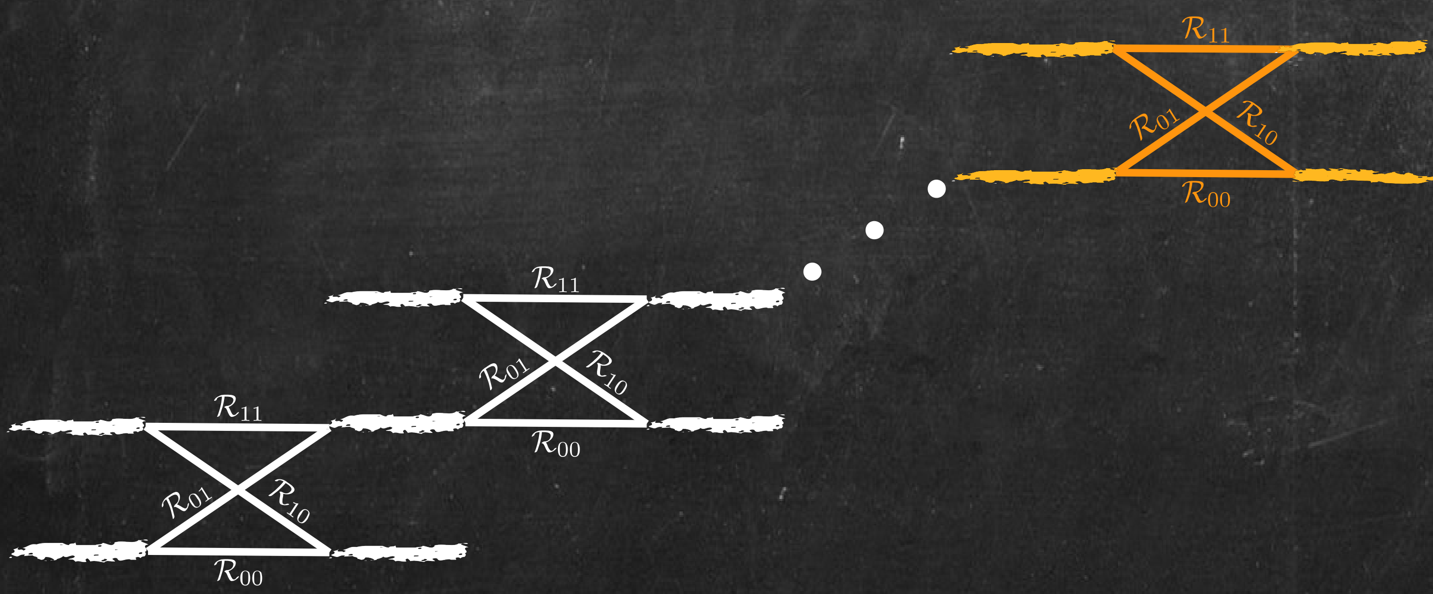
One can proceed iteratively to arbitrary N:

$$\Phi_1 \rightarrow \Phi_2 \rightarrow \cdots$$



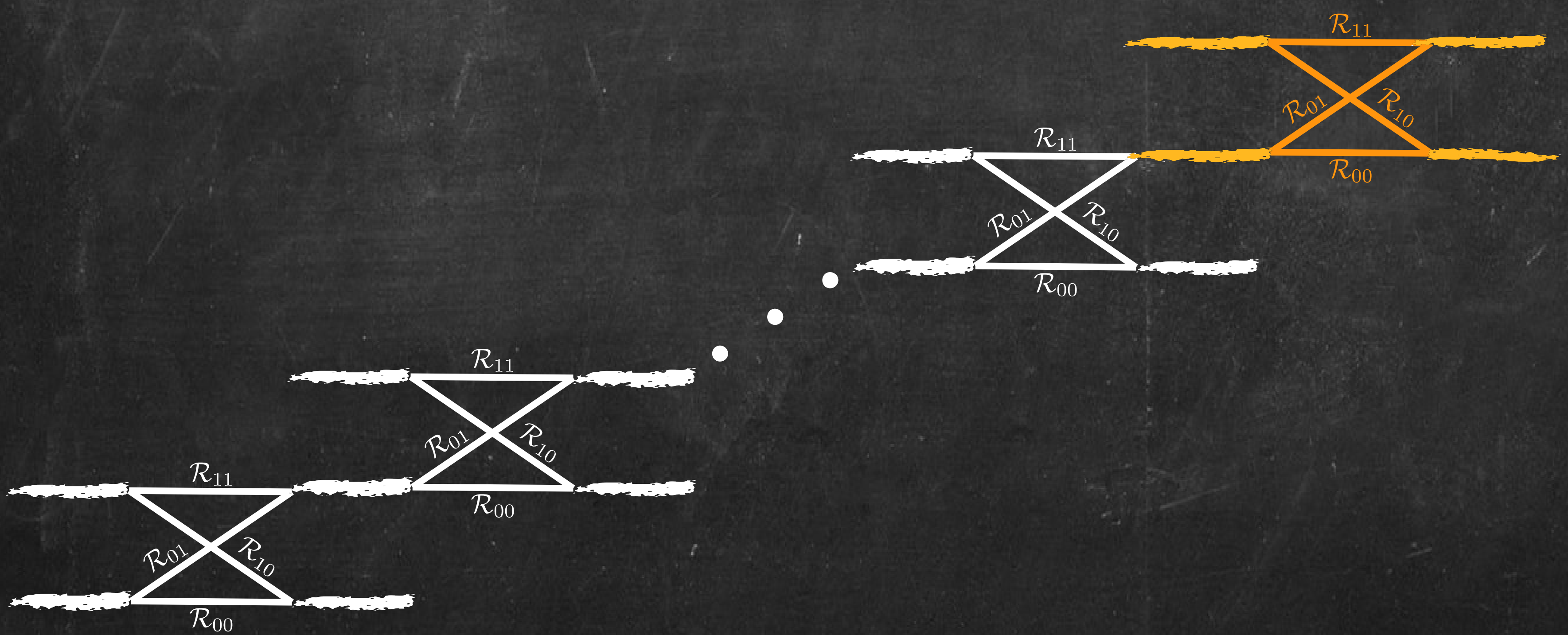
One can proceed iteratively to arbitrary N:

$$\Phi_1 \to \Phi_2 \to \cdots \to \Phi_{N-1}$$



One can proceed iteratively to arbitrary N:

$$\Phi_1 \to \Phi_2 \to \cdots \to \Phi_{N-1} \to \Phi_N$$



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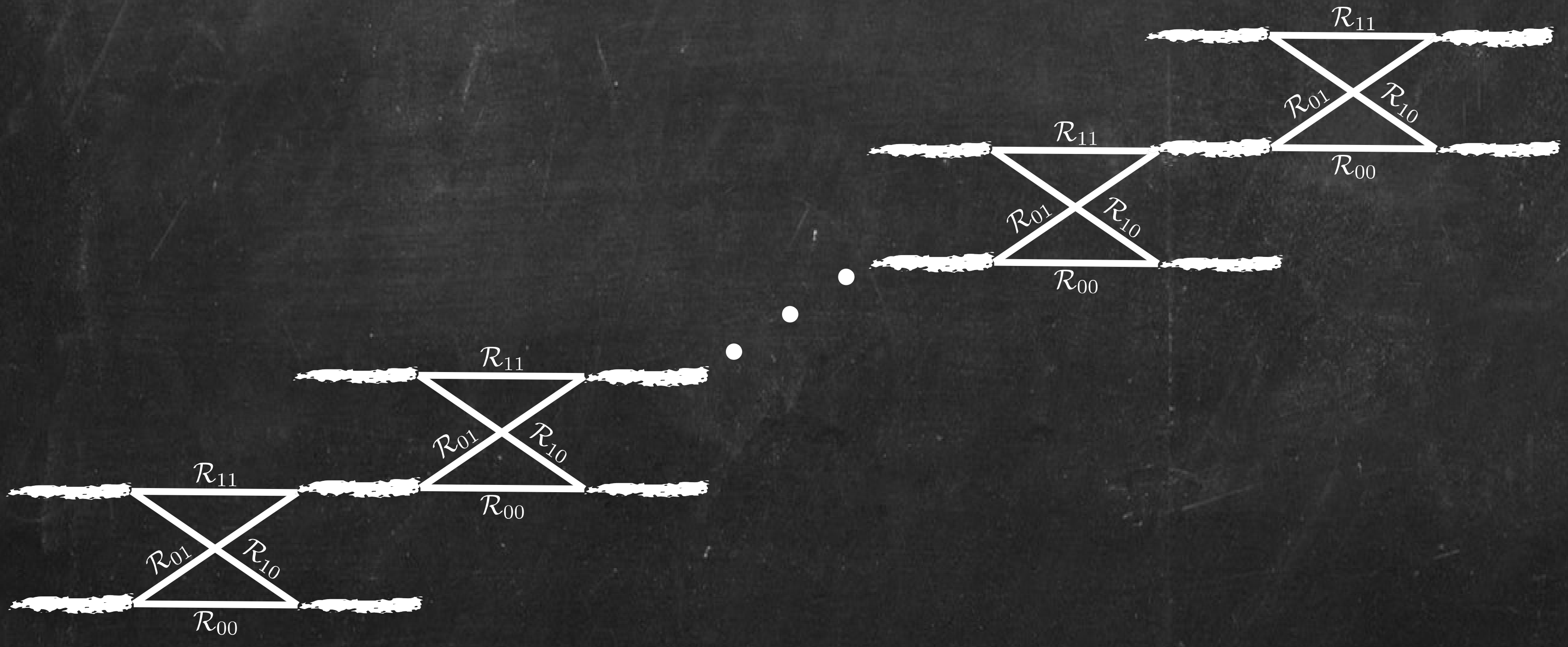
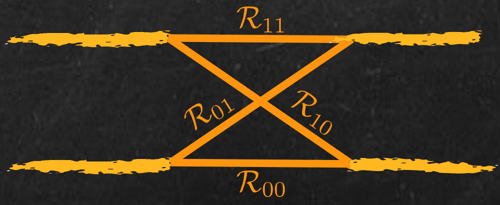
$$\Phi_1 \rightarrow \Phi_2 \rightarrow \cdots \rightarrow \Phi_{N-1} \rightarrow \Phi_N$$



Note: for work > 0 (distillation)

one can proceed similarily:

$\Phi_1 \to \Phi_2$



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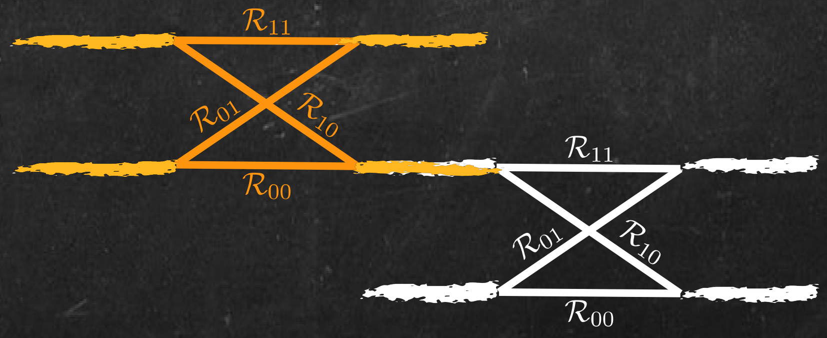
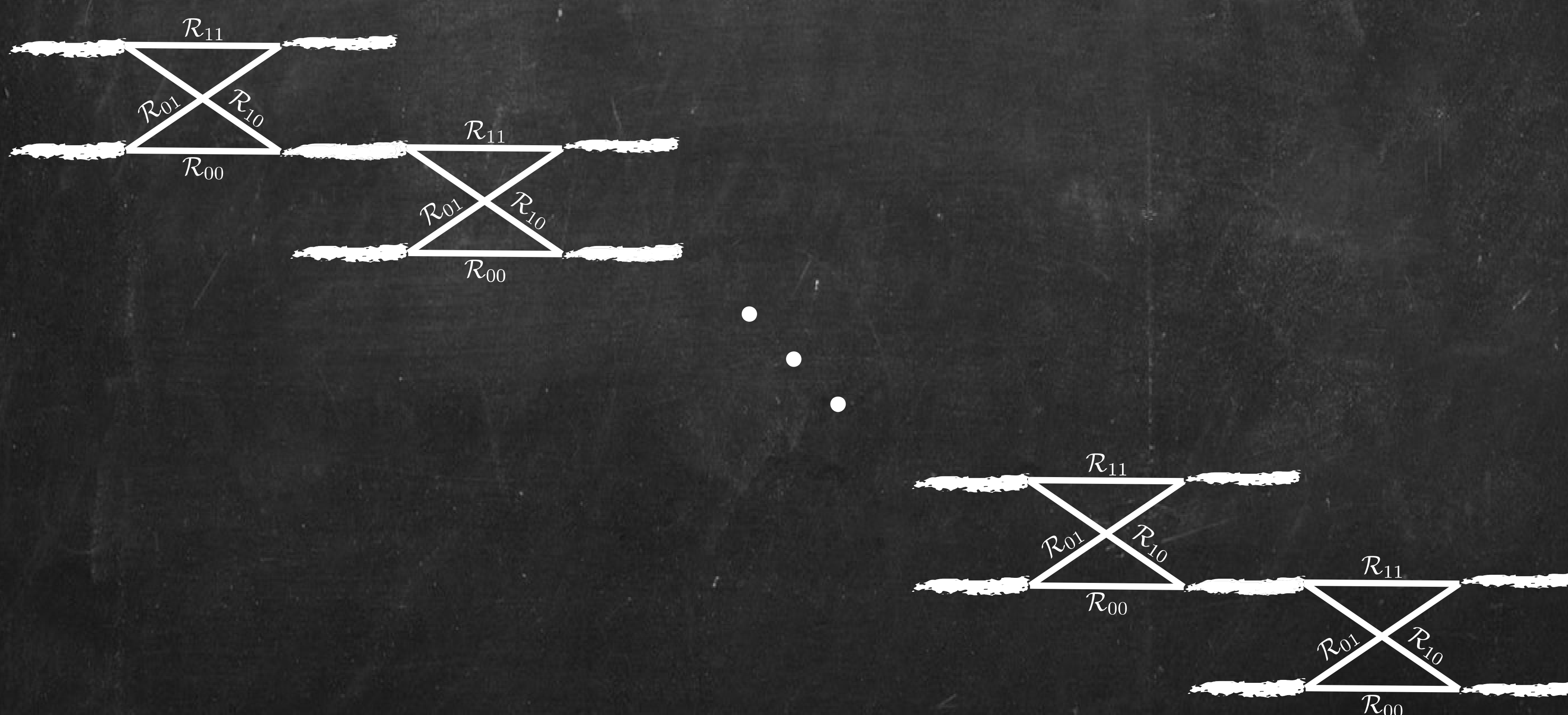


Note: for work > 0 (distillation)

one can proceed similarily:

$$\Phi_1 \to \Phi_2 \cdots \to \Phi_N$$



What about work fluctuations?

Conditional average work

$\langle w(\epsilon) \rangle_{s'} =$

Conditional average work

$$\langle w(\epsilon)\rangle_{s'} = \sum_{work} p(work|s') \cdot work$$

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$$= \sum_{k,k'} p(k,k'|s') \cdot w \cdot (k' - k)$$

Conditional average work

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$$= \sum_{k,k'} p(k,k'|s') \cdot w \cdot (k'-k)$$

$$= \frac{1}{p(s')} \sum_{k,k'} p(k,k',s') \cdot w \cdot (k'-k)$$

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$$= \frac{1}{p(s')} \sum_{k,k'} \sum_{s} p(s) \cdot p(k) \cdot p(s'k'|sk) \cdot w \cdot (k'-k)$$

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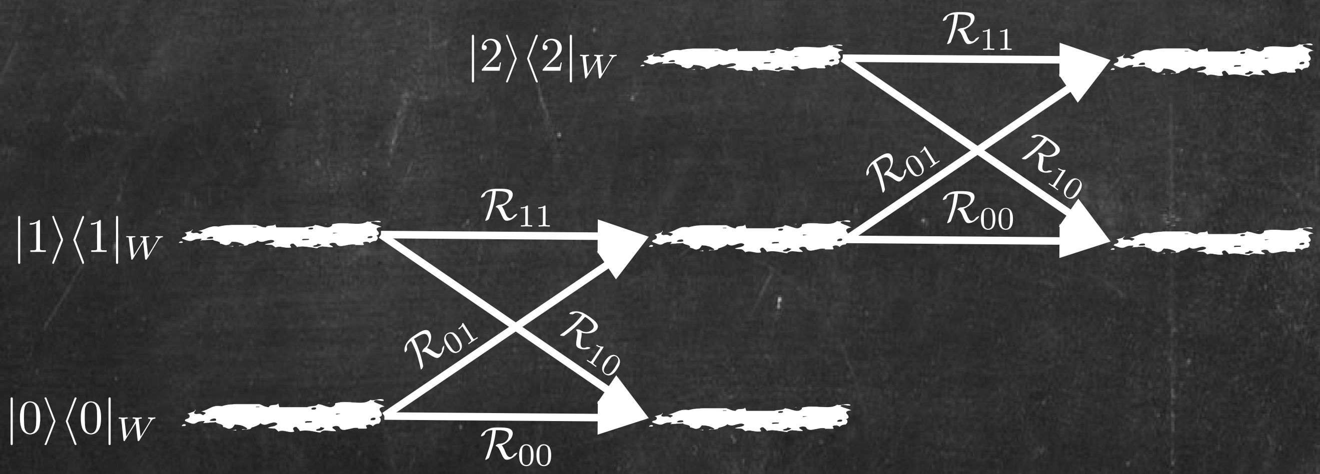
Consider a 3-level weight

Conditional average work

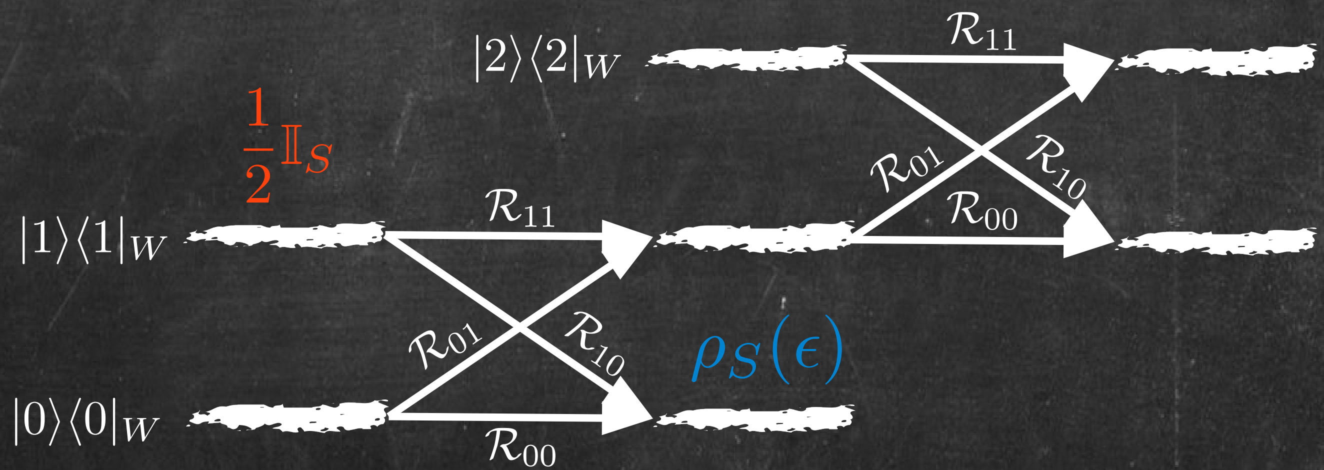
$$\langle w(\epsilon)\rangle_{s'} = \frac{1}{p(s')}\sum_{k,k'}\sum_{s} p(s)\cdot p(k)\cdot p(s'k'|sk)\cdot w\cdot(k'-k)$$



One can start in any state above ground state.

Conditional average work

$$\langle w(\epsilon)\rangle_{s'} = \frac{1}{p(s')} \sum_{k,k'} \sum_{s} p(s) \cdot p(k) \cdot p(s'k'|sk) \cdot w \cdot (k' - k)$$



Then for any $k > 0$ $\quad p(s'k'|sk) = p(s'|s)\,\delta_{k',k+1}$

Conditional average work

$$\langle w(\epsilon)\rangle_{s'} = \frac{1}{p(s')}\sum_{k,k'}\sum_{s} p(s)\cdot p(k)\cdot p(s'k'|sk)\cdot w\cdot(k'-k)$$

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$$\langle w(\epsilon)\rangle_{s'} = \frac{1}{p(s')}\sum_{k,k'}\sum_{s} p(s)\cdot p(k)\cdot p(s'k'|sk)\cdot w\cdot(k'-k)$$

$$= \frac{1}{p(s')}\sum_{k}\sum_{s} p(s)\cdot p(k)\cdot p(s'|s)\cdot w$$

Then for any $k > 0$ $\quad p(s'k'|sk) = p(s'|s)\,\delta_{k',k+1}$

Conditional average work

$$\langle w(\epsilon)\rangle_{s'} = \frac{1}{p(s')}\sum_{k,k'}\sum_{s} p(s)\cdot p(k)\cdot p(s'k'|sk)\cdot w\cdot(k'-k)$$

$$= \frac{1}{p(s')}\sum_{k}\sum_{s} p(s)\cdot p(k)\cdot p(s'|s)\cdot w$$

$$= w\,\frac{1}{p(s')}\sum_{s} p(s',s)$$

Then for any $k > 0$ $\quad p(s'k'|sk) = p(s'|s)\,\delta_{k',k+1}$

Conditional average work

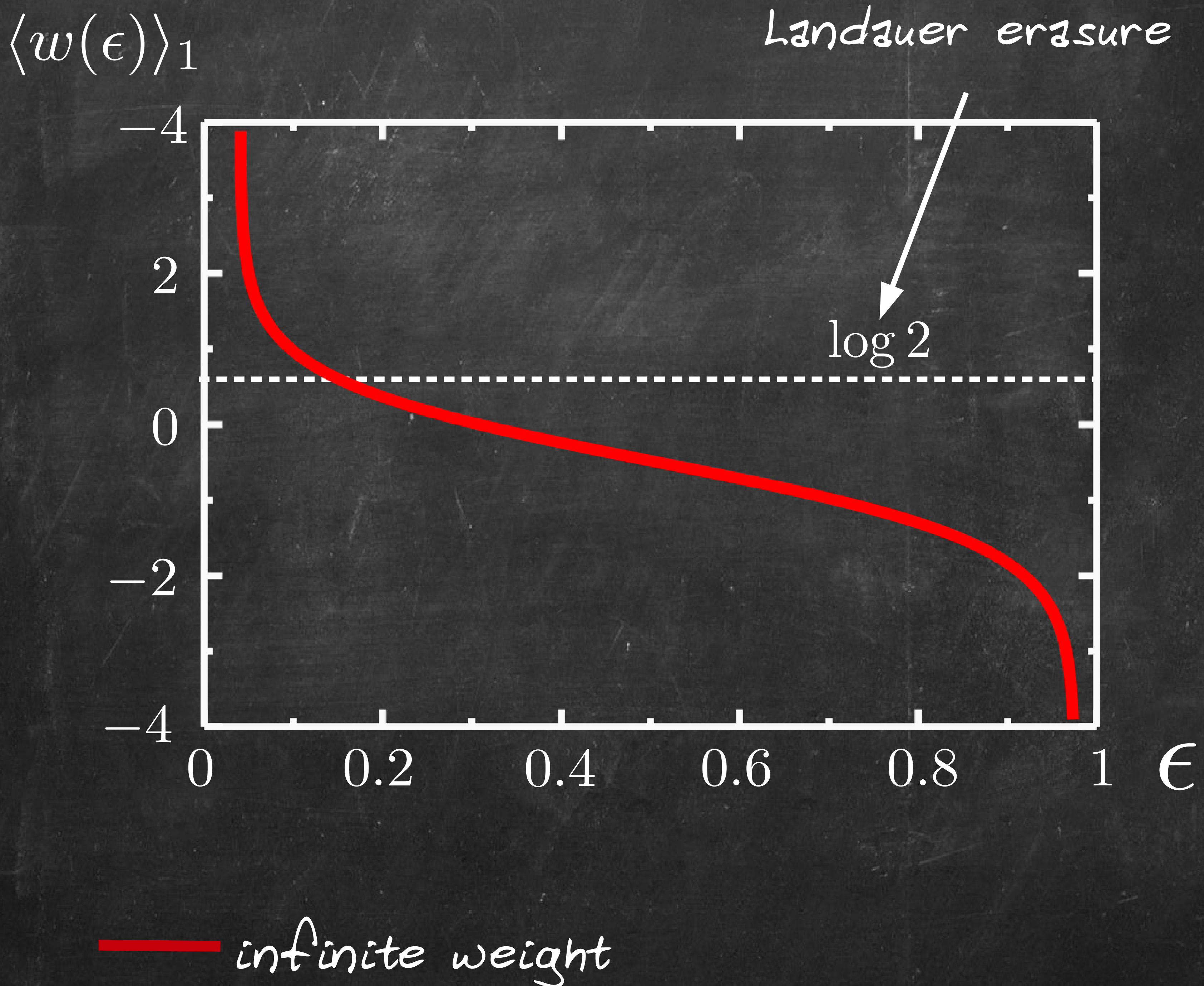
$$\langle w(\epsilon)\rangle_{s'} = \frac{1}{p(s')}\sum_{k,k'}\sum_{s} p(s)\cdot p(k)\cdot p(s'k'|sk)\cdot w\cdot(k'-k)$$

$$= \frac{1}{p(s')}\sum_{k}\sum_{s} p(s)\cdot p(k)\cdot p(s'|s)\cdot w$$

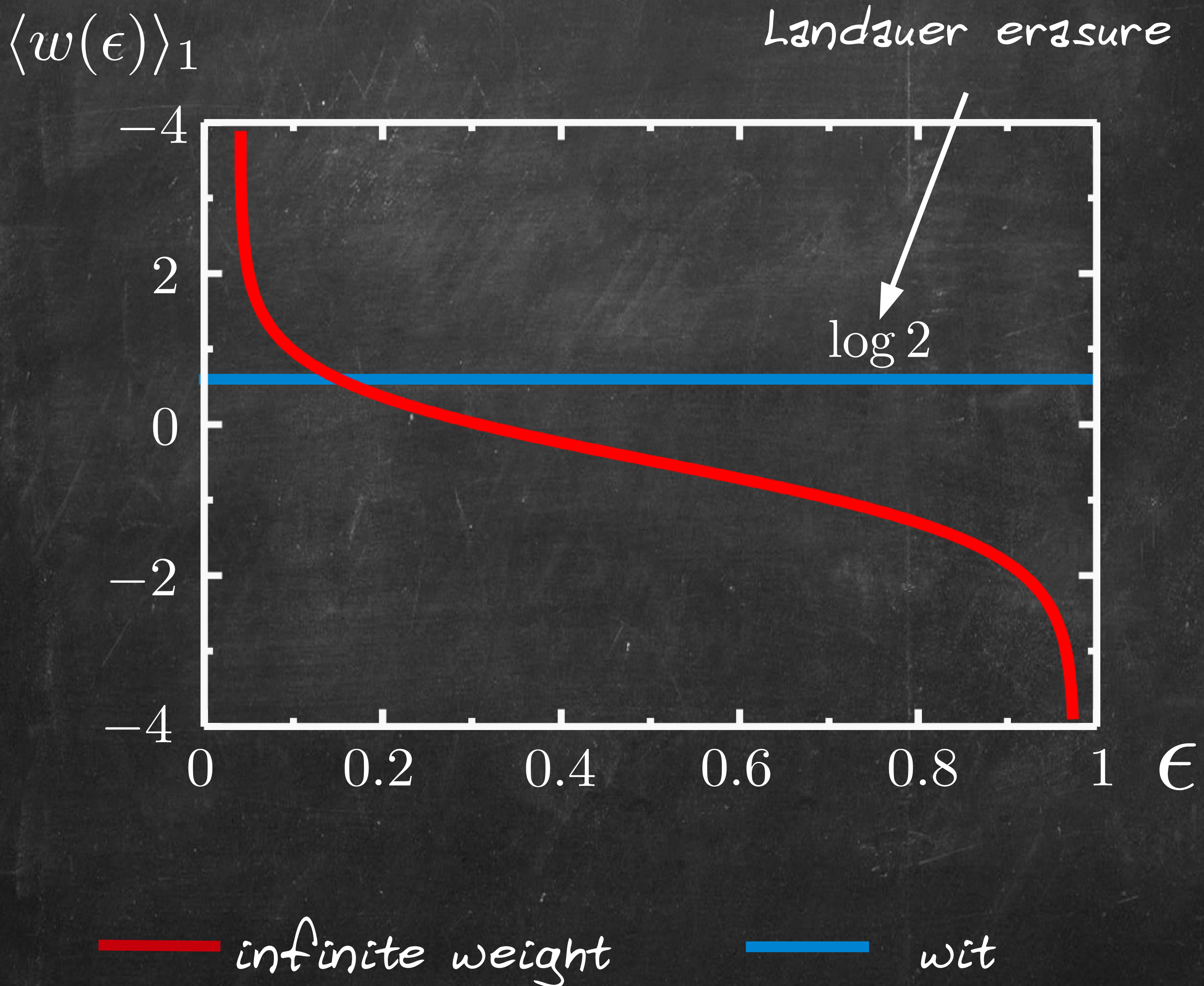
$$= w\,\frac{1}{p(s')}\sum_{s} p(s',s)$$

$$= w$$

Then for any $k > 0$ $\quad p(s'k'|sk) = p(s'|s)\,\delta_{k',k+1}$



$\langle w(\epsilon) \rangle_1$
Landauer erasure
log 2
−4
2
0
−2
−4
0
0.2
0.4
0.6
0.8
1
ϵ
infinite weight



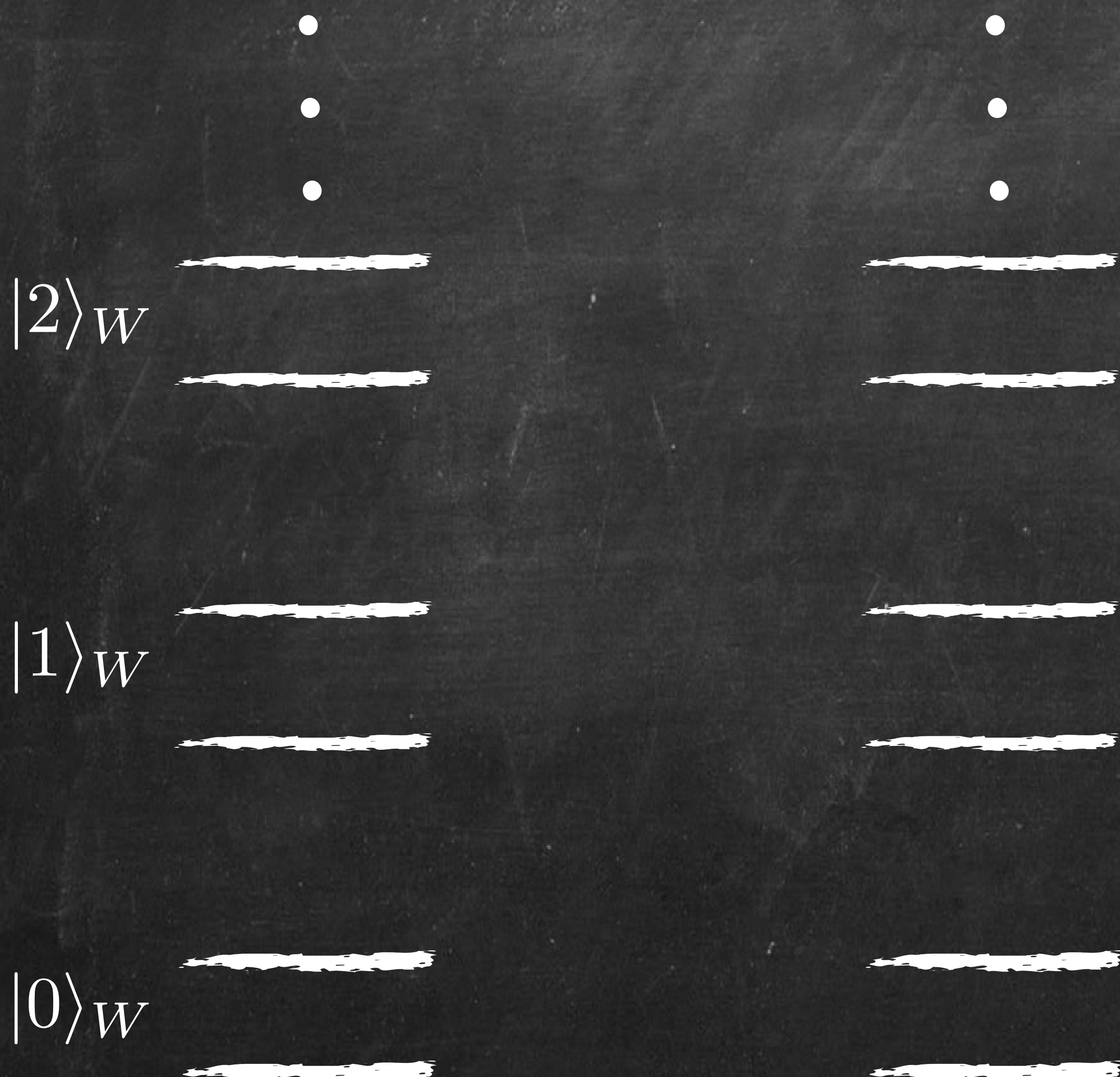
$\langle w(\epsilon)\rangle_1$
Landauer erasure
$\log 2$
4
2
0
−2
−4
0
0.2
0.4
0.6
0.8
1
ϵ
infinite weight
wit

Fluctuations

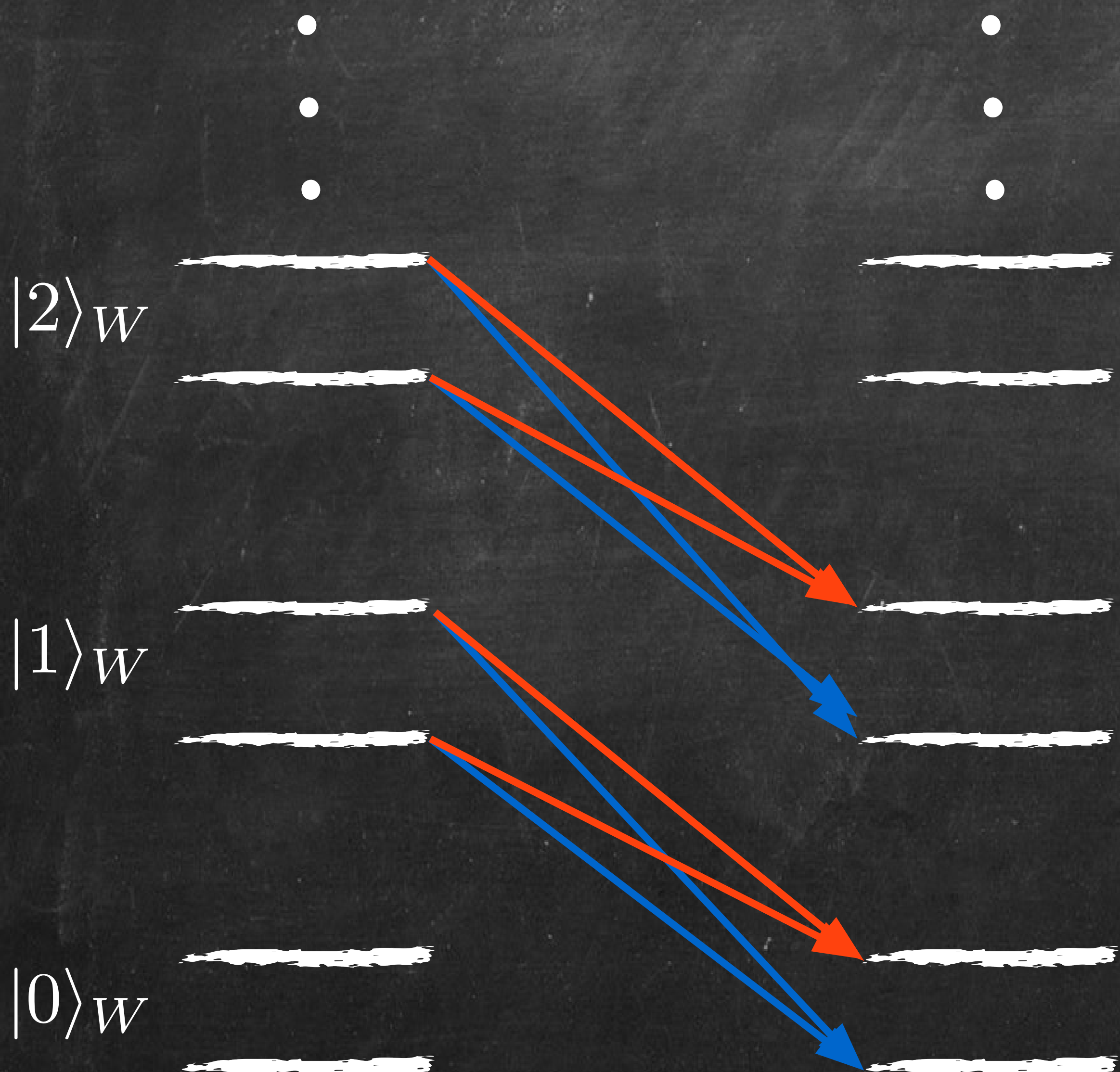
What happened to fluctuations?

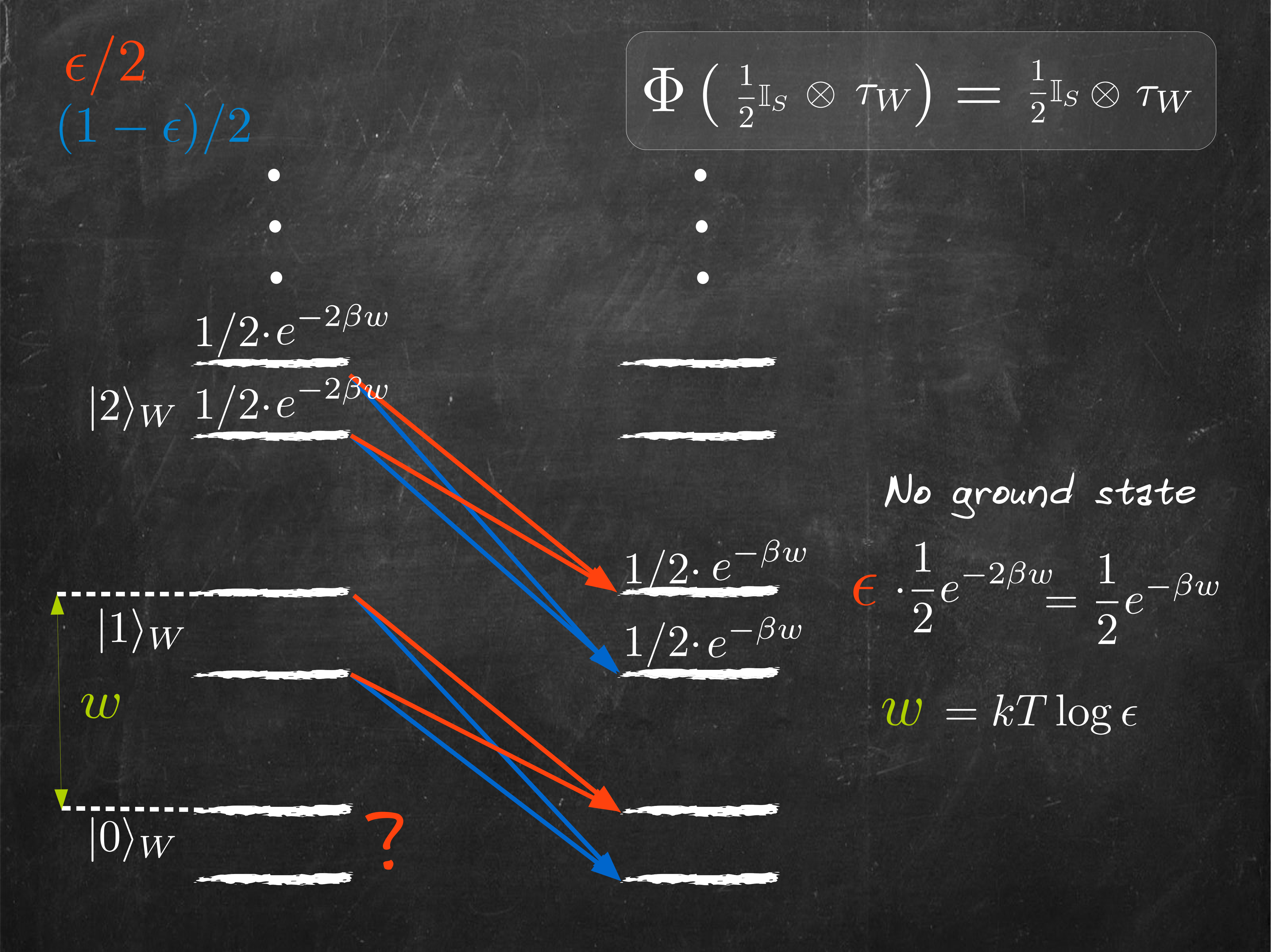
Fluctuations

Fluctuations



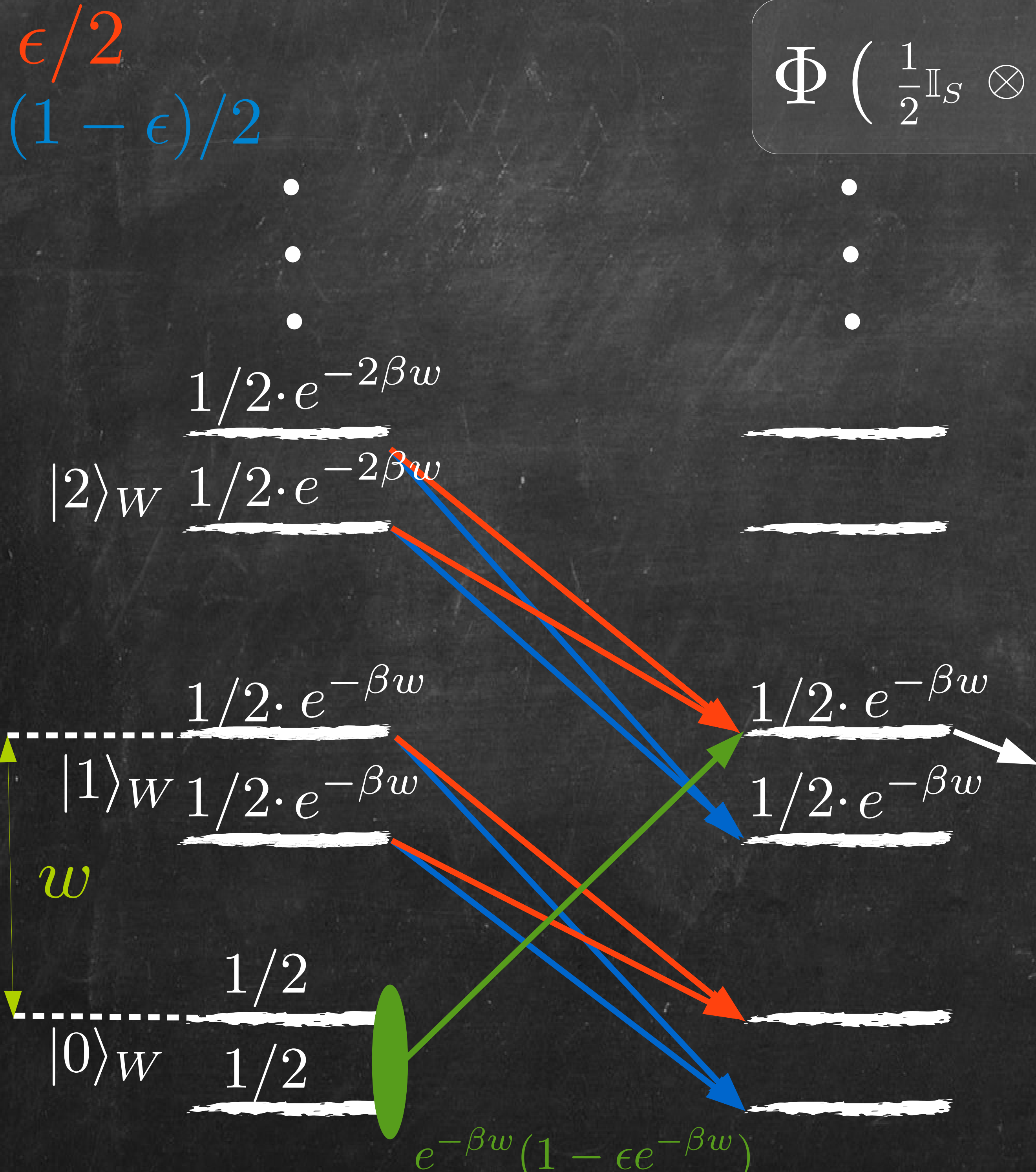
Fluctuations





$\epsilon/2$
$(1-\epsilon)/2$
$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes \tau_W\right) = \frac{1}{2}\mathbb{I}_S \otimes \tau_W$
$1/2 \cdot e^{-2\beta w}$
$|2\rangle_W$ $1/2 \cdot e^{-2\beta w}$
$|1\rangle_W$
w
$|0\rangle_W$
?
$1/2 \cdot e^{-\beta w}$
$1/2 \cdot e^{-\beta w}$
No ground state
$\epsilon \cdot \frac{1}{2} e^{-2\beta w} = \frac{1}{2} e^{-\beta w}$
$w = kT \log \epsilon$

$$\Phi\left(\frac{1}{2}\mathbb{I}_S \otimes \tau_W\right) = \frac{1}{2}\mathbb{I}_S \otimes \tau_W$$



With ground state

$$\frac{1}{2}e^{-\beta w}(1-\epsilon e^{-\beta w}) + \frac{1}{2}\epsilon\, e^{-2\beta w} = \frac{1}{2}e^{-\beta w}$$

$$w = kT \log 2$$

What if one wants to start with energies other than

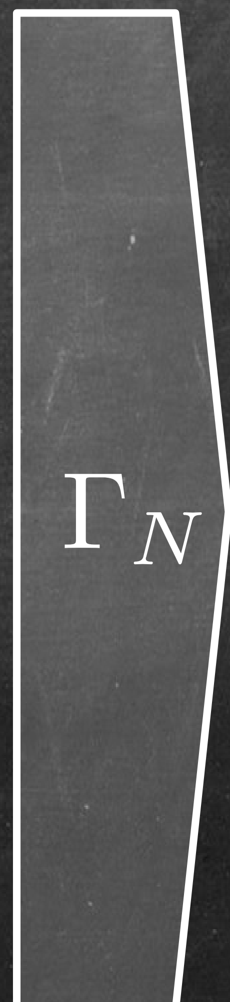
$w, \; 2w, \; 3w \ldots$?

What if one wants to start with energies other than

$w, 2w, 3w \ldots$?

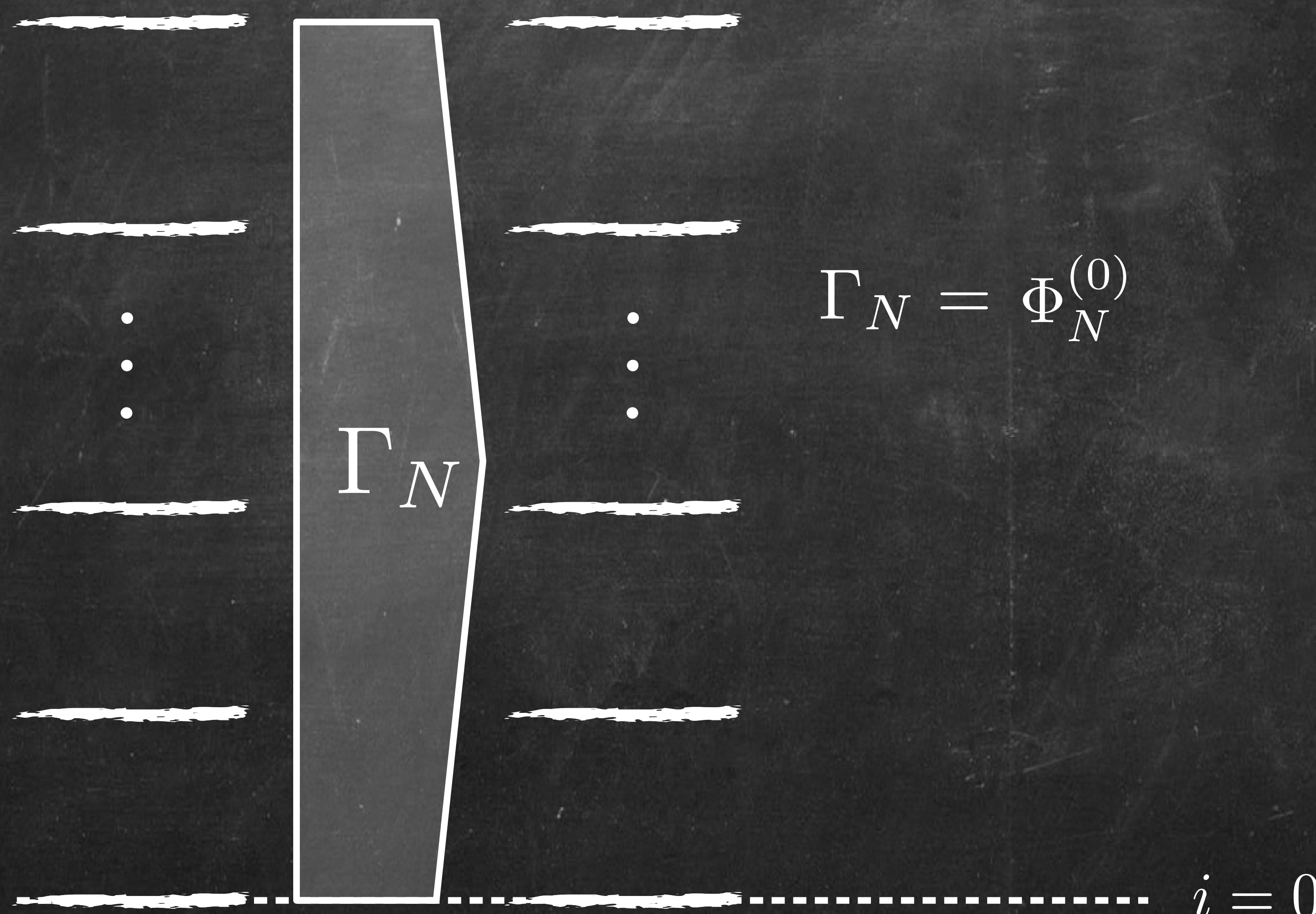
Quasi-continous weight!

Quasi-continous weight:



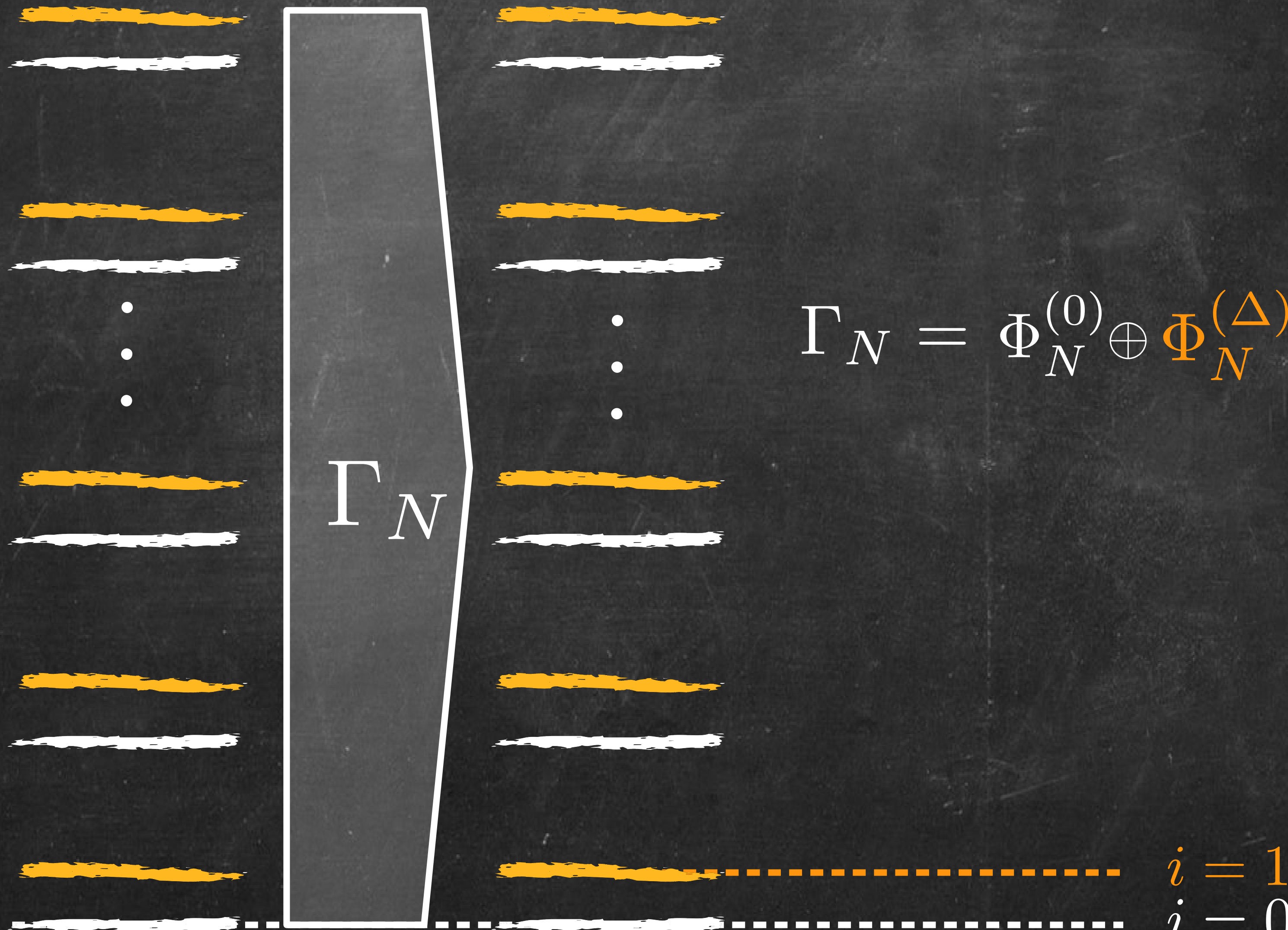
$$\Gamma_N =$$

Quasi-continous weight:



$$\Gamma_N = \Phi_N^{(0)}$$

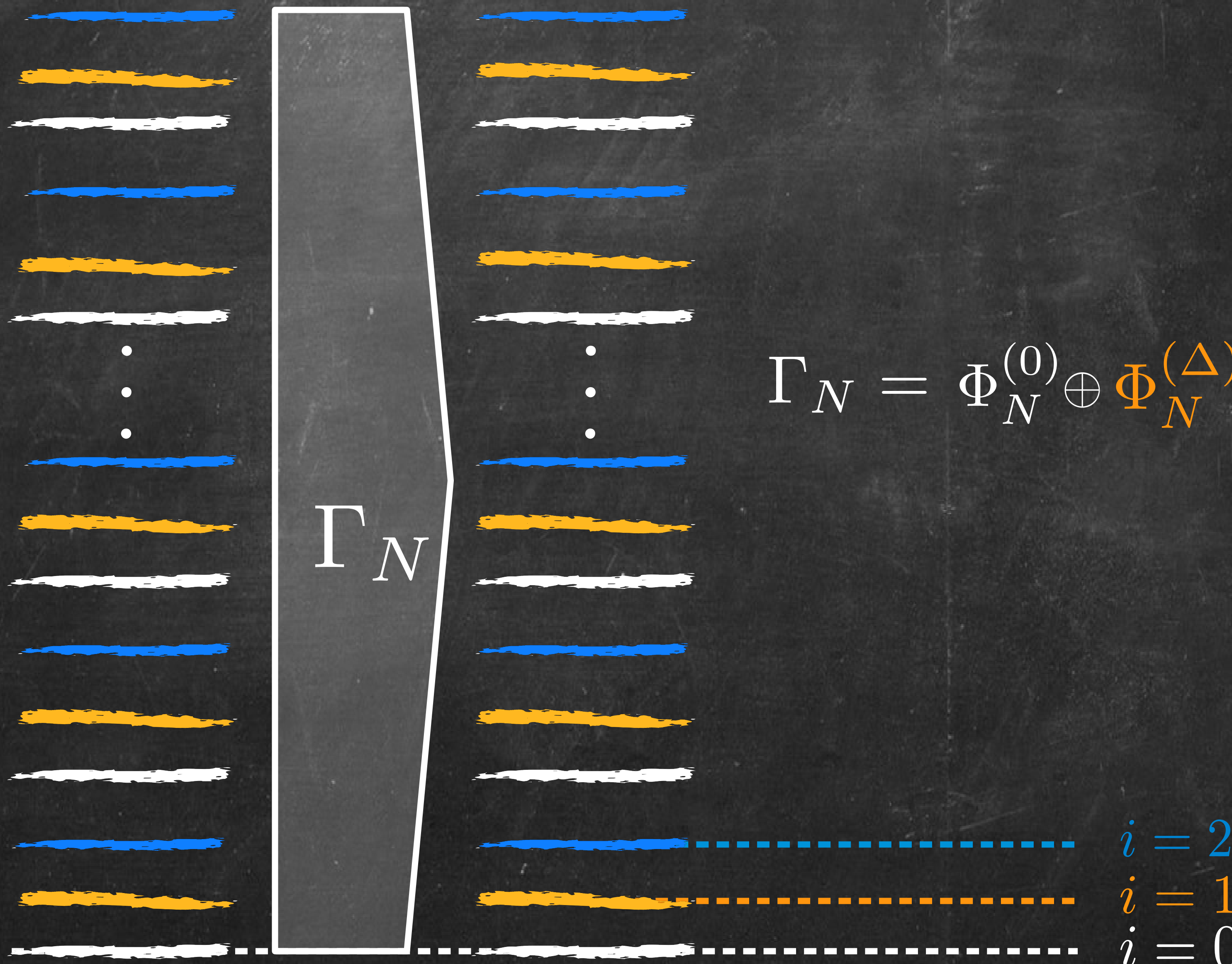
Quasi-continous weight:



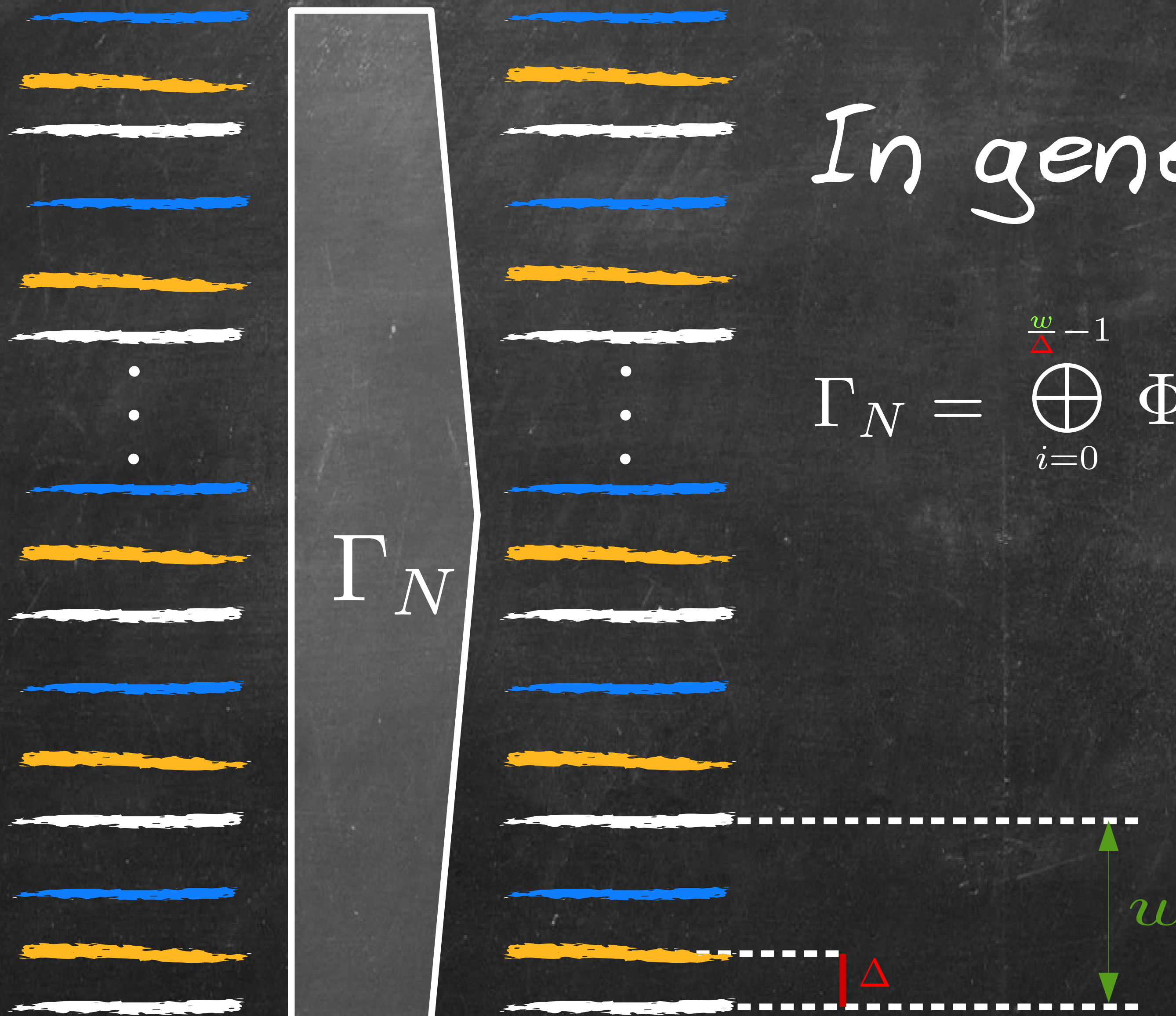
$$\Gamma_N = \Phi_N^{(0)} \oplus \Phi_N^{(\Delta)}$$

Quasi-continous weight:

$$\Gamma_N = \Phi_N^{(0)} \oplus \Phi_N^{(\Delta)} \oplus \Phi_N^{(2\Delta)}$$



Quasi-continous weight:



In general:

$$\Gamma_N = \bigoplus_{i=0}^{\frac{w}{\Delta}-1} \Phi_N^{(i\Delta)}$$

Summary

- One can extend transformation defined on „wit” to arbitrary (also infinite) N-level weights.
- One can get rid of work fluctuations in the battery by violating (just a little) translational invariance.
- Perfect erasure is possible.

Summary

- One can extend transformation defined on „wit” to arbitrary (also infinite) N-level weights.
- One can get rid of work fluctuations in the battery by violating (just a little) translational invariance.
- Perfect erasure is possible.
- Final message:
- Fluctuations can be hidden in the ground state of the battery.

Quantum Metrology and Thermodynamics

2 x PhD + 1 x Postdoc

positions

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Thank you!