Autonomous Quantum Machines & Finite sized Clocks

Institute Henri Poincare , 10/05/2017

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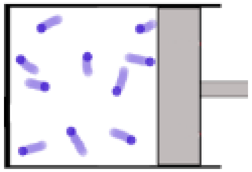
ArXiv 1607.04591

How to perform unitaries with Quantum Clocks?

- Control in Macro. thermodynamics:



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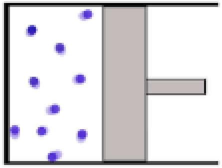

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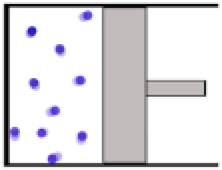

- Control in Q. thermodynamics:

$$\rho_s$$

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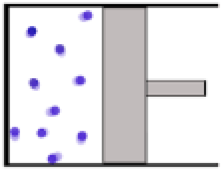

- Control in Q. thermodynamics:

$$\rho_s \quad \rightarrow \quad U\rho_s U^\dagger \qquad \text{where} \quad U = e^{-i\hat{H}(t)}$$

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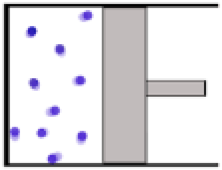

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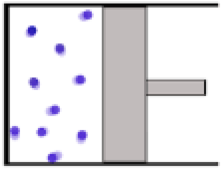

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    - $\Delta S = 0$
    - $\Delta E = 0$

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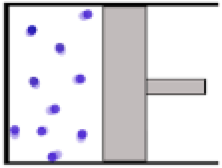

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- Are there hidden costs?

  - RT based on U:
    - $\Delta S = 0$
    - $\Delta E = 0$

  - Model explicitly control unit:

$$\rho_{sc}(t) = e^{-it\hat{H}_{sc}}\rho_s \otimes \rho_c \, e^{it\hat{H}_{sc}}$$

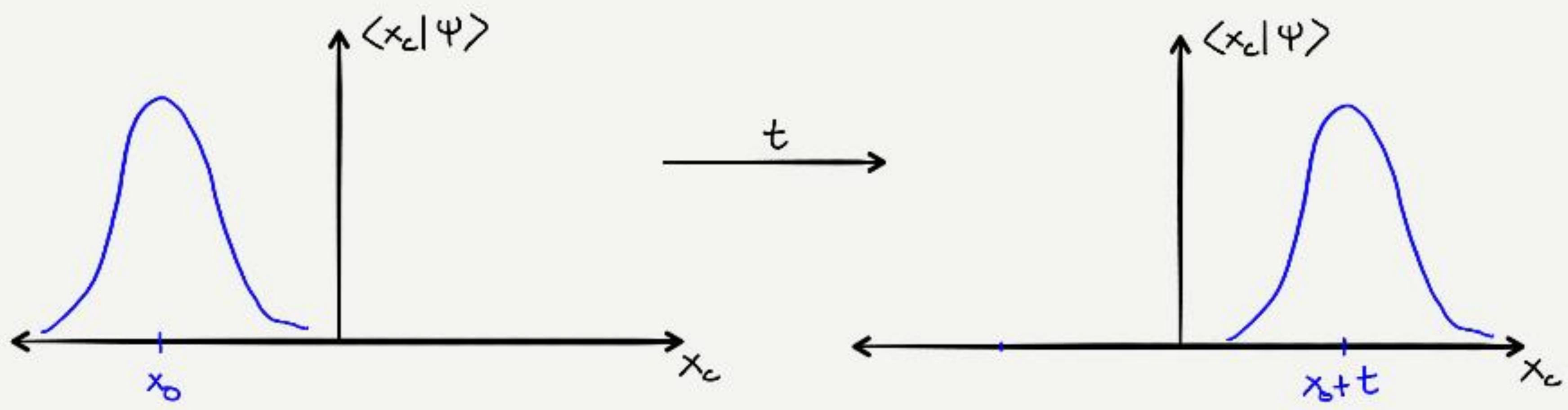
  - Therefore let ρ_c be a quantum clock

What is an Idealised Quantum Clock?

- Idealised Clocks: how they work

$$\hat{H}_c = \hat{P}$$

$$\langle x|\Psi(t)\rangle = \langle x - t|\Psi(0)\rangle$$



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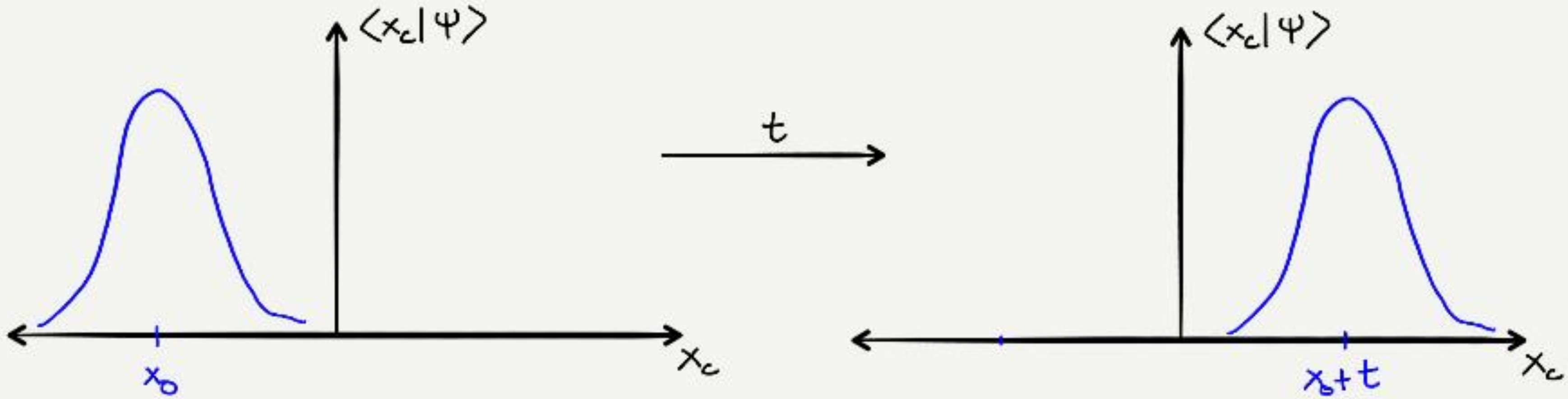
Achievable only in Infinite energy limit



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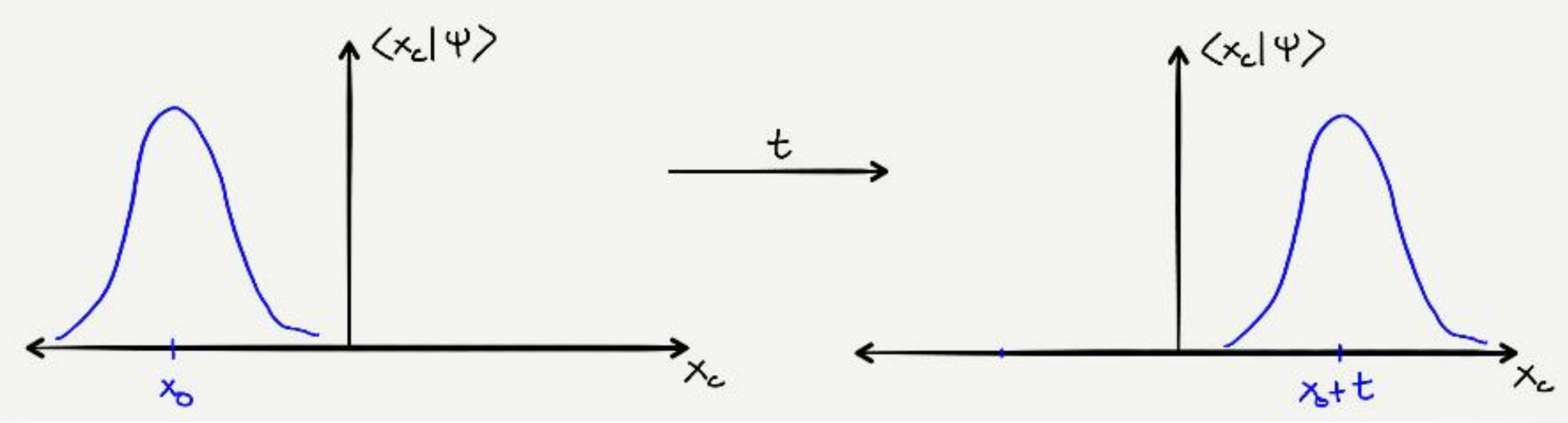

Not a useful clock yet!

What is an Idealised Quantum Clock?

- Idealised Clocks: how they work

$$\hat{H}_c = \hat{P} + V(\hat{x}_c) \qquad V : \mathbb{R} \to \mathbb{R}$$

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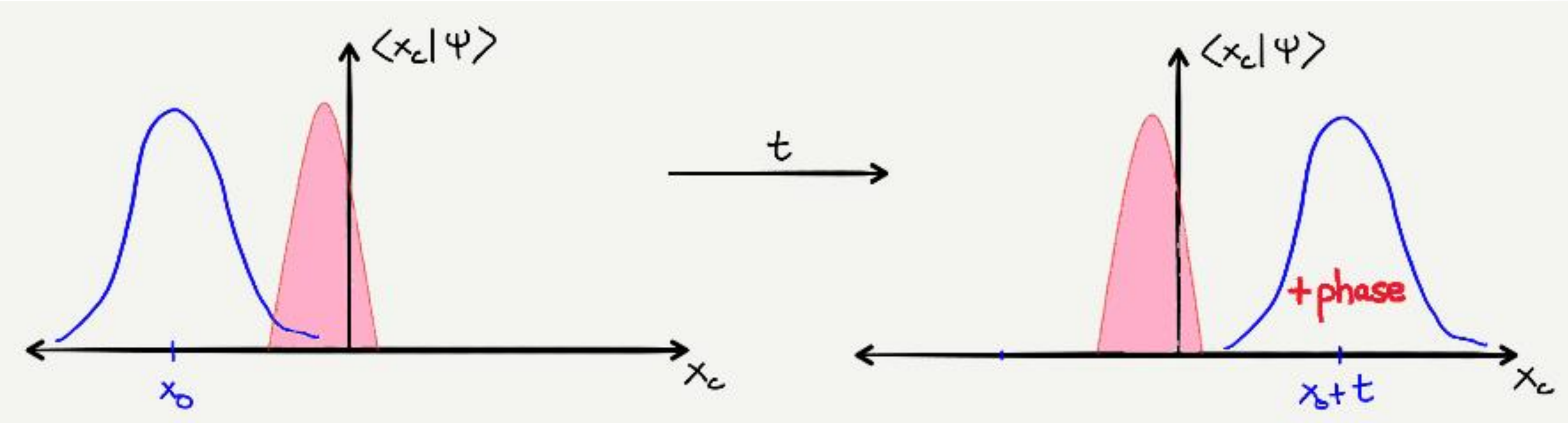
$$\hat{H}_c = \hat{P} + V(\hat{x}_c) \qquad V : \mathbb{R} \to \mathbb{R}$$

$$\langle x_c | \Psi(t) \rangle = \langle x_c - t | \Psi(0) \rangle \exp\left(-i \int_{x_c - t}^{x_c} dx \, V(x) \right)$$



- Zero clock back-reaction, zero error. This in an artefact of infinite energy

What is an Idealised Quantum Clock?

- How to make a process autonomous

- Given $|\phi\rangle_s$, want $U(t) = e^{-i\int_0^t \hat{H}_s^{int}(x')dx'}$
- Construct *t*-independent:

$$\hat{H}_{sc} = \mathbb{1}_s \otimes \hat{p}_c + \hat{H}_s^{int}(\hat{x}_c).$$

- Initial System-Clock state:

$$|\Phi(t=0)\rangle_{sc} = \int_{\mathbb{R}} dx\, \psi(x)\, |x\rangle_c \otimes |\phi\rangle_s$$

- Solution:

$$|\Phi(t)\rangle_{sc} = \int_{\mathbb{R}} dx\, \psi(x-t)\, |x\rangle_c \otimes \left[e^{-i\int_{x-t}^{x} \hat{H}_s^{int}(x')dx'}\right] |\phi\rangle_s$$

- E.g. if $\psi(x,0) = \delta(x,0)$

$$|\Phi(t)\rangle_s = e^{-i\int_0^t \hat{H}_s^{int}(x')dx'}\, |\phi\rangle_s = U(t)\, |\phi\rangle_s$$

History of Idealised Quantum Clocks

- Time in Quantum Mechanics

- W. Pauli: *does there exist a perfect clock in Q.M.?* A *self-adjoint operator* $\hat{t}_c$ *and a Hamiltonian* $\hat{H}_c$ *s.t. in the Heisenberg pic,* $d\hat{t}/dt = 1$*?*

No, since only solution
$\hat{t}_c = \hat{x}, \ \hat{H}_c = \hat{P}$

- Thermodynamics with Idealized Clocks

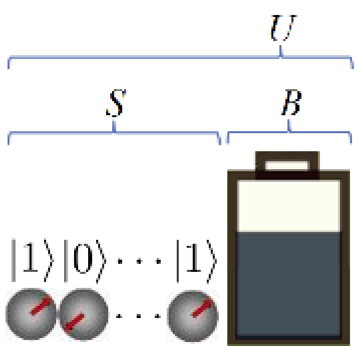
- Clock-driven Thermal engines:

  [A. Malabarba, A. J. Short, P. Kammerlander, NJP (2015)]

  Proves that the 2nd law holds in an autonomous setting with $\hat{H}_c = \hat{P}$

- Catalytic Coherences: [J. Abert, PRL (2014)]

  Proves that Q. coherences in energy are catalytic when one has access to a battery.



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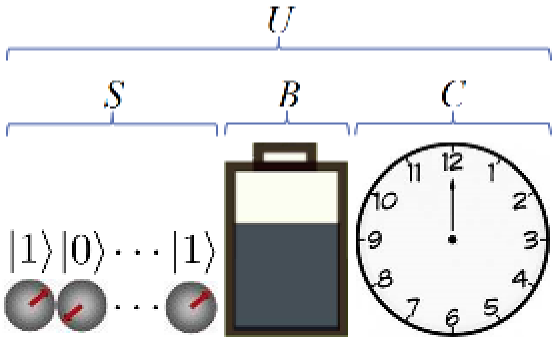

Problems with Idealised Quantum clocks

-Infinite energy approximation disguises true costs

- They never degrade: i.e. zero back reaction on the clocks
- Can Perform any unitry instantaneously with zero error
- Have infinite capacity to embezzle work
- Can tell the time perfectly for ever

Finite clocks

- Salecker-Wigner & Peres Clocks [1]

$$H = \sum_{n=0}^{d-1} n\omega \, |n\rangle\langle n| \, ,$$

Angle basis: $$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i2\pi nk/d} \, |n\rangle \, ,$$

[1]: *Quantum Limitations of the Measurement of Space-Time Distances*, (1958)
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Clocks and time in quantum Mechanics

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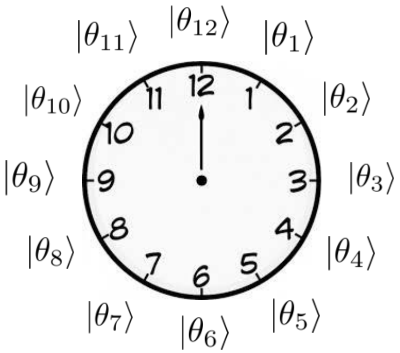
Clock period: $T_0 = \frac{2\pi}{\omega}$

$d = 12$



Initial Clock state: $|\theta_{12}\rangle$

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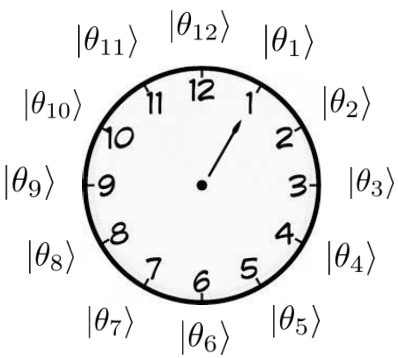

At $t = T_0 \dfrac{1}{d}$: $\quad \mathrm{e}^{-\mathrm{i}tH_c} \, |\theta_{12}\rangle = |\theta_1\rangle$

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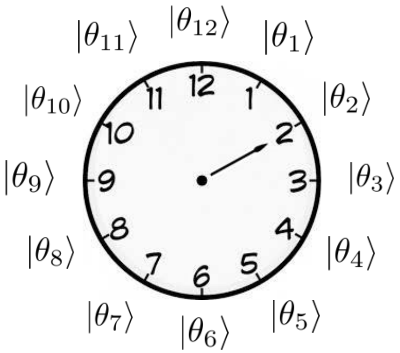
At $t = T_0 \frac{2}{d}$: $$\mathrm{e}^{-\mathrm{i}tH_c} \, |\theta_{12}\rangle = |\theta_2\rangle$$

$d = 12$



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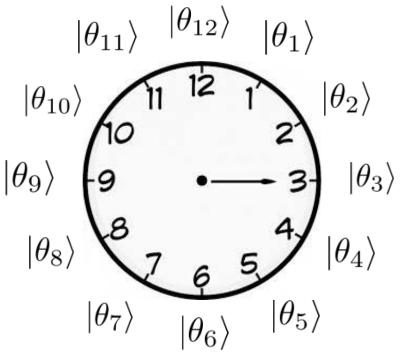

At $t = T_0 \dfrac{3}{d}$: $\quad \mathrm{e}^{-\mathrm{i}tH_c} \, |\theta_{12}\rangle = |\theta_3\rangle$

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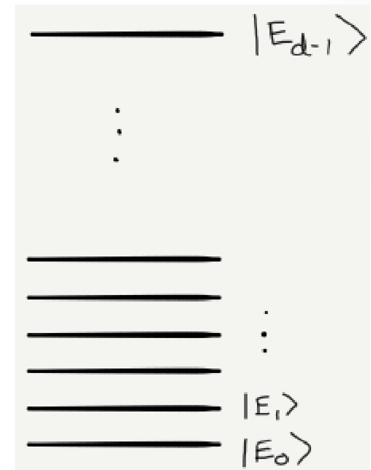
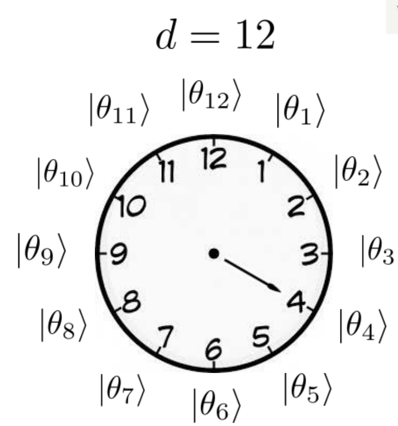


Angle basis: $$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i2\pi nk/d} \, |n\rangle \,,$$

At $t = T_0 \frac{4}{d}$: $$\mathrm{e}^{-\mathrm{i}tH_c} \, |\theta_{12}\rangle = |\theta_4\rangle$$



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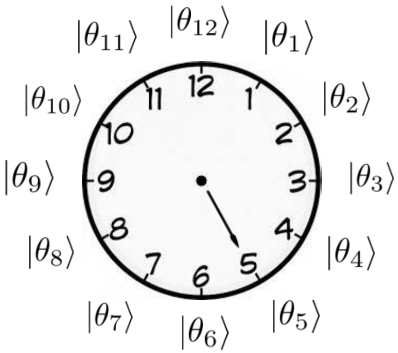

At $t = T_0 \frac{5}{d}$: $\mathrm{e}^{-\mathrm{i}tH_c} \, |\theta_{12}\rangle = |\theta_5\rangle$

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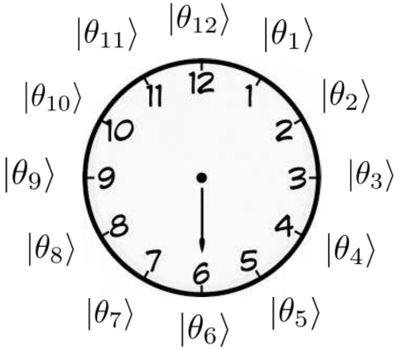

At $t = T_0 \dfrac{6}{d}$: $\quad \mathrm{e}^{-\mathrm{i}tH_c} \, |\theta_{12}\rangle = |\theta_6\rangle$

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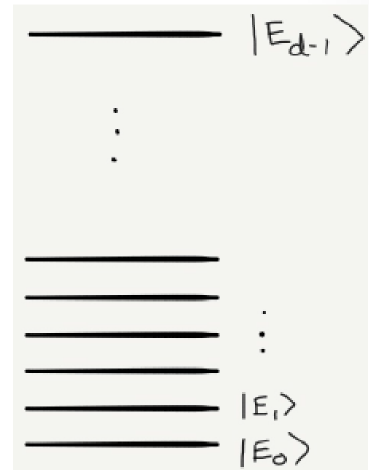

Angle basis: $$|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} e^{-i2\pi nk/d} \, |n\rangle \,,$$

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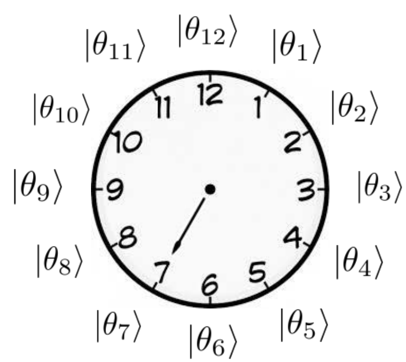

At $t = T_0 \frac{7}{d}$: $$\mathrm{e}^{-\mathrm{i}tH_c} \, |\theta_{12}\rangle = |\theta_7\rangle$$

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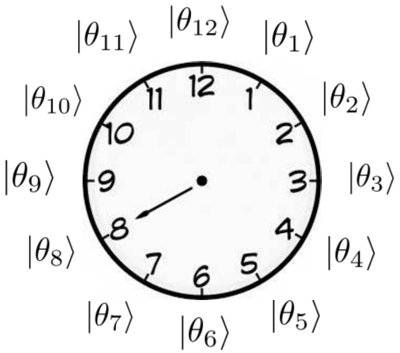

At $t = T_0 \frac{8}{d}$: $\quad e^{-itH_c} \, |\theta_{12}\rangle = |\theta_8\rangle$

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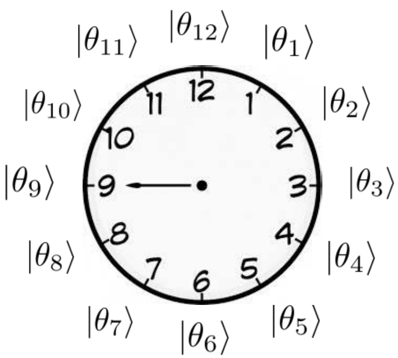
$d=12$



At $t=T_0\dfrac{9}{d}$: $\quad \mathrm{e}^{-\mathrm{i}tH_c}\,|\theta_{12}\rangle = |\theta_9\rangle$

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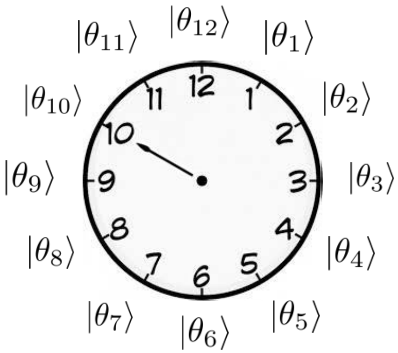

At $t = T_0 \dfrac{10}{d}$: $\quad e^{-itH_c} \, |\theta_{12}\rangle = |\theta_{10}\rangle$

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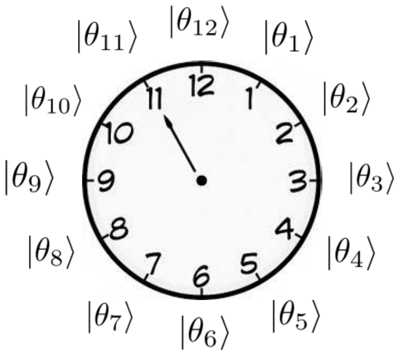

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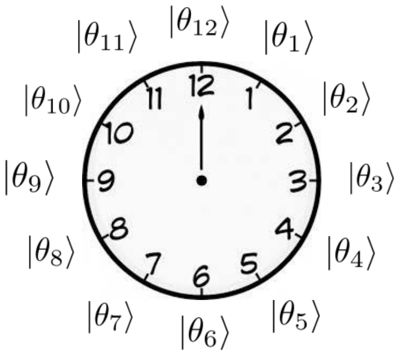

At $t = T_0 \dfrac{12}{d}$: $$\mathrm{e}^{-\mathrm{i}tH_c} \, |\theta_{12}\rangle = |\theta_{12}\rangle$$

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Angle States are not good Quantum Clocks

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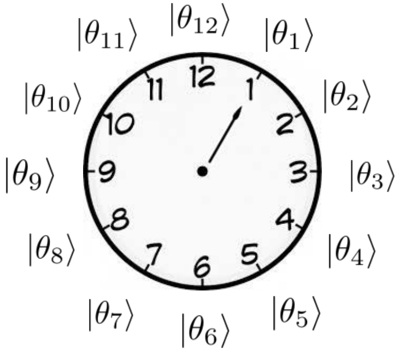

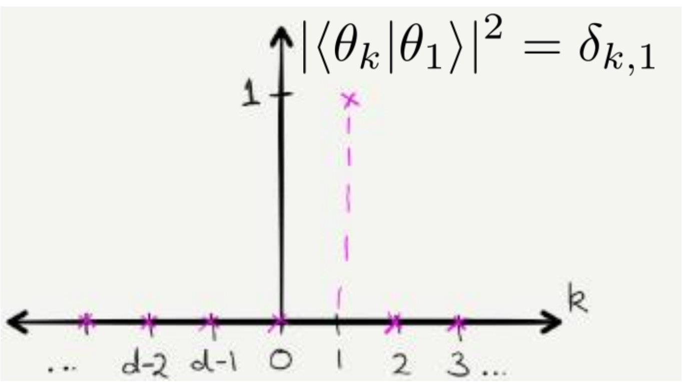

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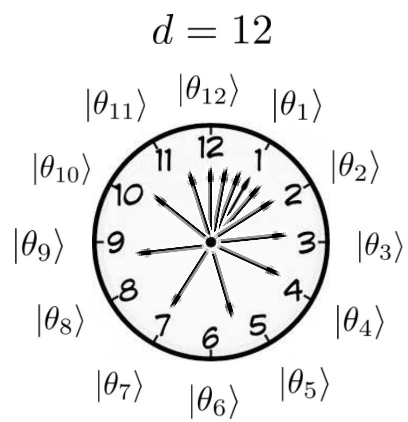
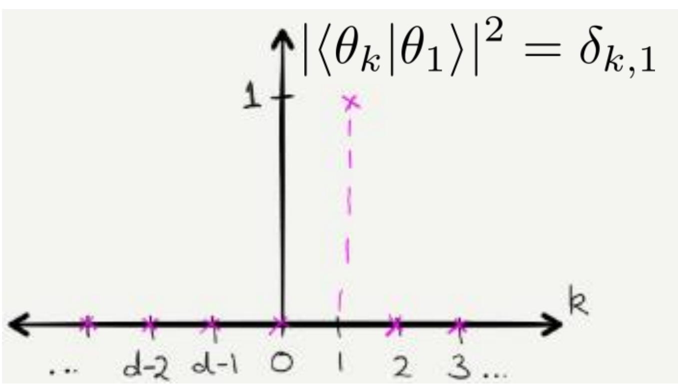
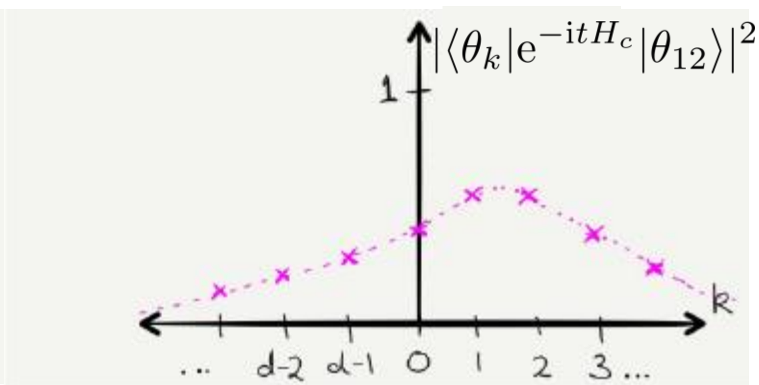
At $t = (1+\frac{1}{2})\frac{T_0}{d} \quad e^{-itH_c}\,|\theta_{12}\rangle = |\psi\rangle$







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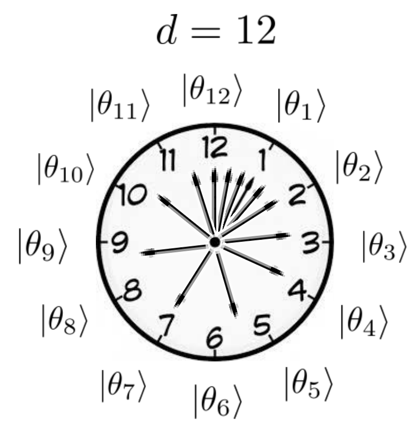
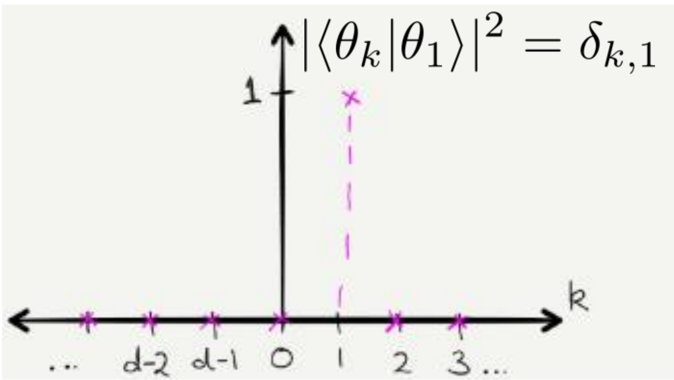
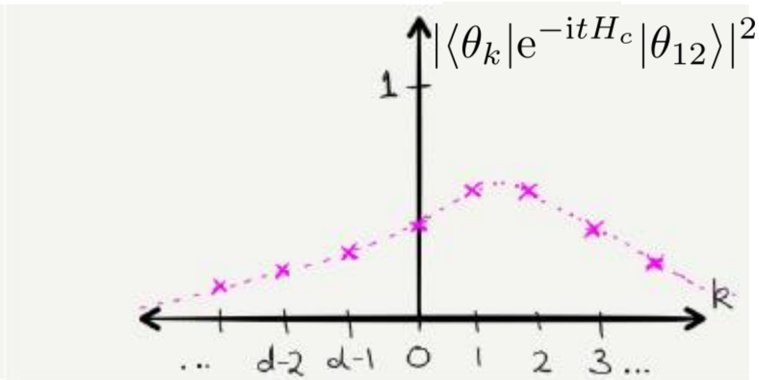
Angle States are not good Quantum Clocks

- Salecker-Wigner & Peres Clocks [1]



At $t = (1 + \frac{1}{2})\frac{T_0}{d} \quad \mathrm{e}^{-\mathrm{i}tH_c}|\theta_{12}\rangle = |\psi\rangle$





- Standard deviation of $|\langle\theta_k|\mathrm{e}^{-\mathrm{i}tH_c}|\theta_{12}\rangle|^2$ scales as $\sigma^2 \sim d$

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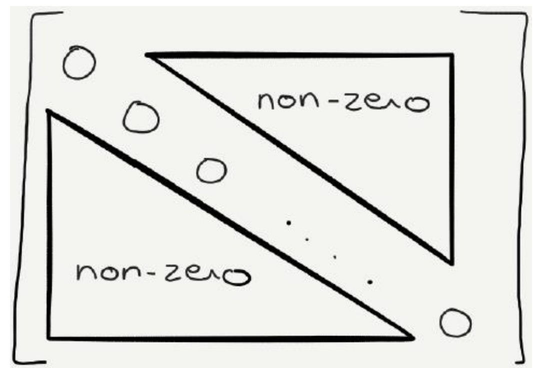
Angle States are not good Quantum Clocks

- Commutator:

    -Recall Idealised Clock: $\hat{H}_c = \hat{x},\ \hat{t}_c = \hat{p}$. Therefore $[\hat{H}_c, \hat{t}_c] = i$

    -Angle States: $\hat{t}_c = \sum_k k\, |\theta_k\rangle\langle\theta_k|$

$$\langle\theta_k|\, [\hat{H}_c, \hat{t}_c]\, |\theta_k\rangle = \langle\theta_k|$$



$$|\theta_k\rangle$$

$$= 0$$

Gaussian amplitude Quantum Clocks

- Keep finite Hamiltonian: $H = \sum_{n=0}^{d-1} n\omega \, |n\rangle\langle n| \,,$
- Replace initial angle state:

$$|\Psi(k_0)\rangle = A \sum_{k \in \mathcal{S}(k_0)} e^{-\frac{(k-k_0)^2}{2\sigma^2}} e^{-i2\pi n_0 k/d} \, |\theta_k\rangle$$

Width of mean position

Mean position

Width of mean position

mean energy

Set of d integers centred around k_0

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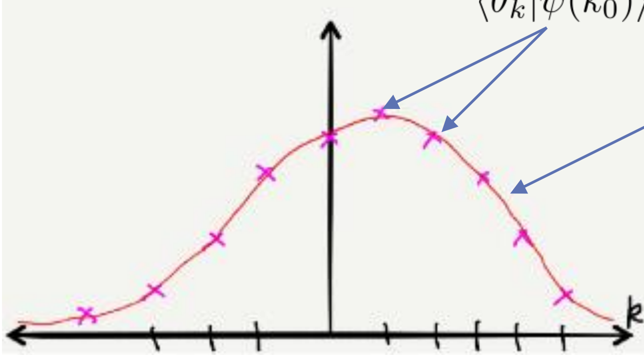
-Analytic continuation:

$$|\Psi(k_0)\rangle = \sum_{k\in\mathcal{S}_d(k_0)} \psi(k_0;k) \, |\theta_k\rangle \;; \qquad \psi(k_0;x) = A e^{-\frac{\pi}{\sigma^2}(x-k_0)^2} e^{i2\pi n_0 (x-k_0)/d}$$



$\langle\theta_k|\psi(k_0)\rangle =$ "wave function"

$\psi(k_0;x) =$ Analytic continuation of "wave function"

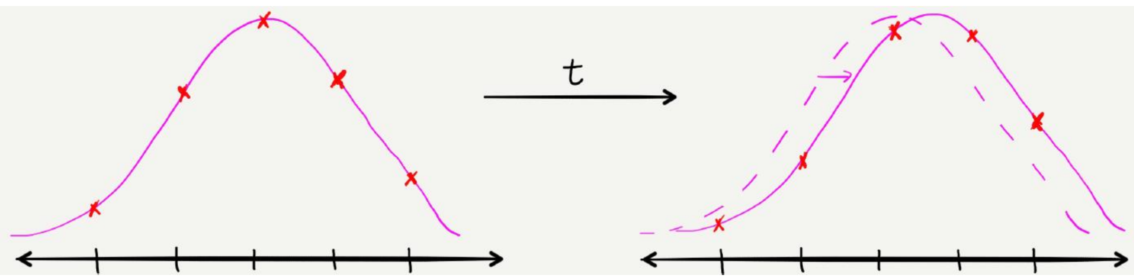
First Result: dynamic localisation

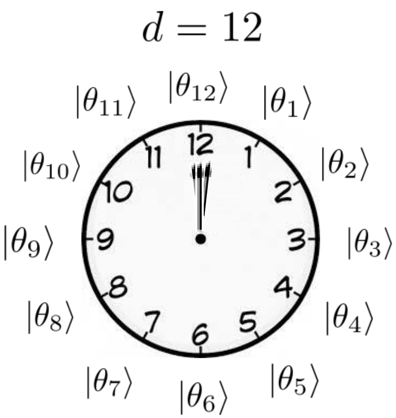
-Recall idealised clock:

$$\Psi(x_c, t) = \Psi(x_c - t, 0)$$

-Gaussian amplitude clock:

$$\langle \theta_k | \, e^{-it\hat{H}_c} \, | \Psi(k_0) \rangle = \psi(k_0 - d\frac{t}{T_0}; k) + \varepsilon_c(t)$$





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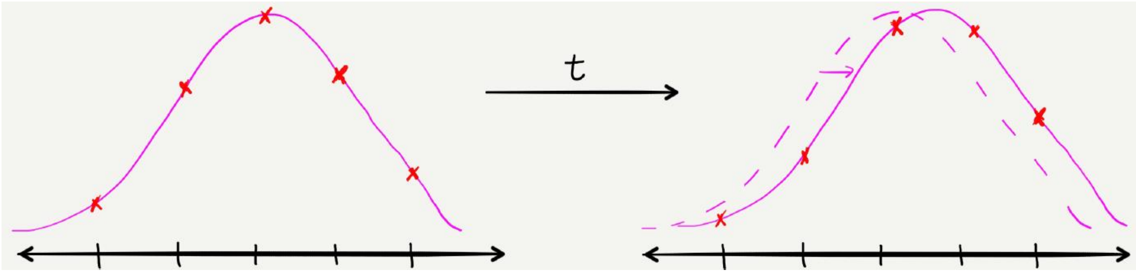

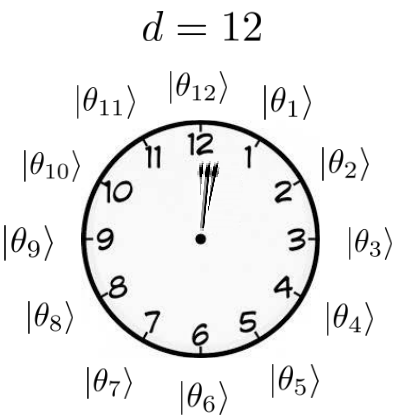

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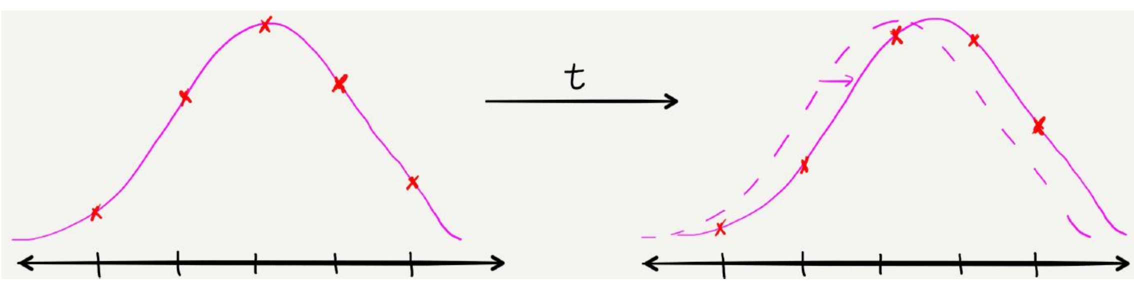

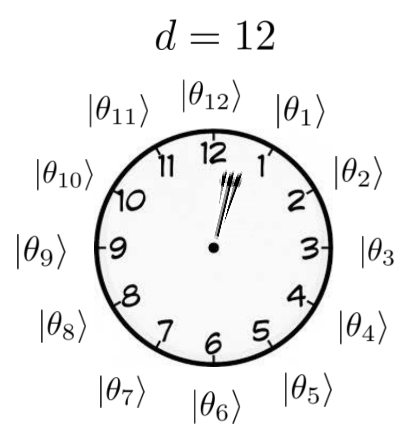

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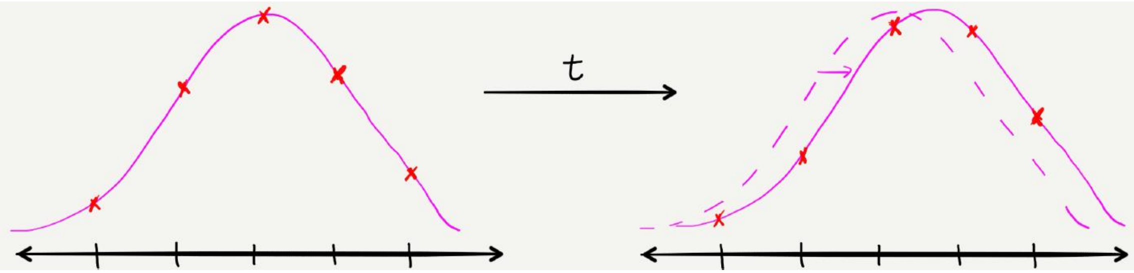

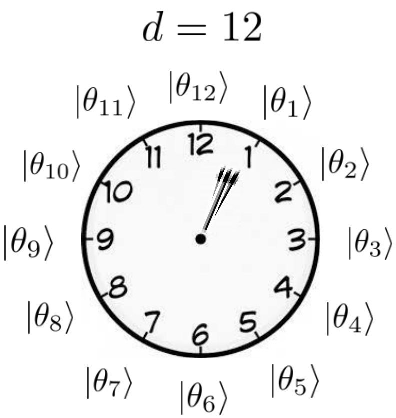

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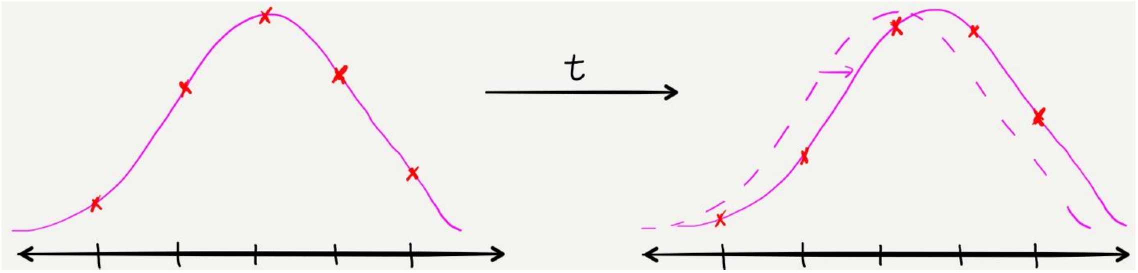

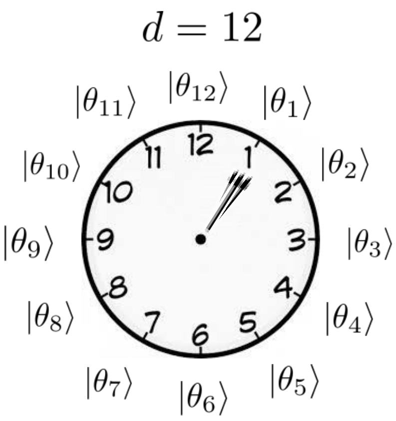

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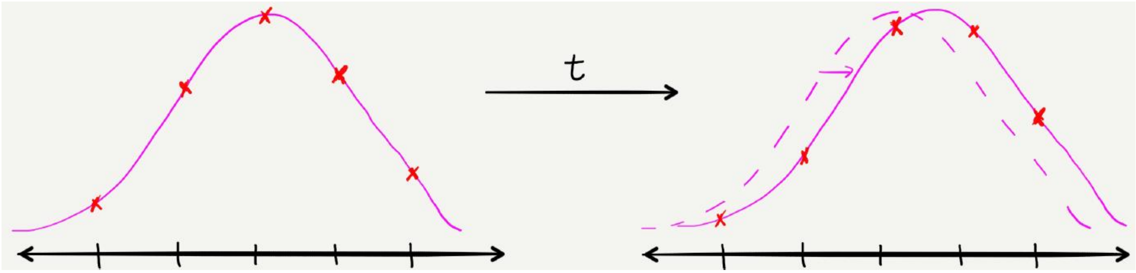

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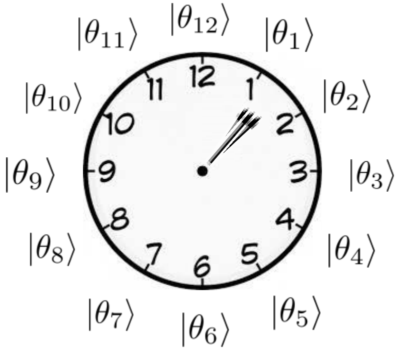

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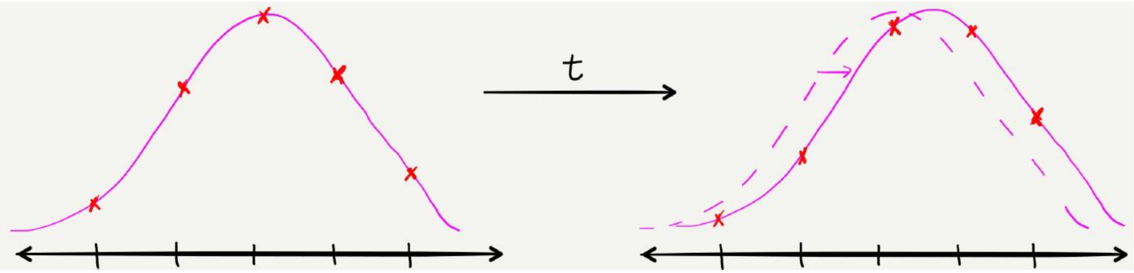

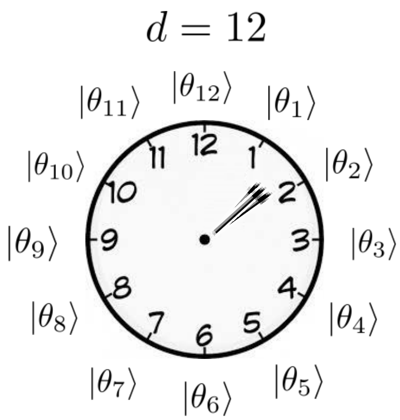

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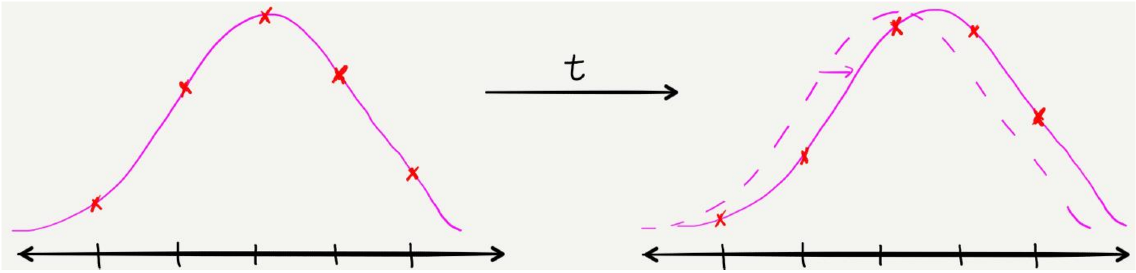

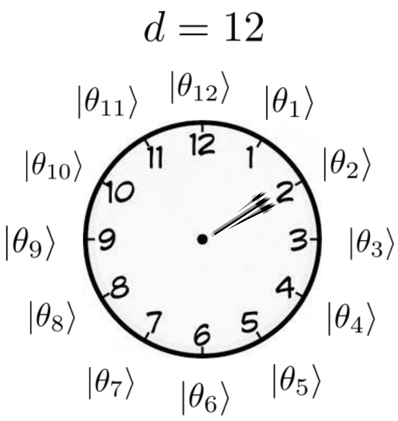

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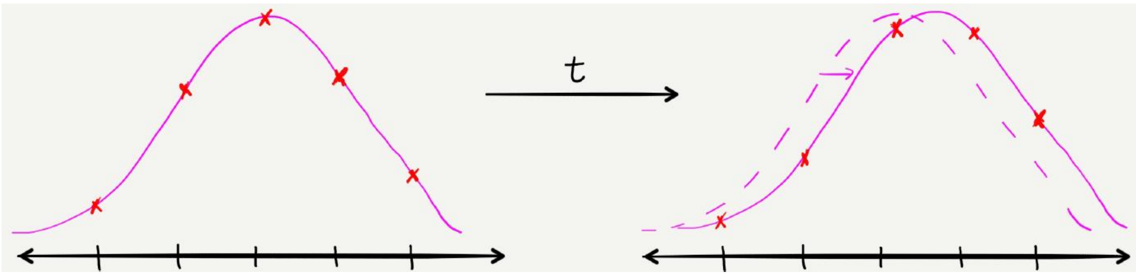

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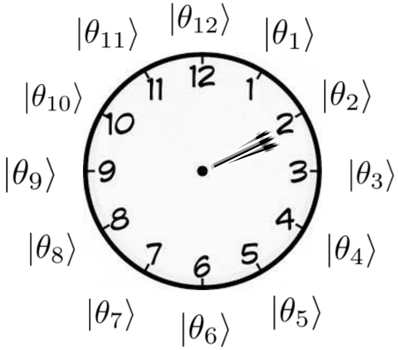

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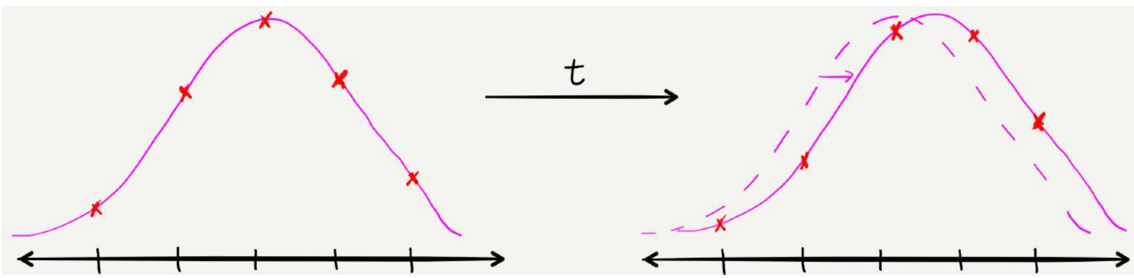

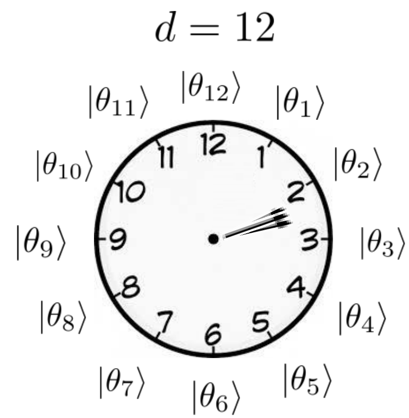

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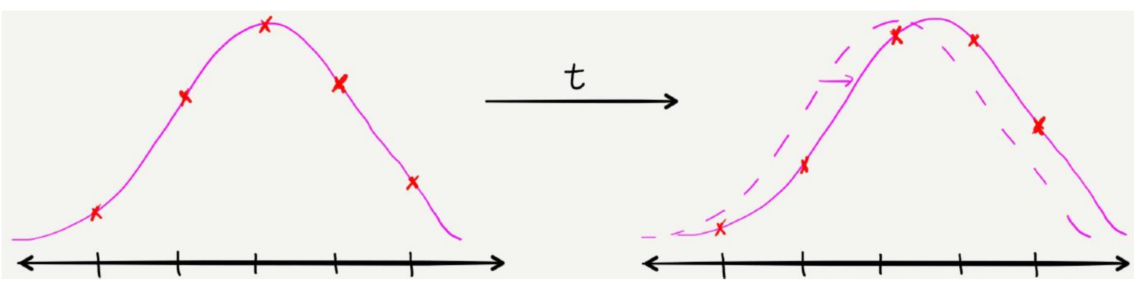

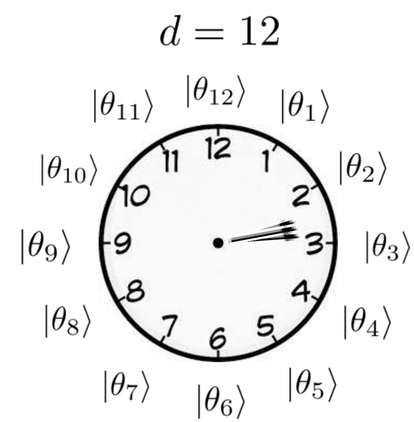

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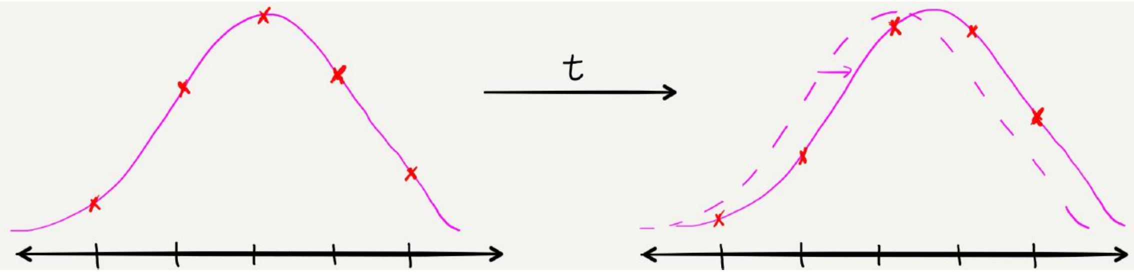

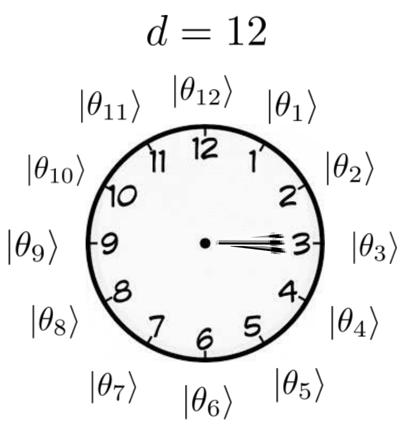

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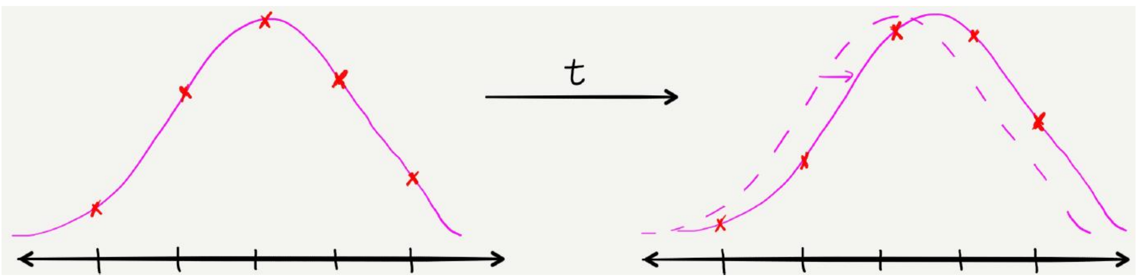

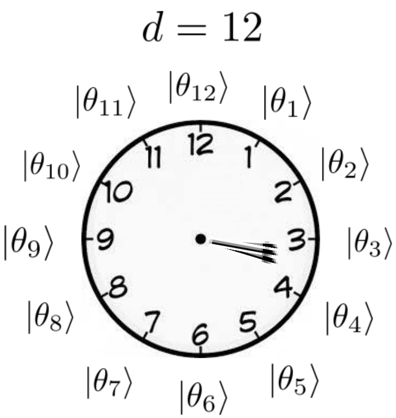

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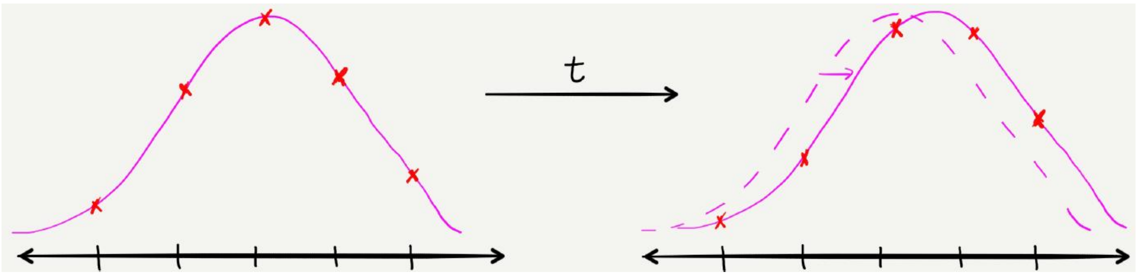

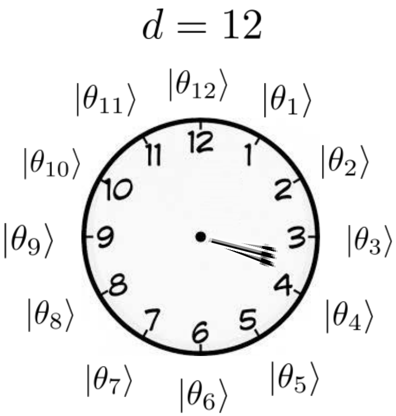

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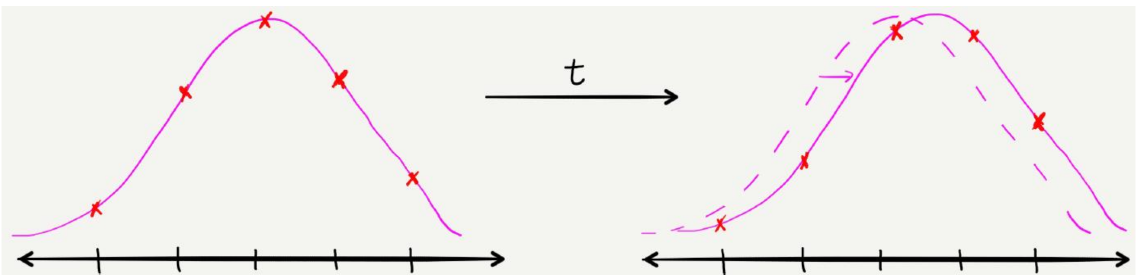

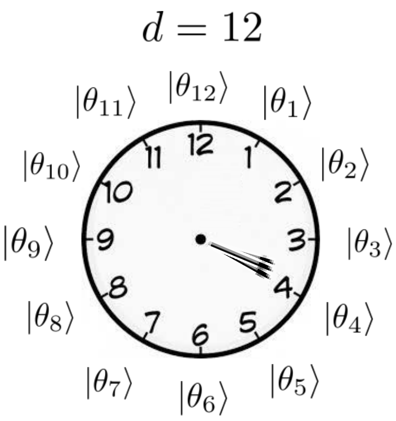

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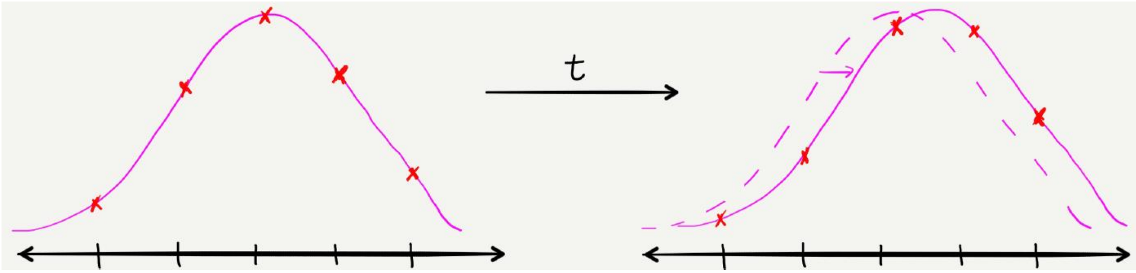

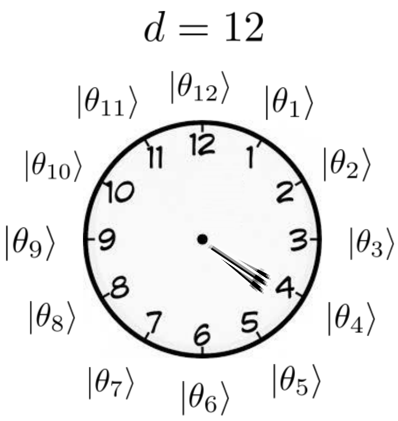

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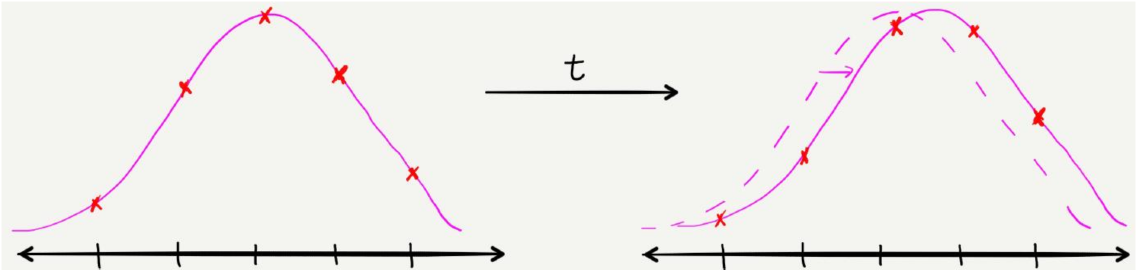

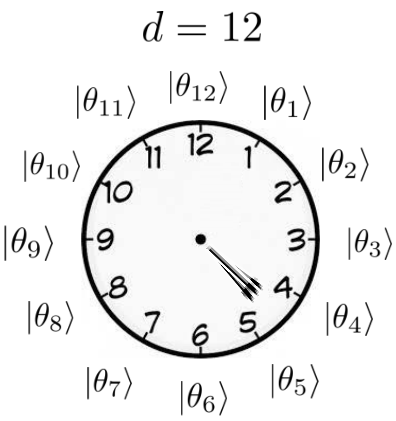

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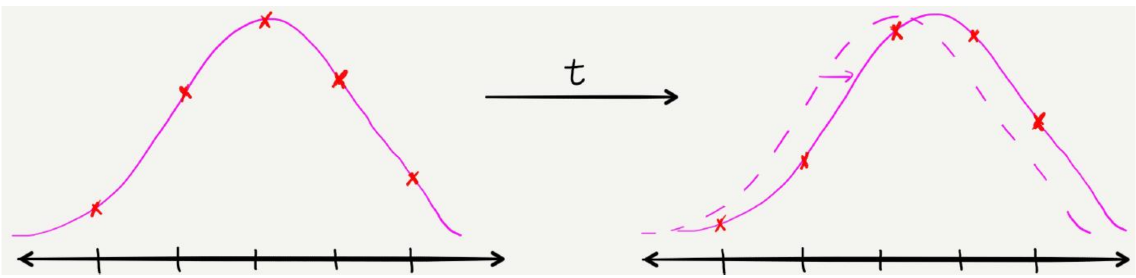

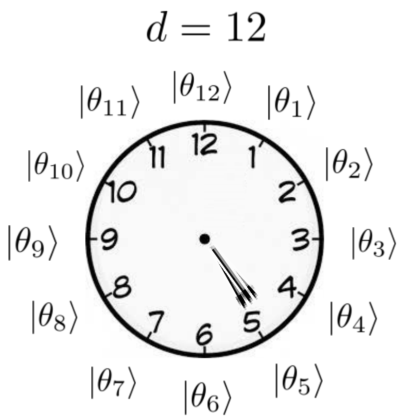

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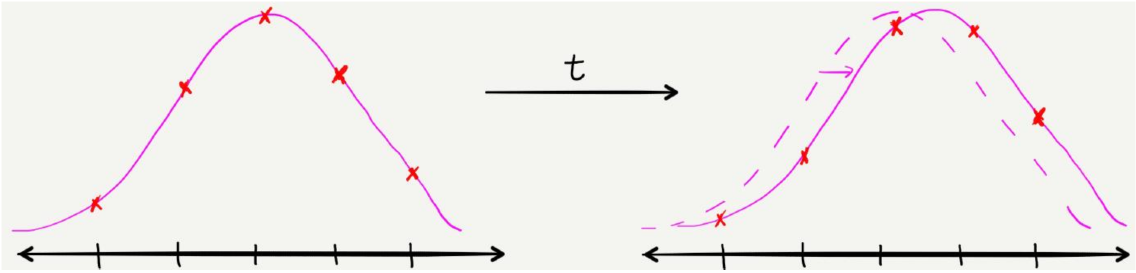

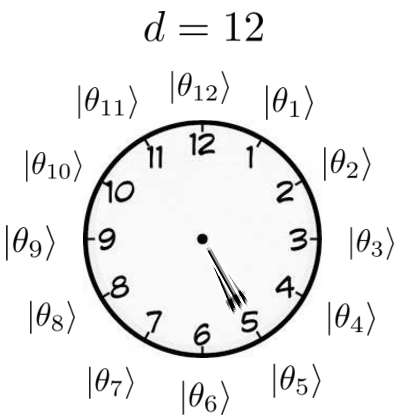

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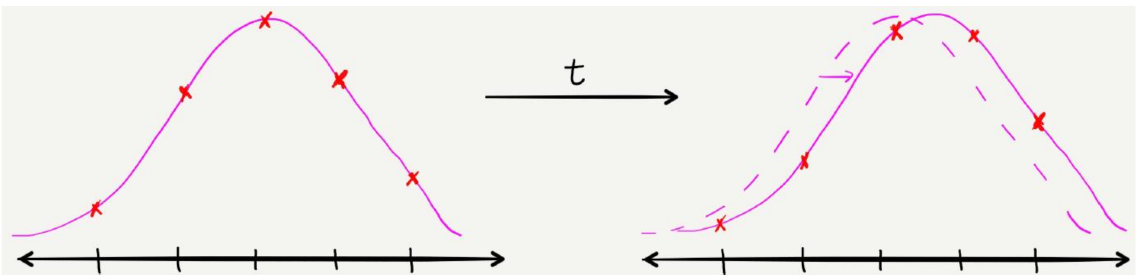

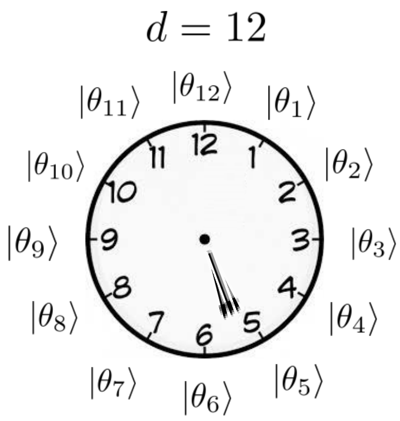

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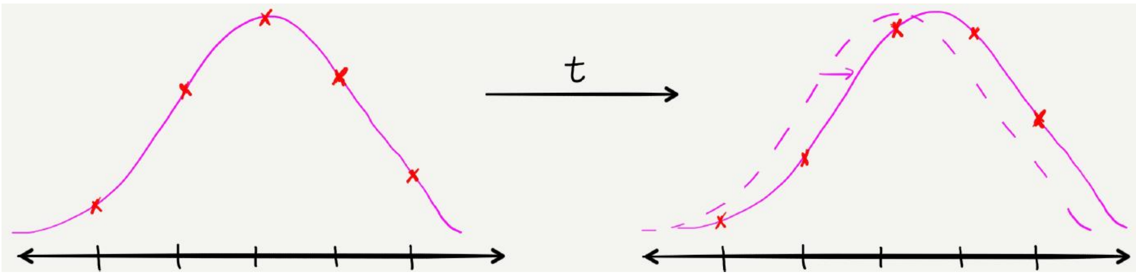

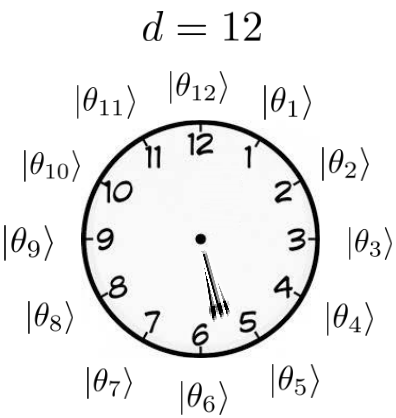

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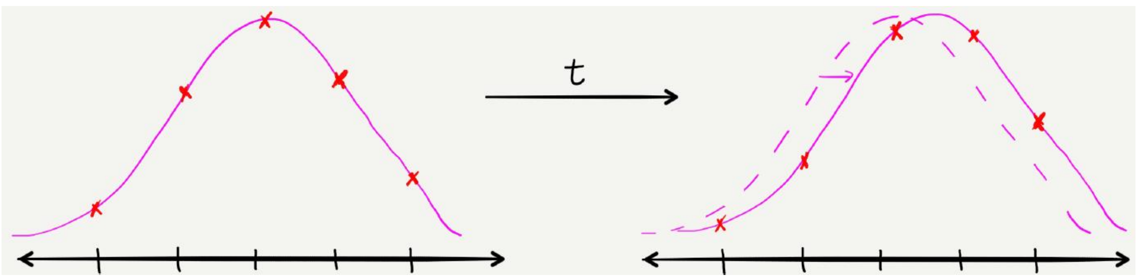

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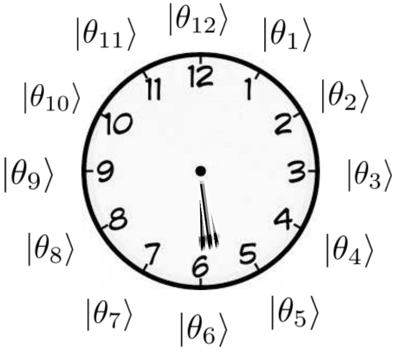

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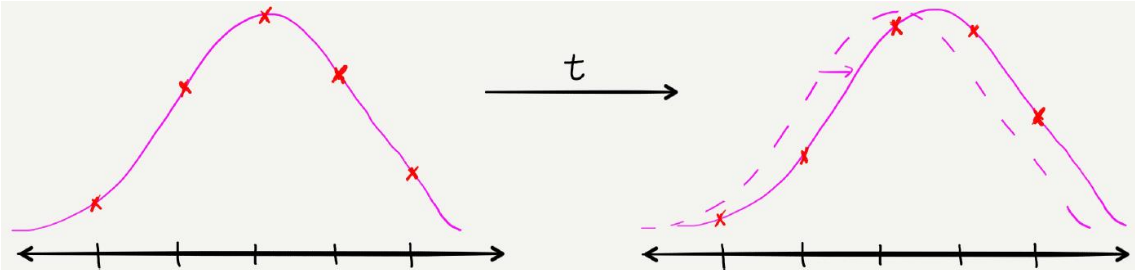

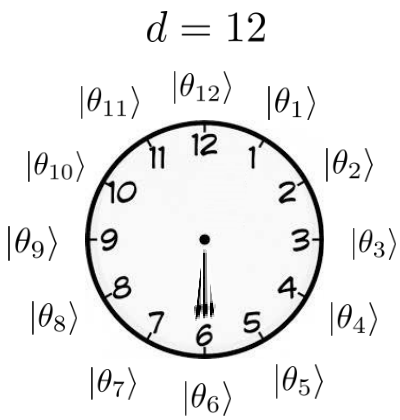

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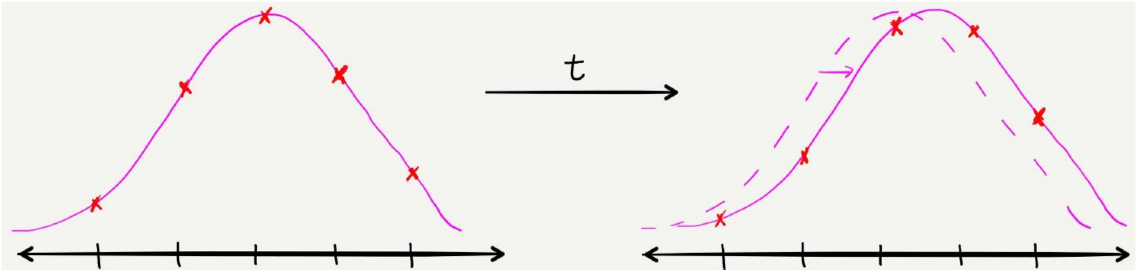

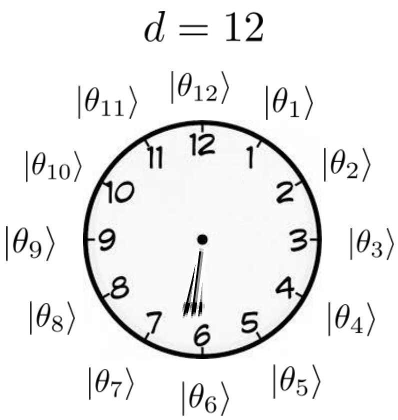

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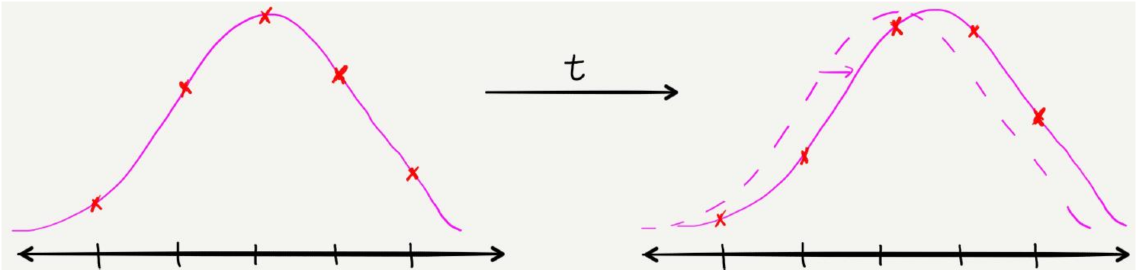

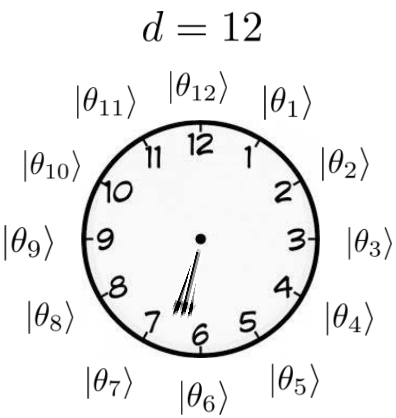

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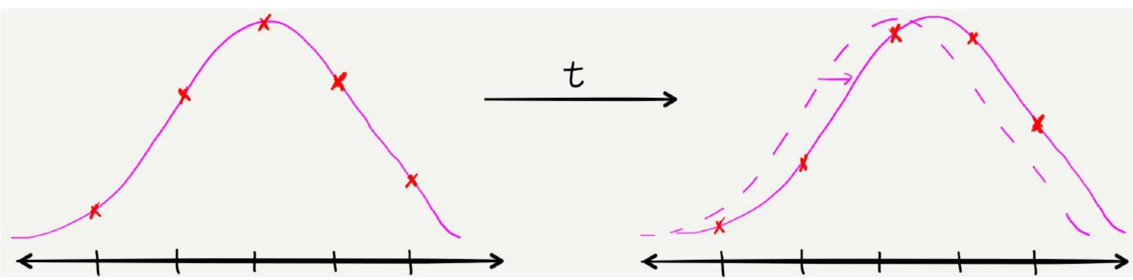

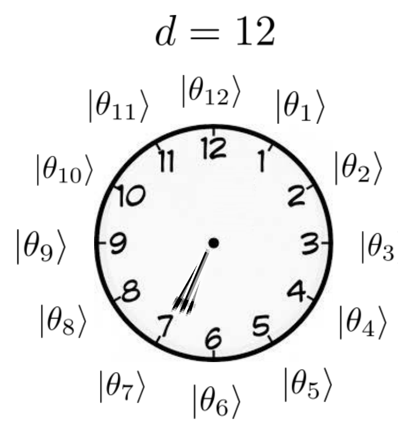

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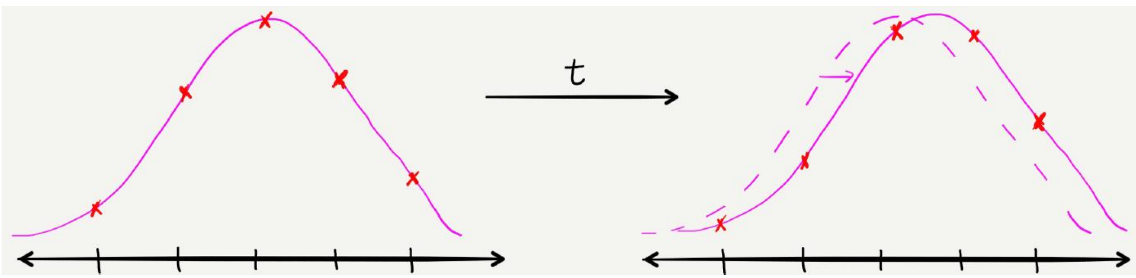

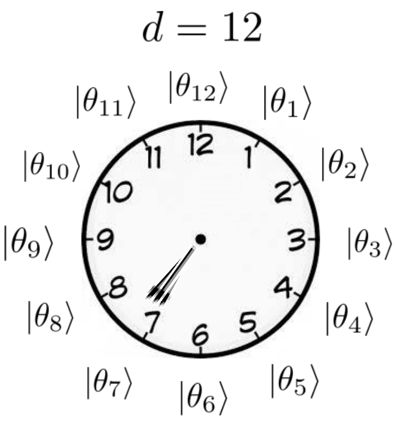

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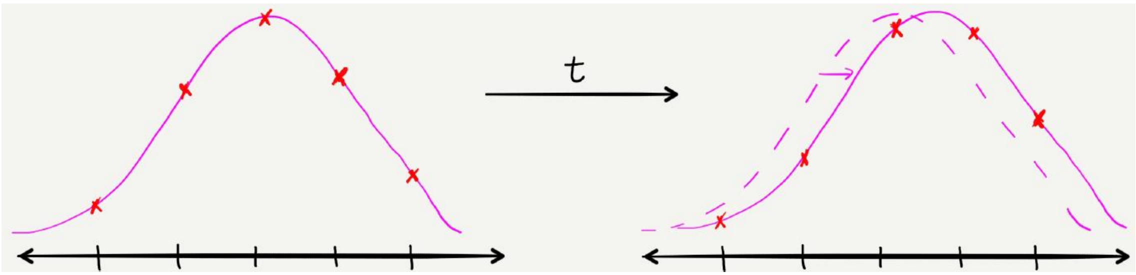

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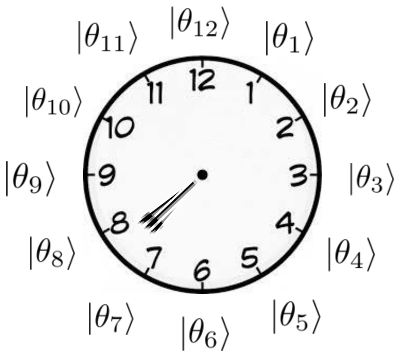

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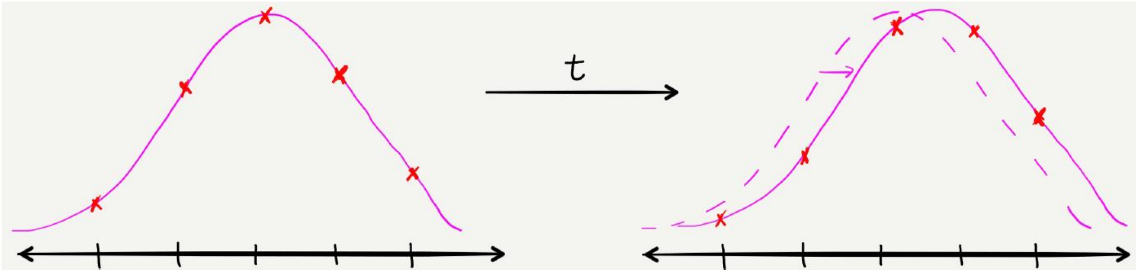

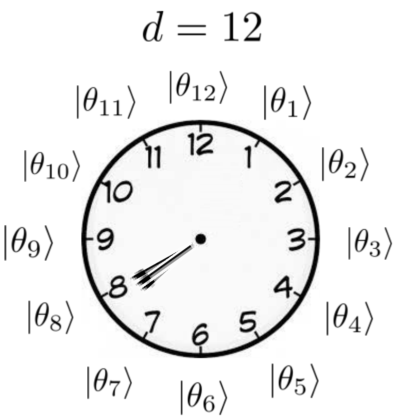

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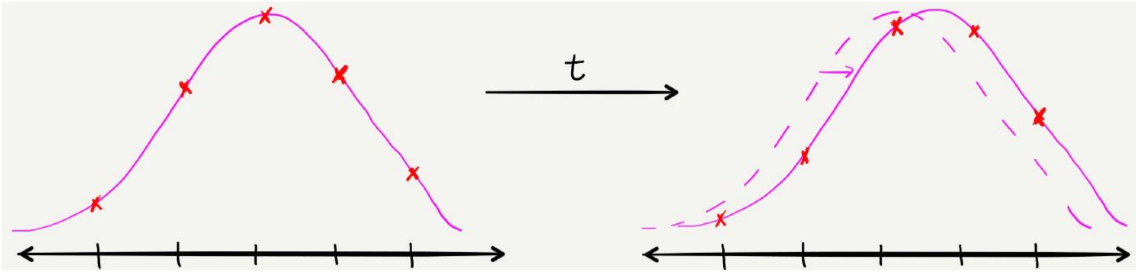

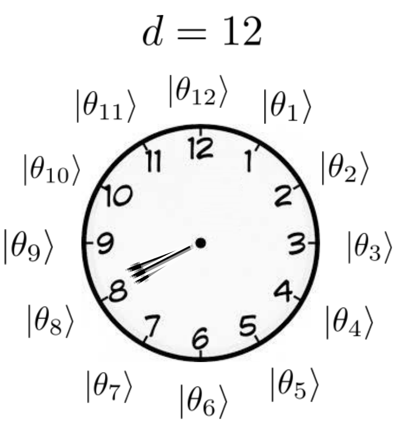

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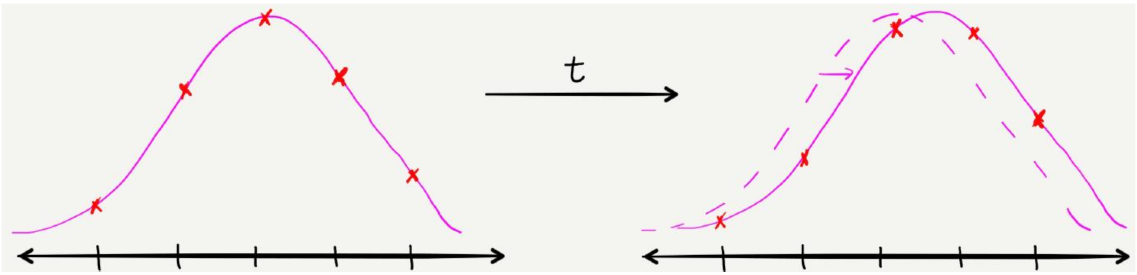

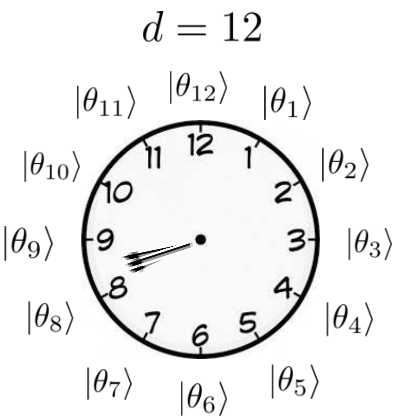

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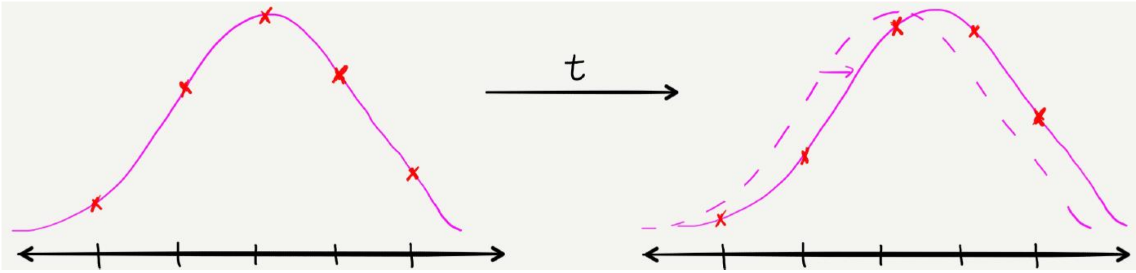

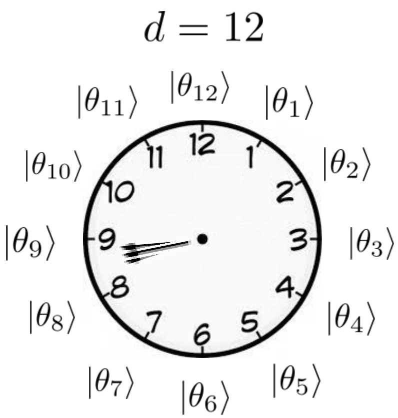

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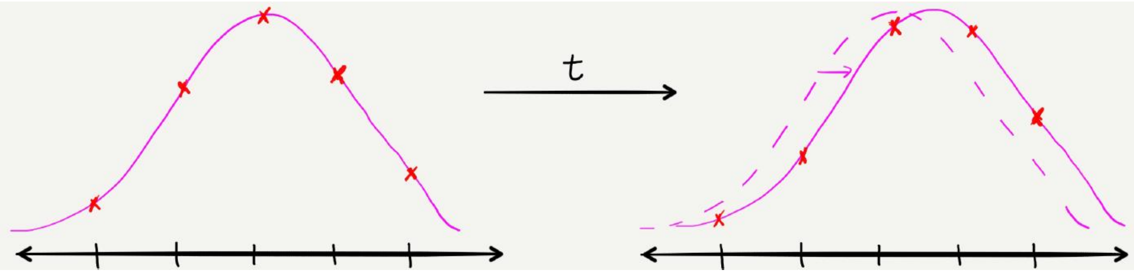

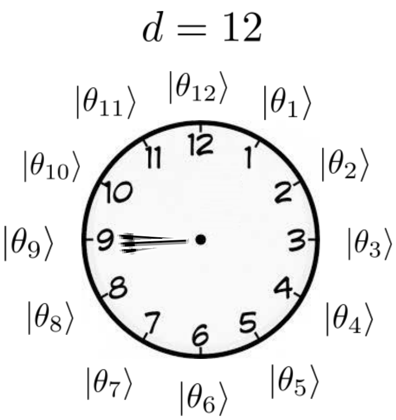

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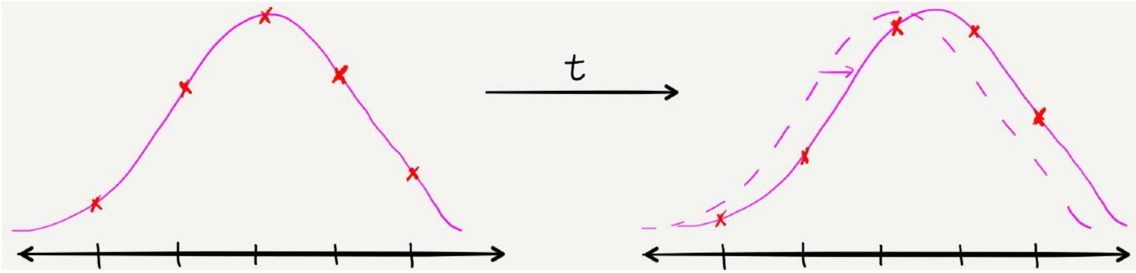

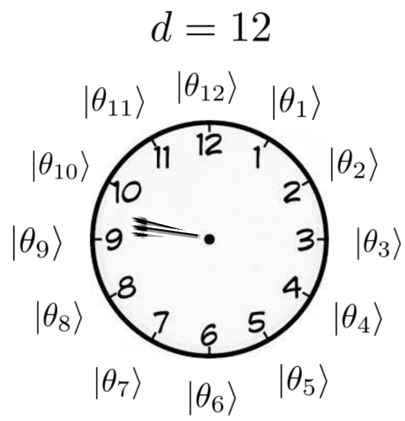

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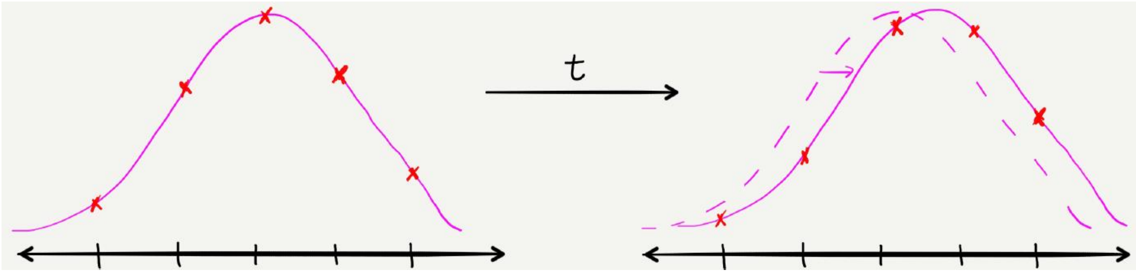

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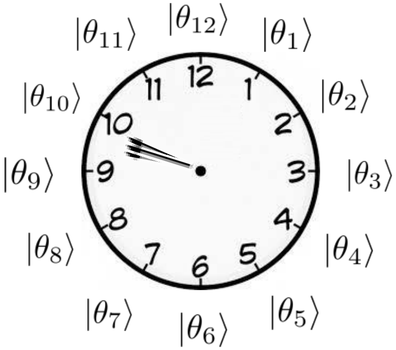

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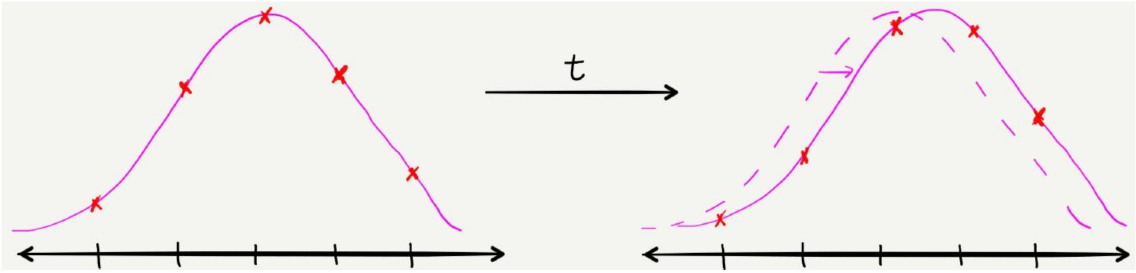

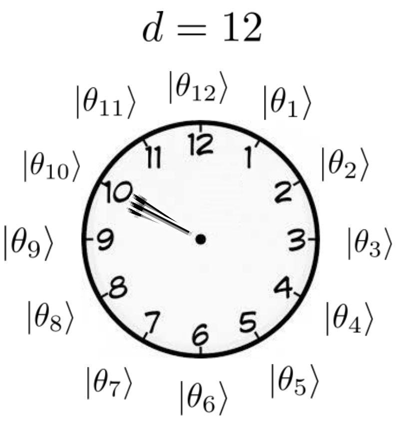

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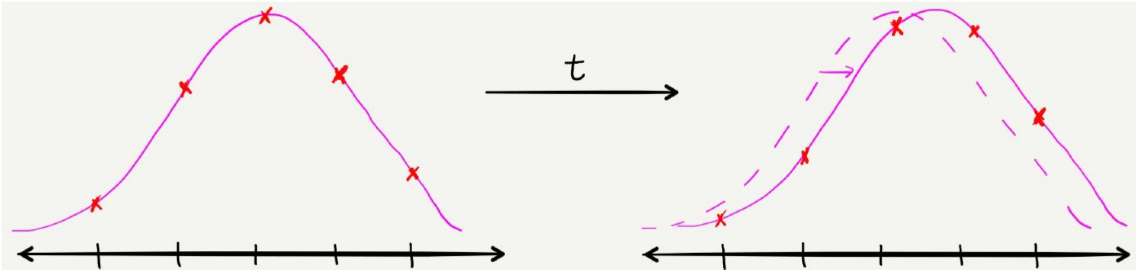

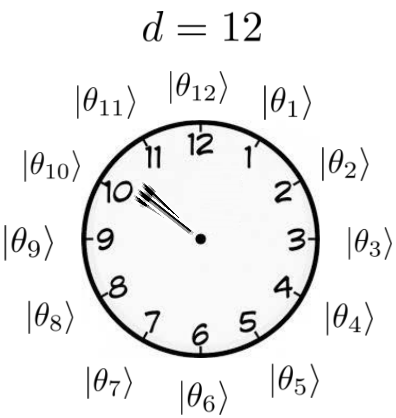

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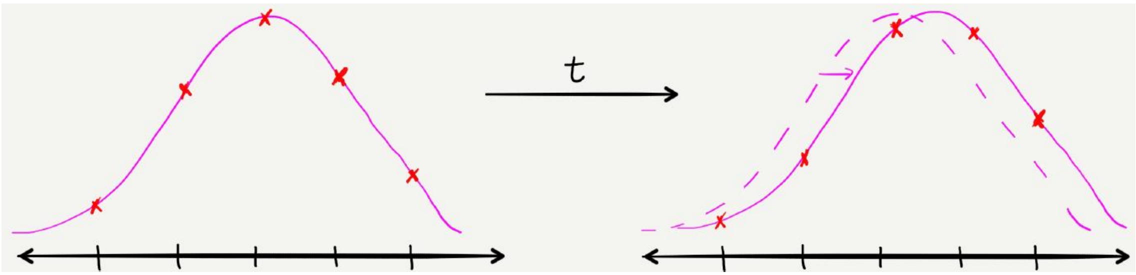

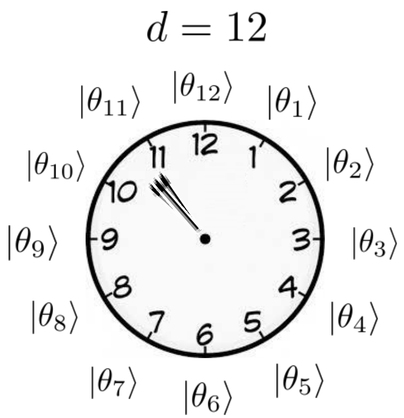

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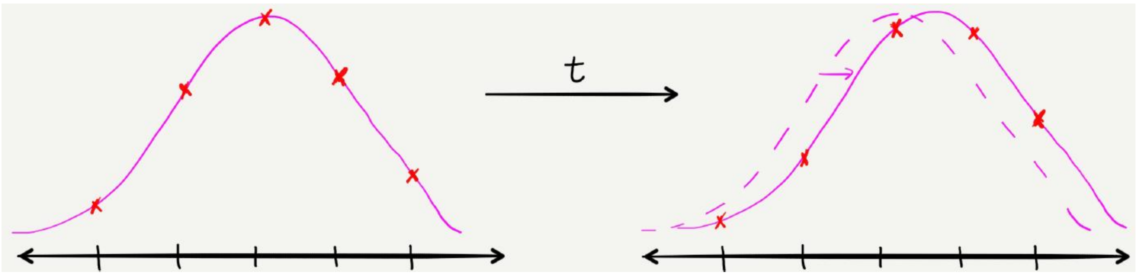

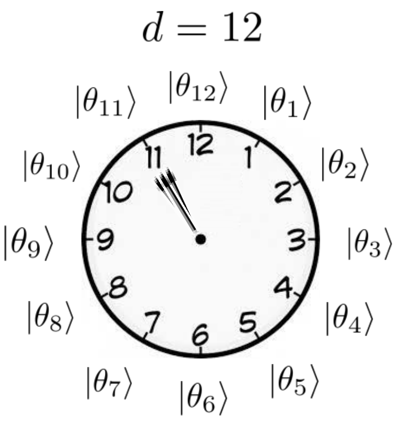

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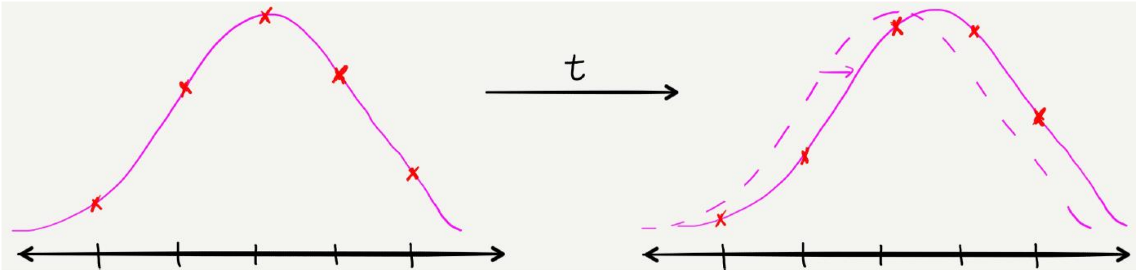

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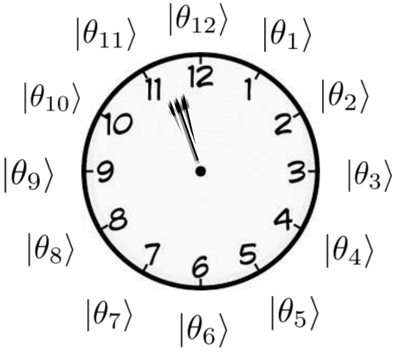

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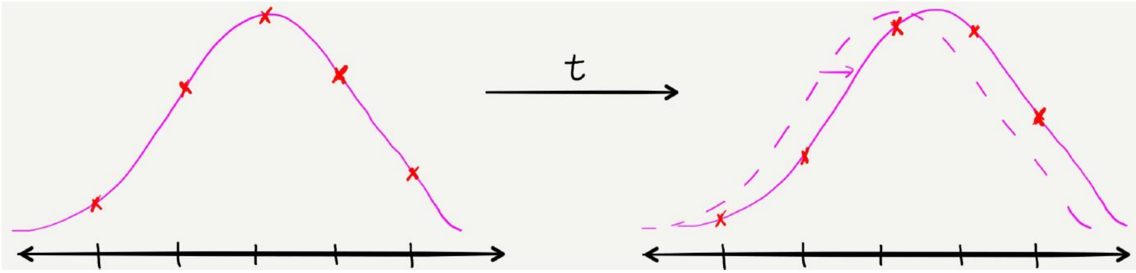

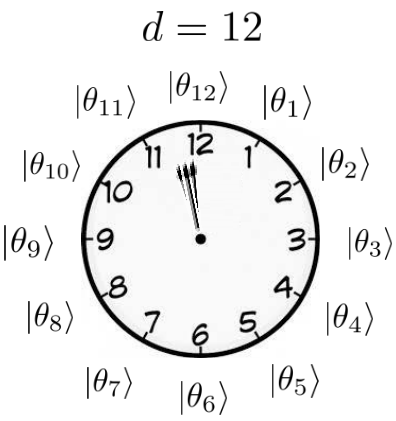

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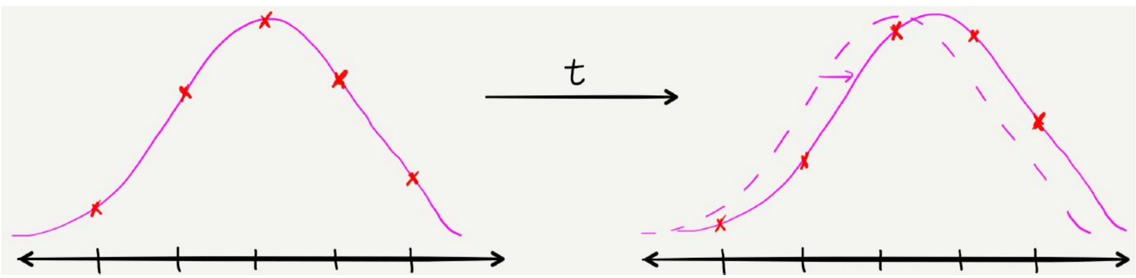

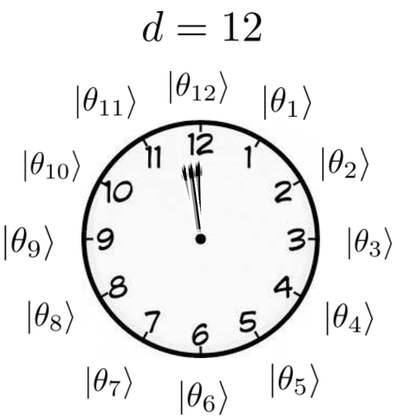

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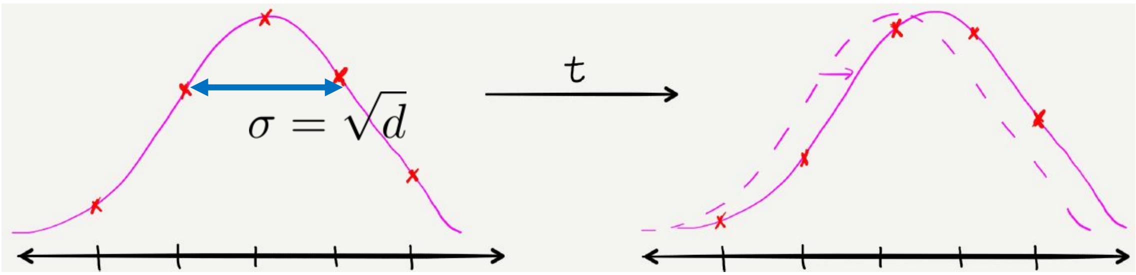

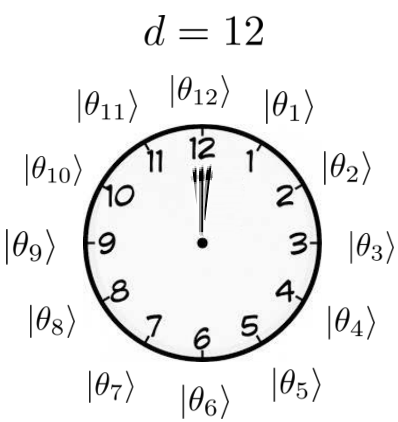

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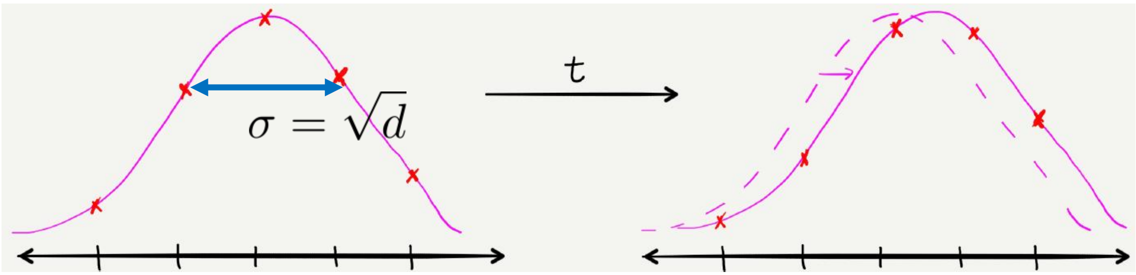

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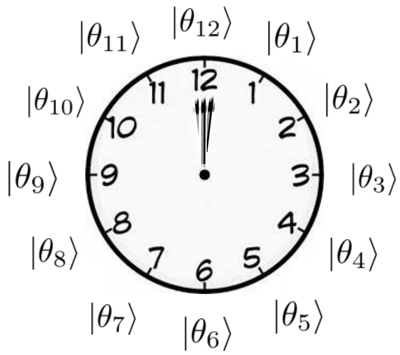

$$|\varepsilon_c(t)| = t \; poly(d) \; e^{-\frac{\pi}{4}d}$$

2^{nd} Result Commutation:

-Recall discrete time operator: $\hat{t}_c = \sum_k k \,|\theta_k\rangle\langle\theta_k|$

$$[\hat{H}_c, \hat{t}_c]\,|\Psi(k_0)\rangle = i\,|\Psi(k_0)\rangle + |\epsilon\rangle\,, \quad \|\,|\epsilon\rangle\|_2 \leq poly(d)\, e^{-\frac{\pi}{4}d}$$

-c.f. angle states

$$\langle\theta_k|\,[\hat{H}_c, \hat{t}_c]\,|\theta_k\rangle = 0$$

-c.f. idealised clock

$$[\hat{P}, \hat{x}_c]\,|\Psi(k_0)\rangle = i\,|\Psi(k_0)\rangle$$

Main Result

-Define Potential:

$$\hat{V} = \sum_{k=0}^{d-1} V_d(k) \, |\theta_k\rangle\langle\theta_k| \,, \qquad V_d(x) = \frac{2\pi}{d} V_0\left(\frac{k}{2\pi d}\right),$$

= Idealised clock

-Dynamics is given by:

$$\langle\theta_k| \, e^{-it(\hat{V}_d+\hat{H}_c)} \, |\Psi(k_0)\rangle = e^{-i\int_{k-td/T_0}^{k} V_d(x)dx} \, \psi(k_0 - d\frac{t}{T_0}; k) + \varepsilon_c(t)$$

Correction term

Main Result

-Define Potential:

V_0 = period 2π

$$\hat{V} = \sum_{k=0}^{d-1} V_d(k)\,|\theta_k\rangle\langle\theta_k|\,, \qquad V_d(x) = \frac{2\pi}{d} V_0\left(\frac{k}{2\pi d}\right), \qquad \int_0^{2\pi} dx V_0(x) = 1$$

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$$\varepsilon_c(t) \leq poly(d)\, t\, e^{-\frac{\pi}{4}\frac{d}{\zeta}}, \qquad \zeta = \left(1 + \frac{0.792\,\pi}{\ln(\pi d)} b\right)^2,$$

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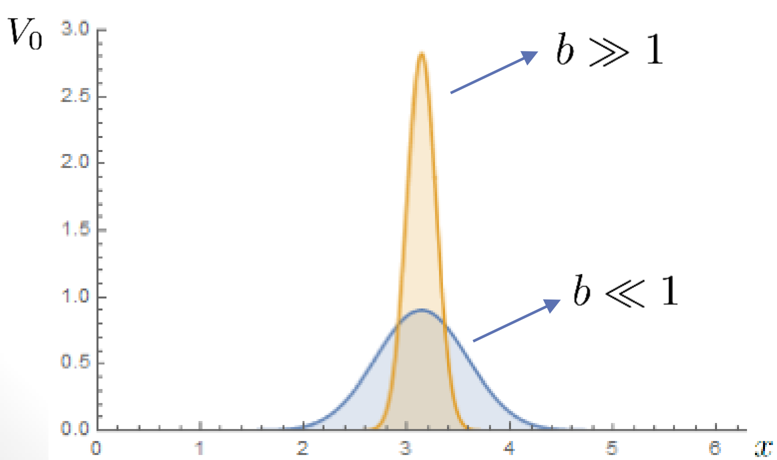


$$b \ge \sup_{k\in\mathbb{N}^+} \left(2 \max_{x\in[0,2\pi]} \left|V_0^{(k-1)}(x)\right|\right)^{1/k}$$

Main Result

-Proof: Similar to before just MUCH more complicated

$$|\bar{\Psi}(k_0,\Delta)\rangle := \sum_{k\in\mathcal{S}_d(k_0)} e^{-i\frac{2\pi}{d}\int_{k-\Delta}^{k} dy V_0(2\pi y/d)}\psi(k_0;k)\,|\theta_k\rangle$$

$$e^{-i\frac{T_0}{d}\hat{H}_c\delta}\,|\bar{\Psi}(k_0,\Delta)\rangle = \sum_{k\in\mathcal{S}_d(k_0)} e^{-i\frac{2\pi}{d}\int_{k-\delta-\Delta}^{k-\delta} dy V_0(2\pi y/d)}\psi(k_0;k-\delta)\,|\theta_k\rangle + |\epsilon\rangle\,,$$

$$\|\,|\epsilon\rangle\,\|_2 < a\delta + b\delta^2$$

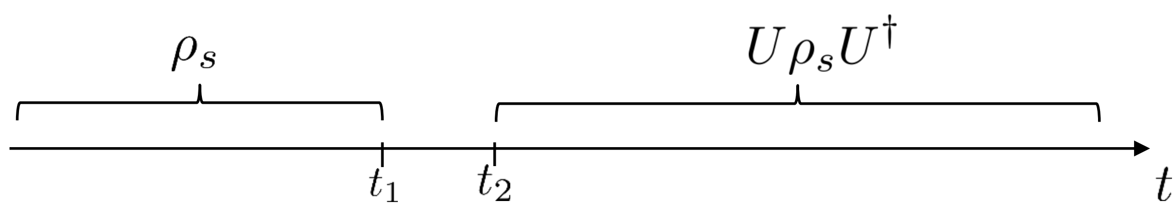
$$e^{-i\frac{T_0}{d}\hat{V}_c\delta}\,|\Phi\rangle = \sum_{k\in\mathcal{S}_d(k_0)} \langle\theta_k|\Phi\rangle\, e^{-i\frac{2\pi}{d}\int_{k-\delta}^{k} dy V_0(2\pi y/d)}\,|\theta_k\rangle + |\epsilon\rangle\,, \quad \|\,|\epsilon'\rangle\,\|_2 < b'\delta^2$$

$$\left(e^{-i\frac{T_0}{d}\delta\hat{V}_d}e^{-i\frac{T_0}{d}\delta\hat{H}_c}\right)^m |\bar{\Psi}(k_0,\Delta)\rangle = |\bar{\Psi}\left(k_0+m\delta,\Delta+m\delta\right)\rangle + |\epsilon^{(m)}\rangle$$

$$\|\,|\epsilon^{(m)}\rangle\,\|_2 < m\Big(a\delta + (b+b')\delta^2\Big)$$

$$e^{-it(\hat{V}_d+\hat{H}_c)} = \lim_{m\to\infty}\left(e^{-i\hat{H}_c t/m}e^{-i\hat{V}_d t/m}\right)^m \qquad \delta = \frac{t}{m}\frac{d}{T_0}$$

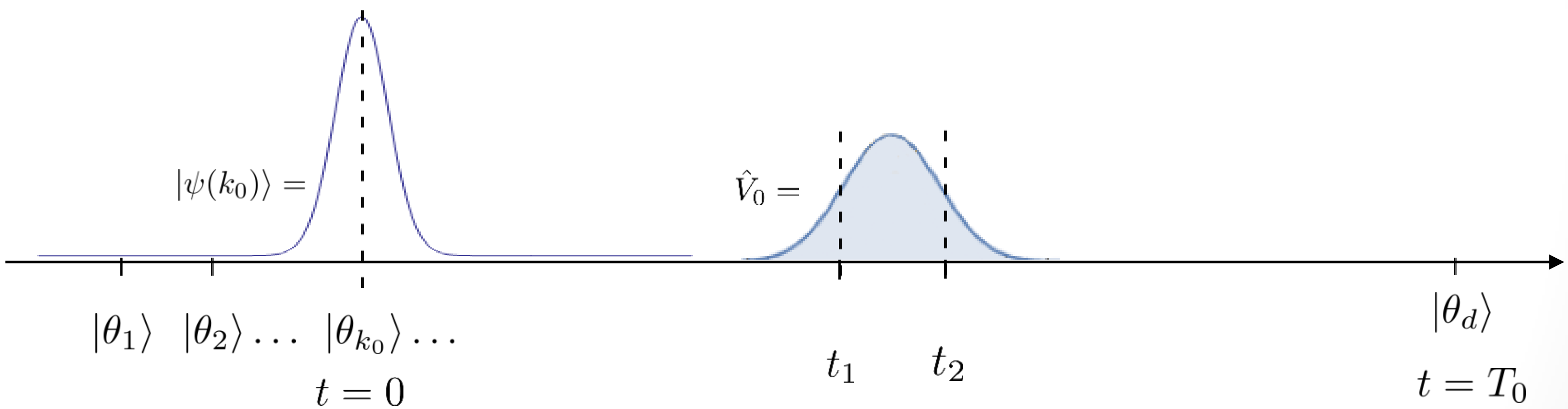
Thermodynamic Consequences



$U = e^{\hat{H}_s^{\text{int}}}$

$$\rho'_{sc}(t) = \left(\rho_s \otimes |\Psi(k_0)\rangle\langle\Psi(k_0)|\right)(t), \qquad \hat{H}_c \otimes \mathbb{1}_s + \hat{V}_0 \otimes \hat{H}_s^{\text{int}}$$

System: ρ_s



Correction term $\mathrm{F}(\rho'_s(t), \rho_s(t)) \leq \sqrt{d_s}\, poly(d)\, e^{-c\sqrt{d}}$

$t \in [0, t_1] \cup [t_2, T_0]$

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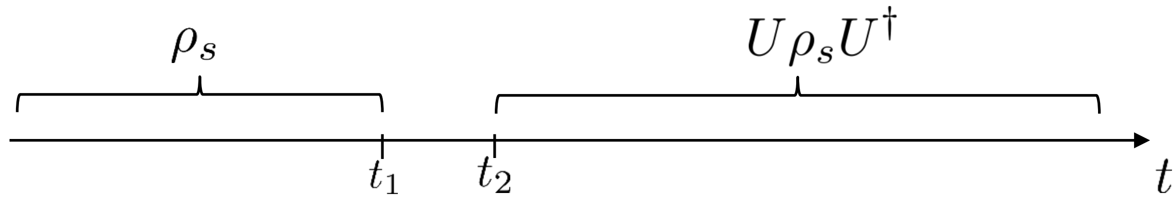
Thermodynamic Consequences



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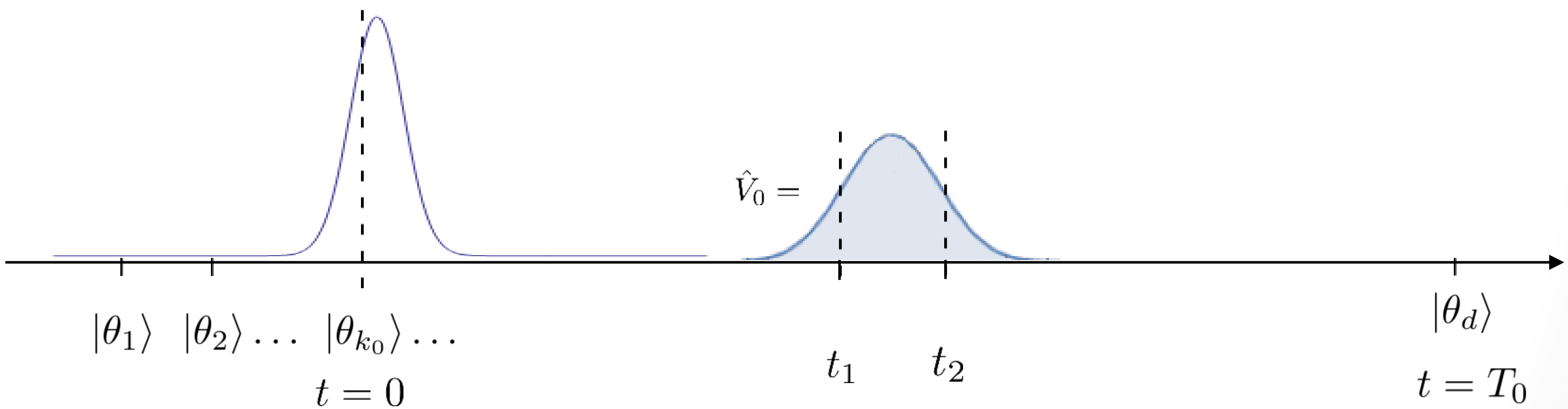

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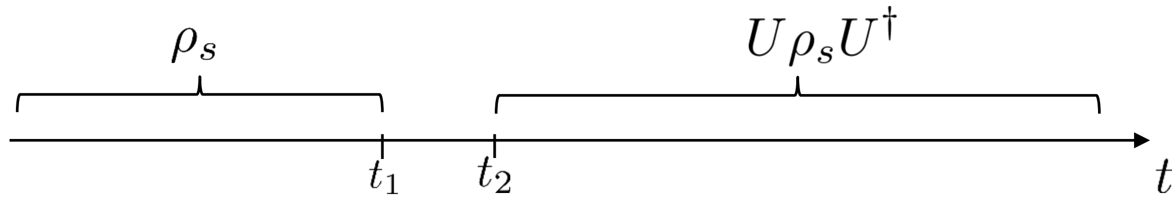

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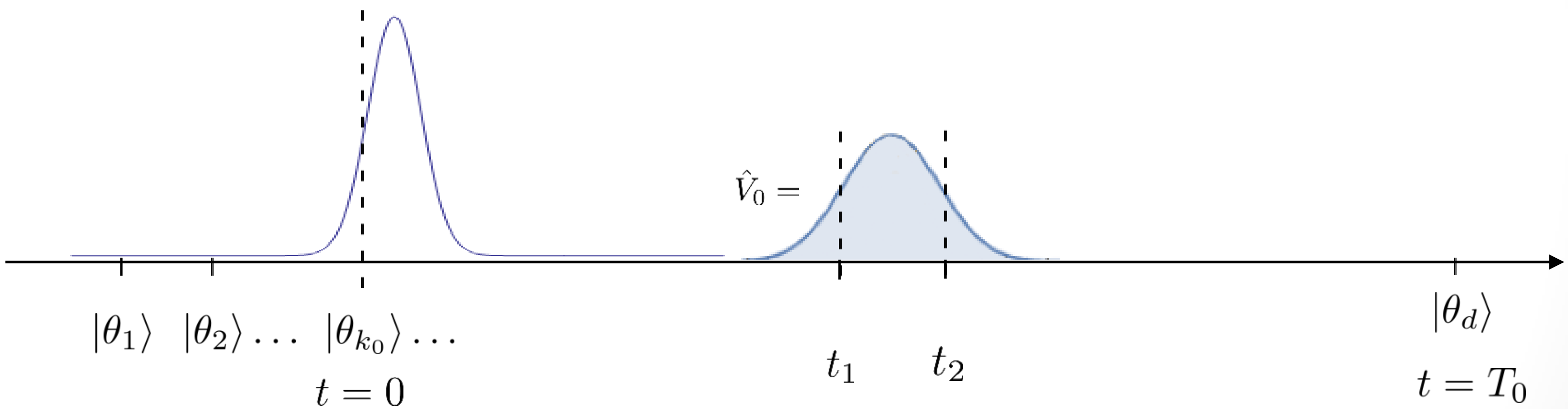

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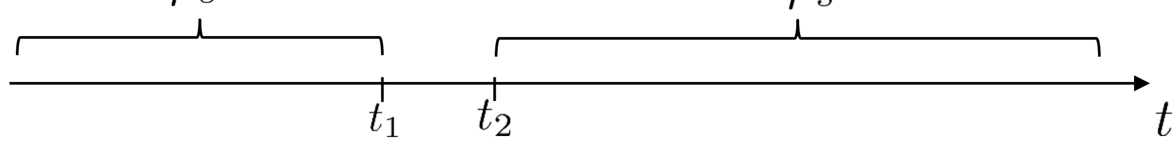

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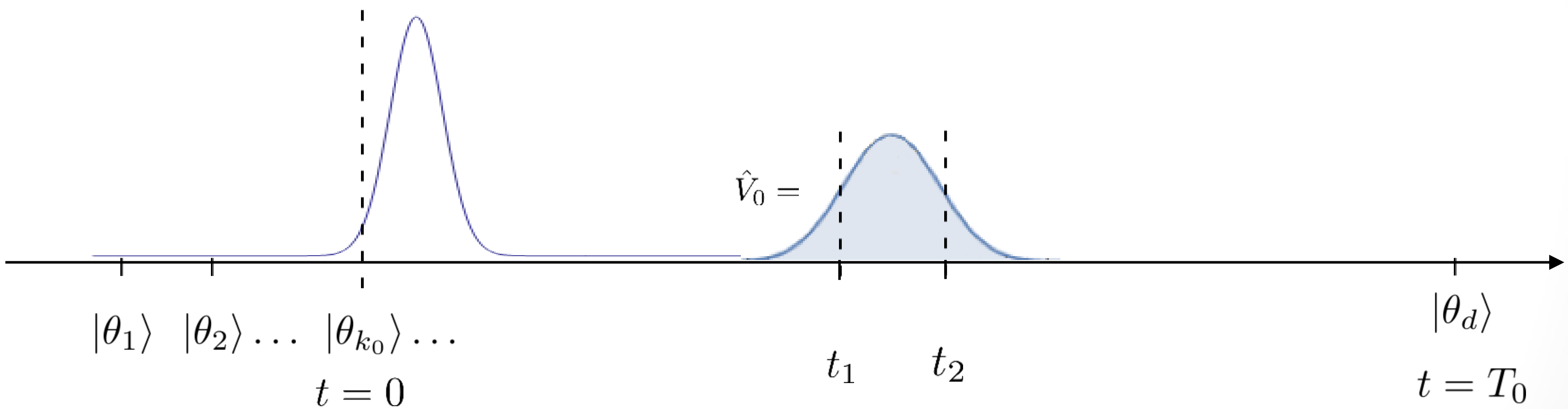

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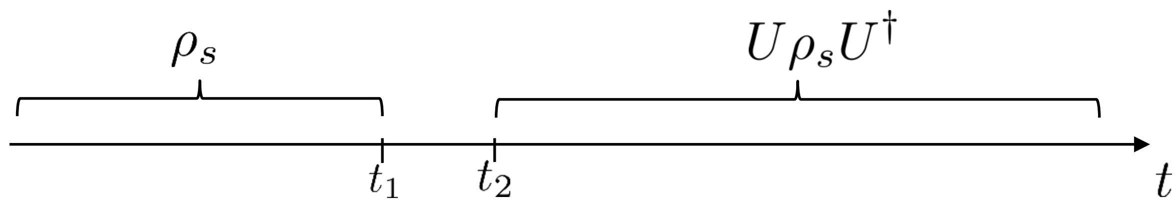

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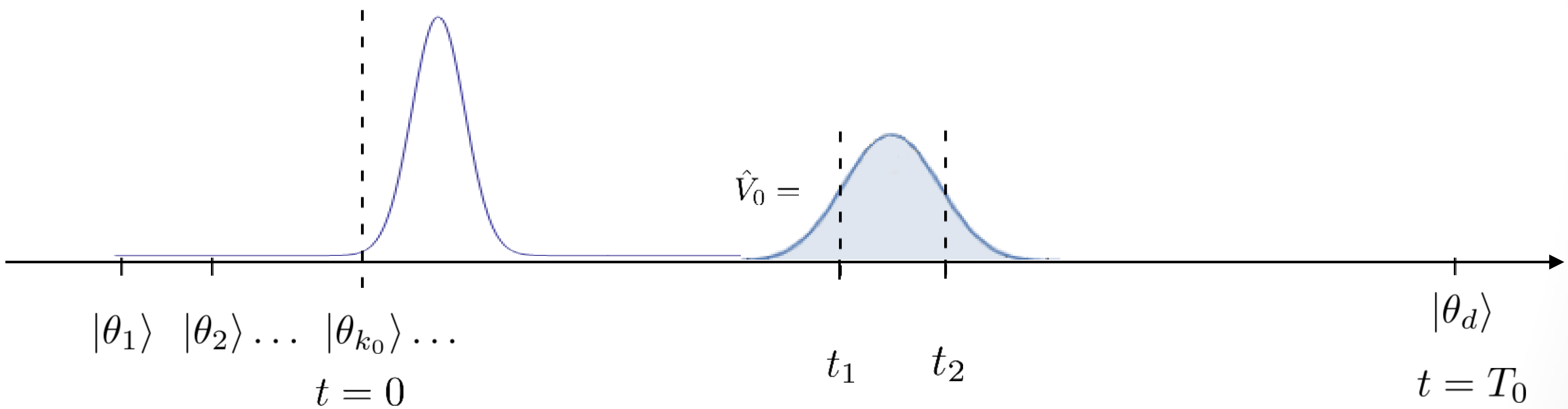

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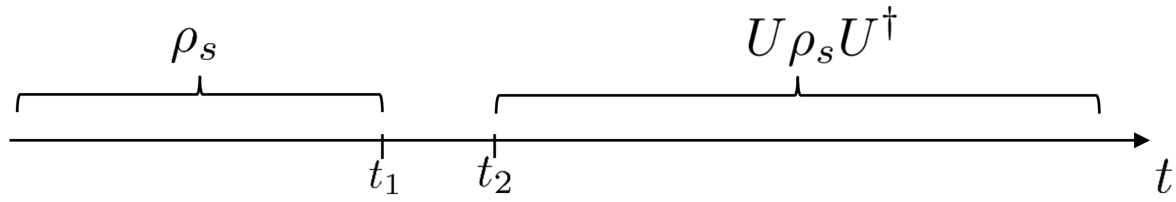

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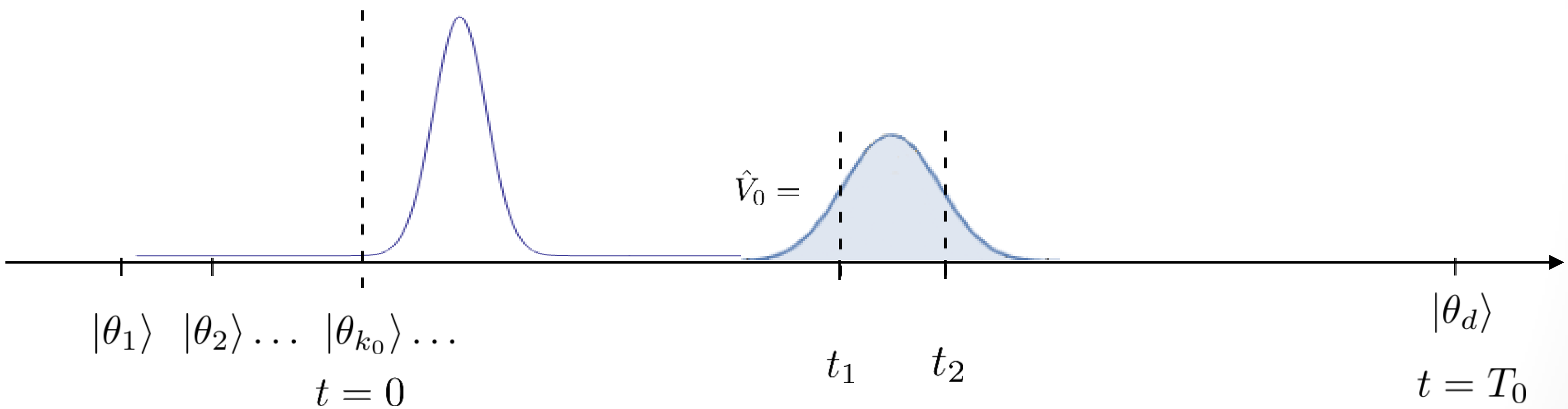

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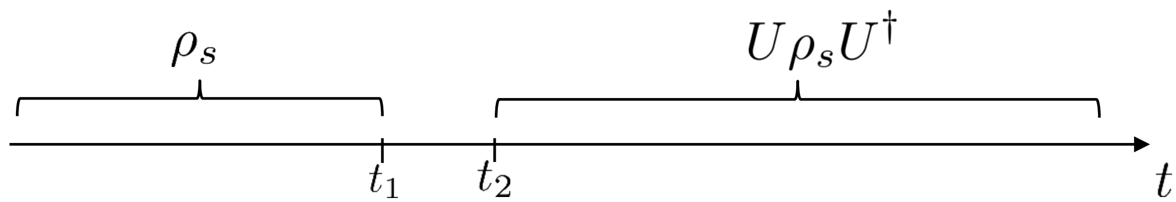

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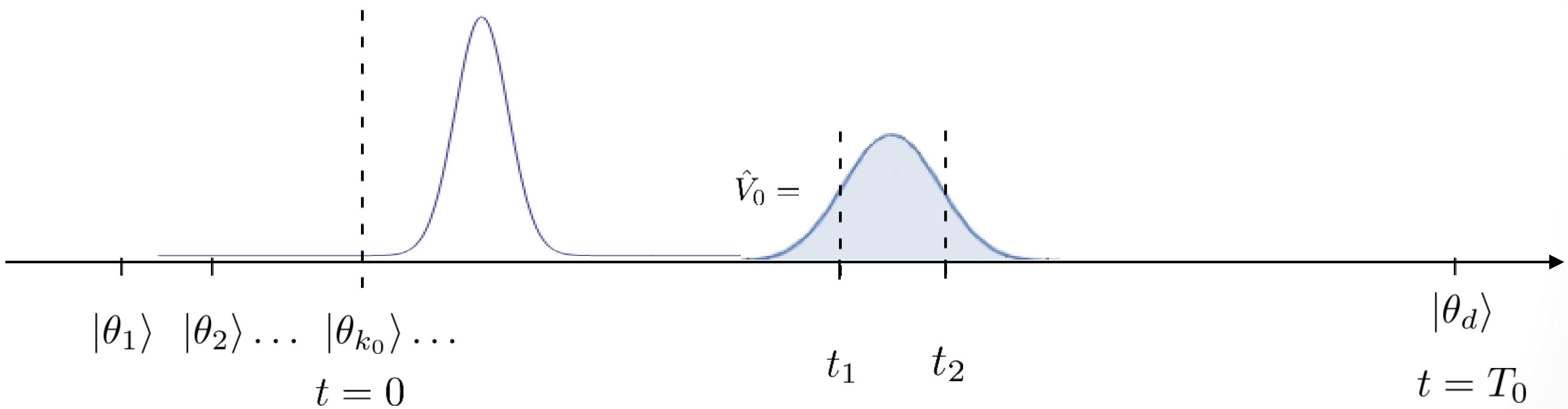

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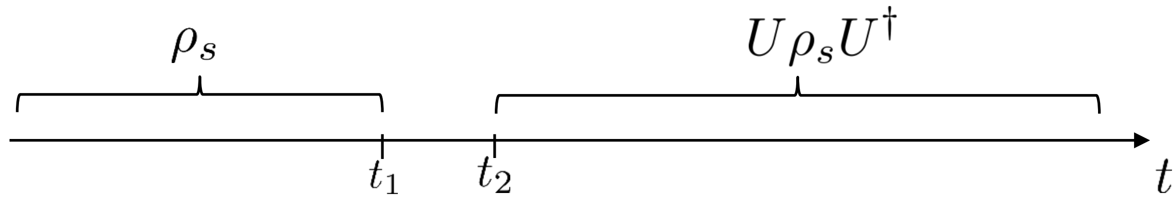

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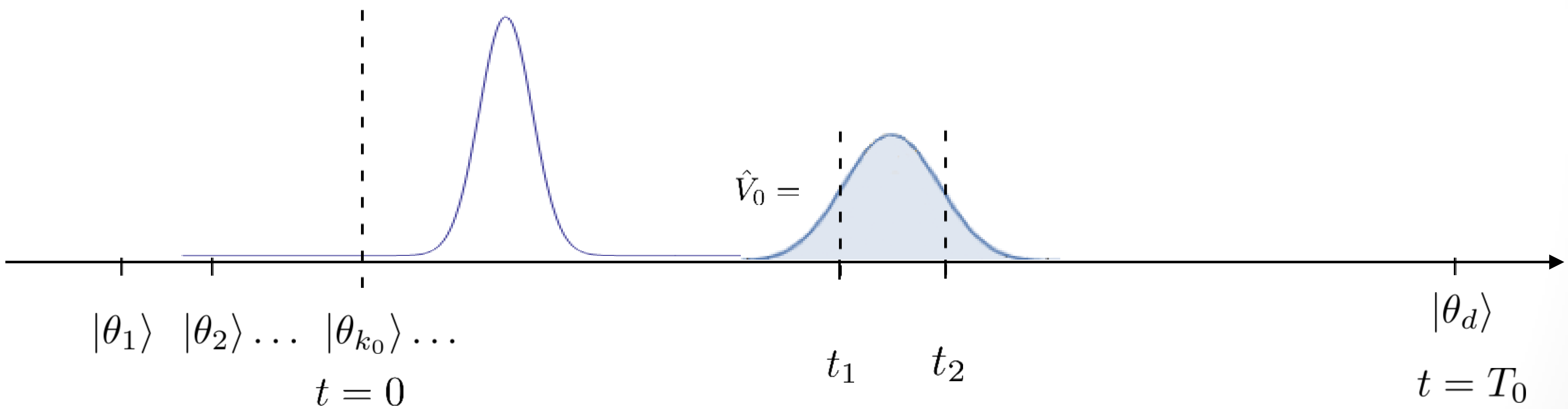

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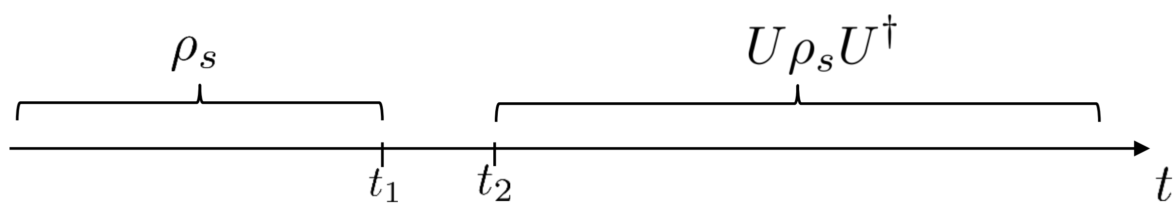

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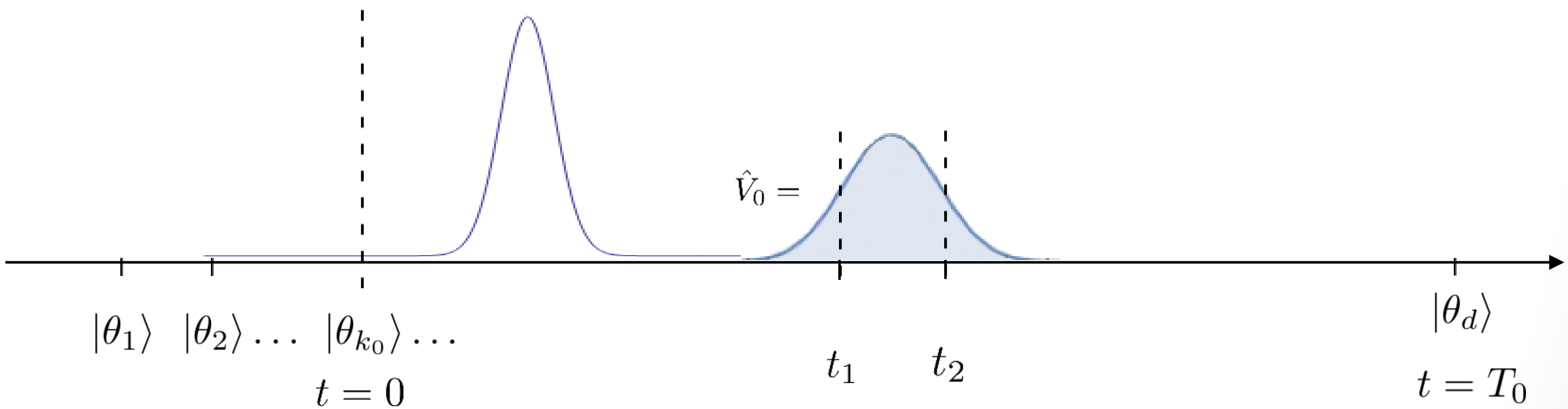

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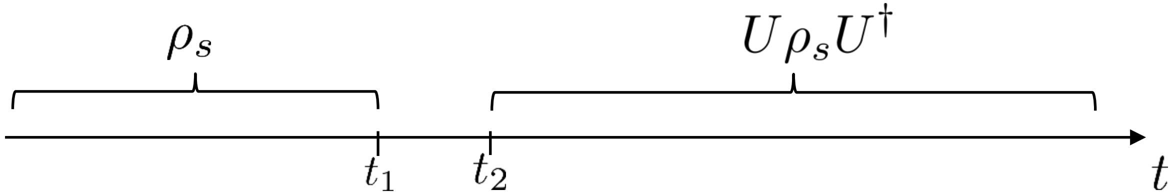

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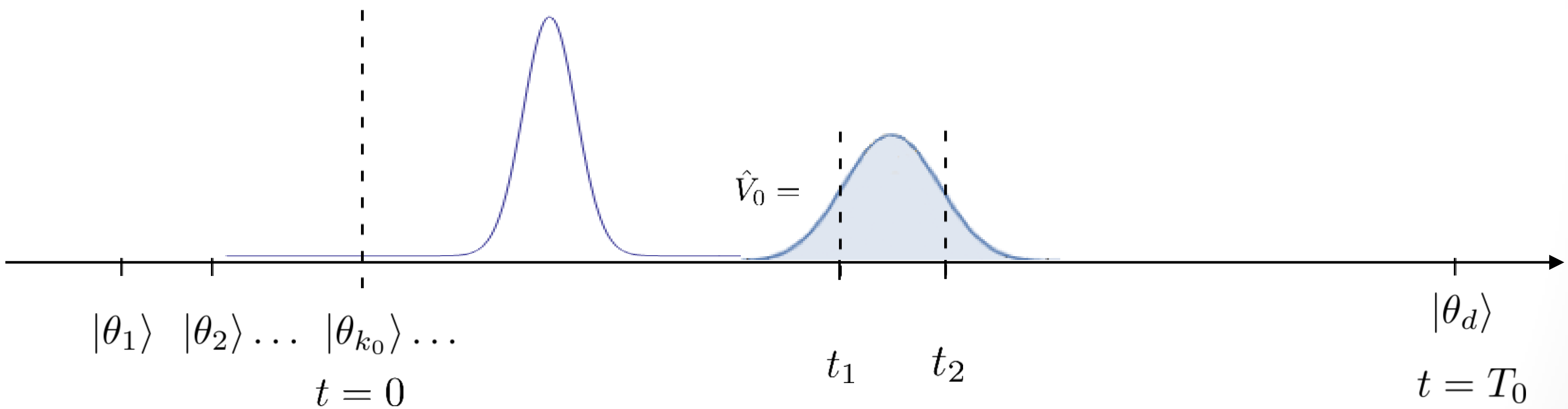

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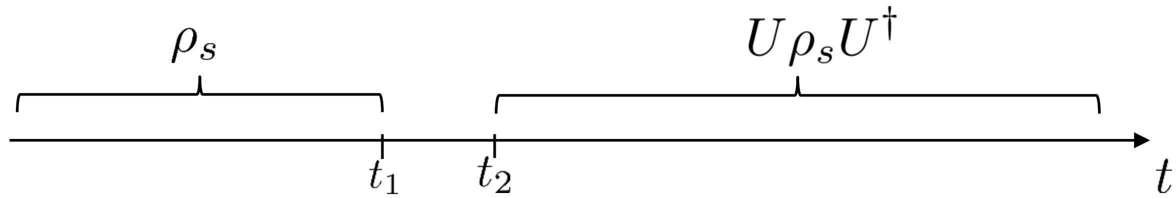

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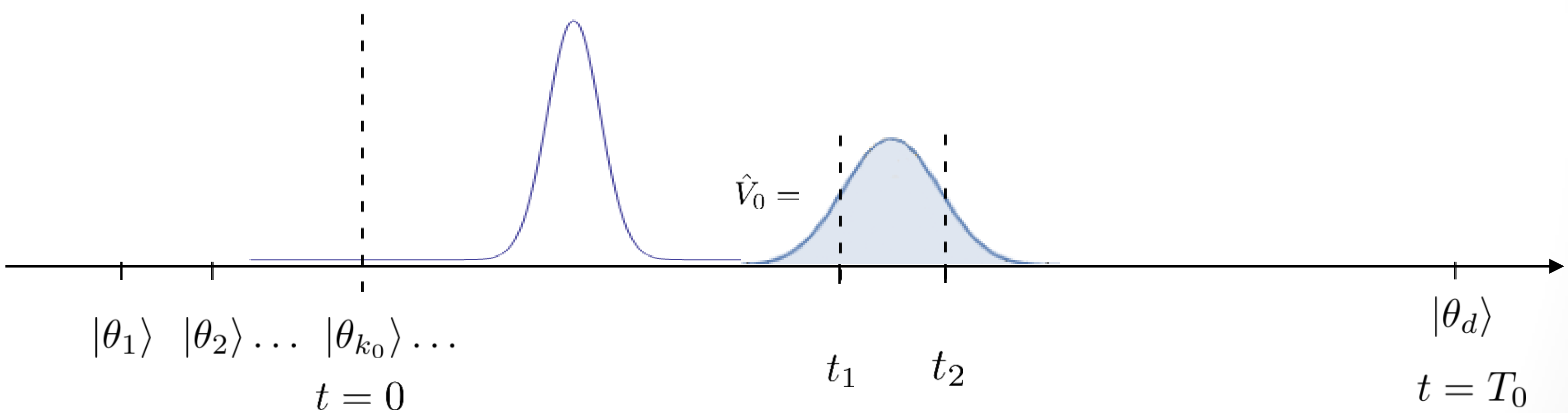

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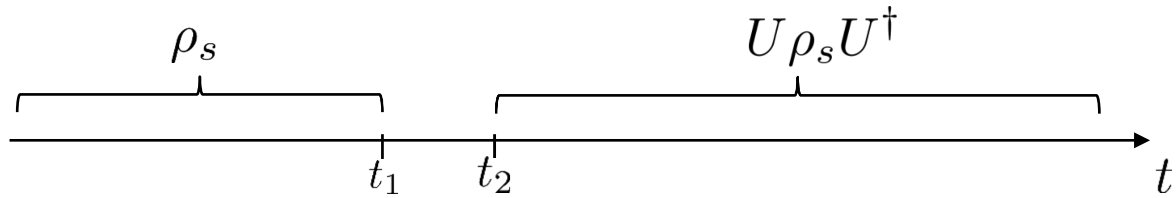

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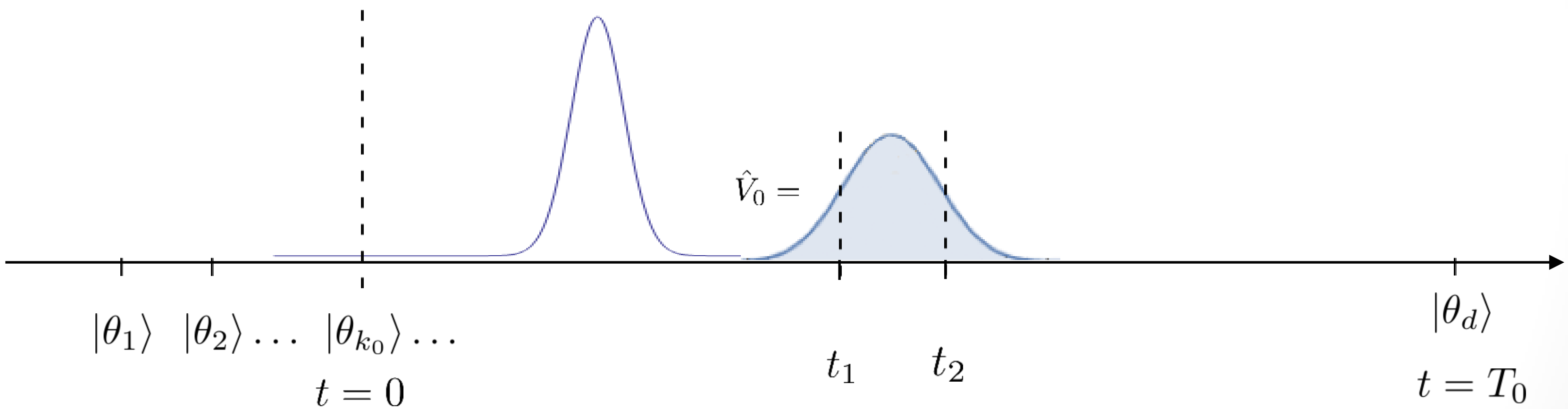

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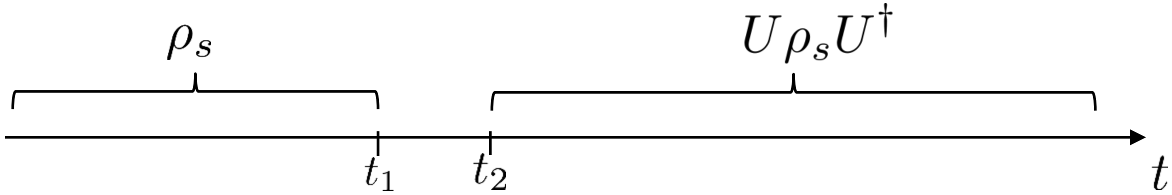

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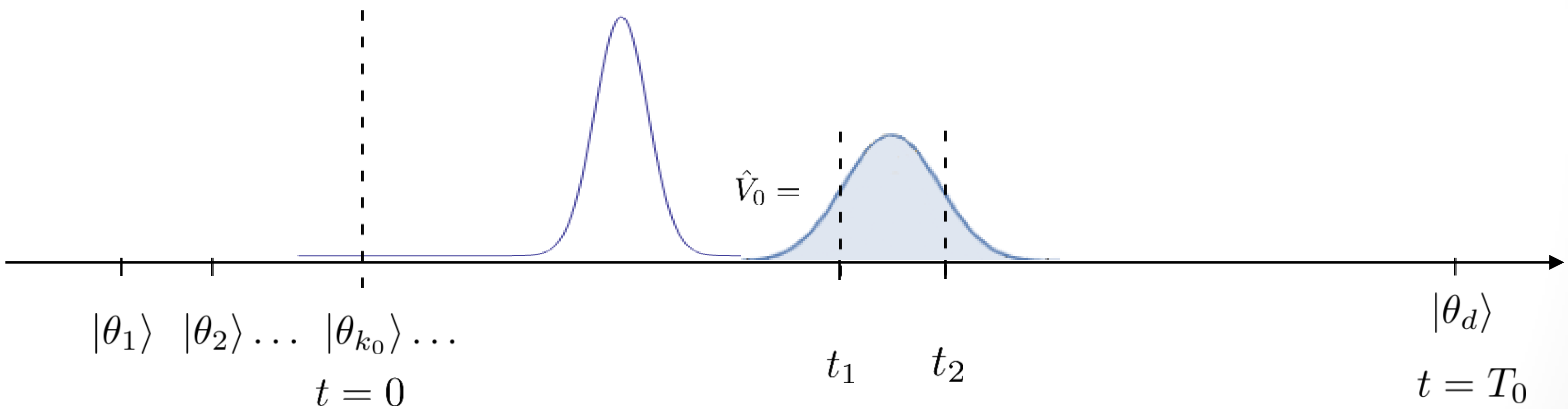

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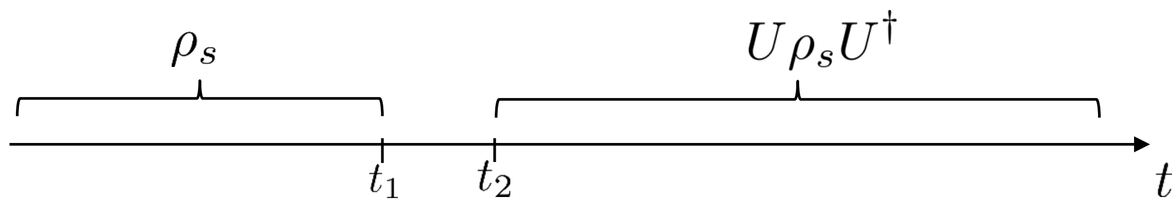

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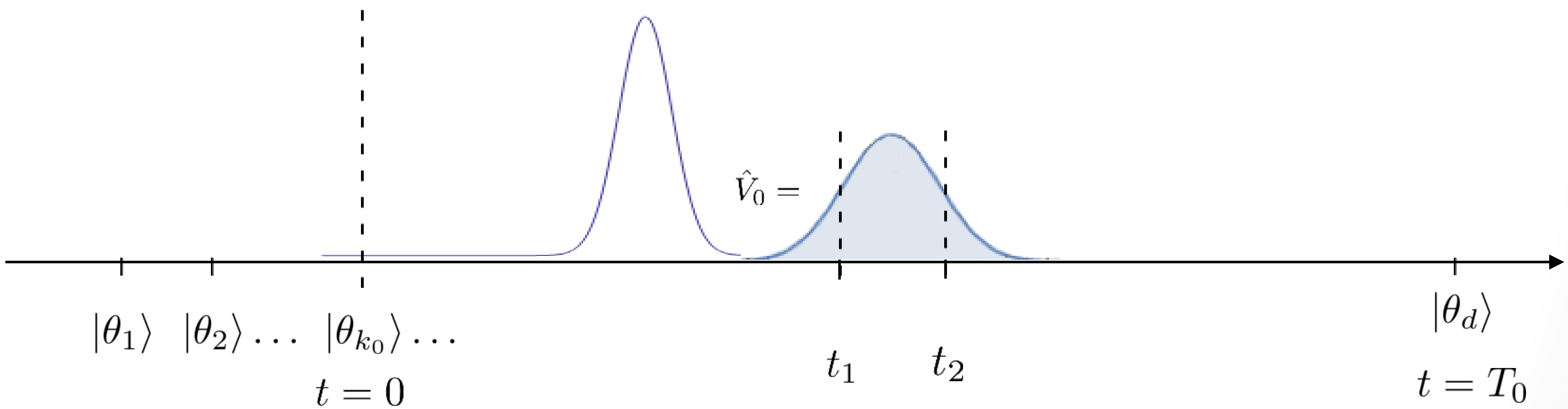

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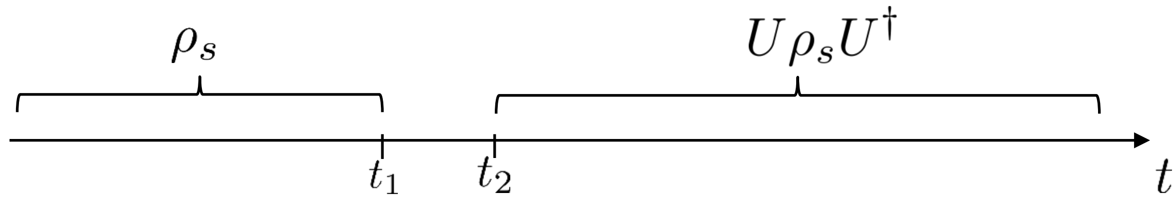

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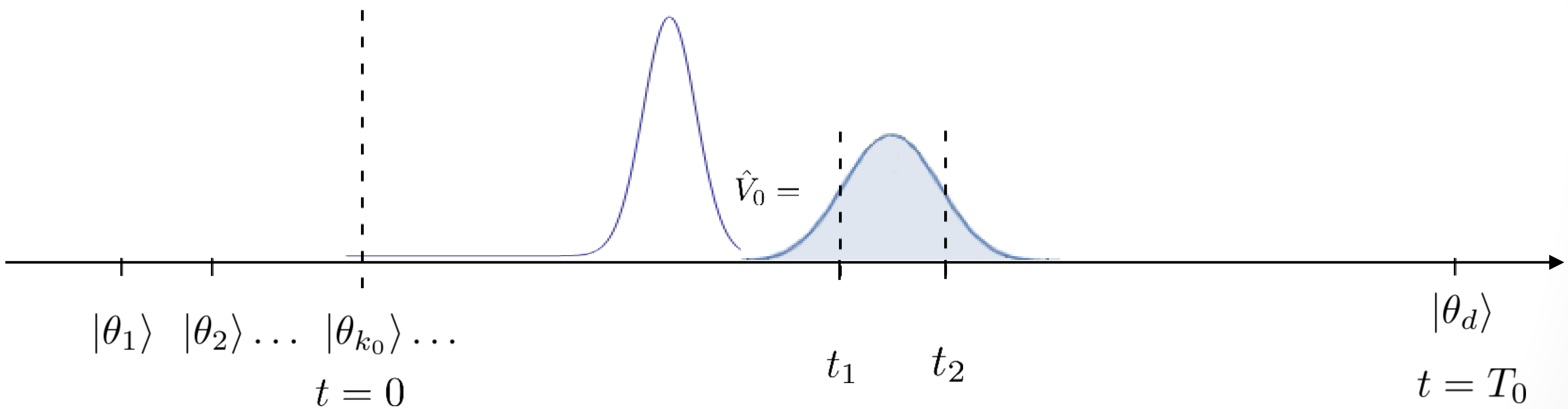

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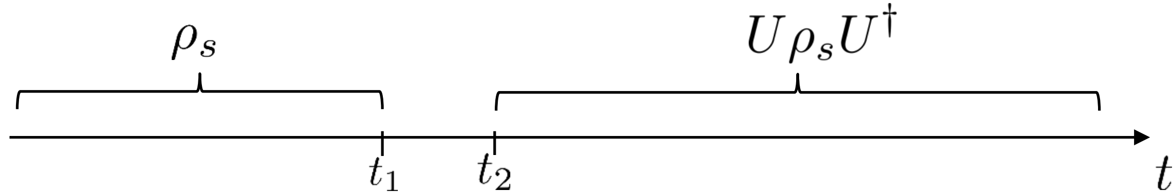

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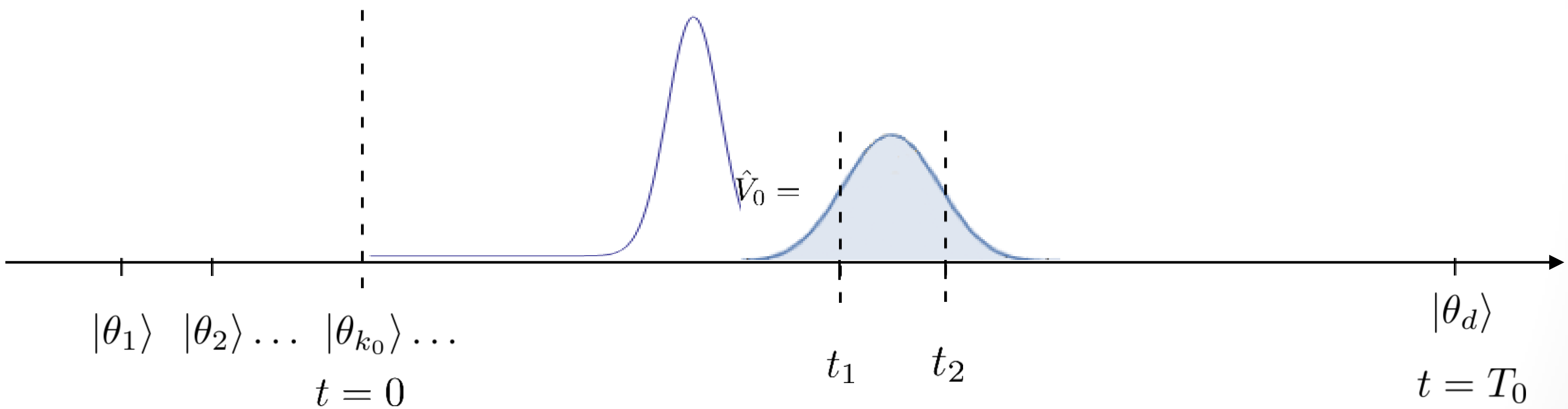

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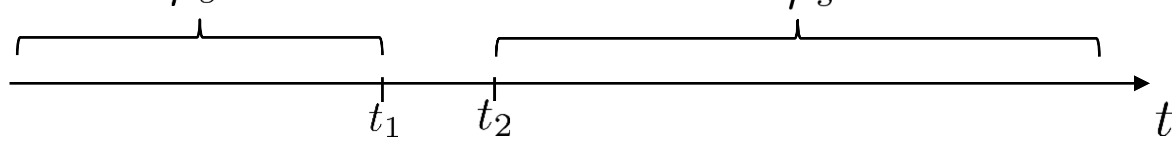

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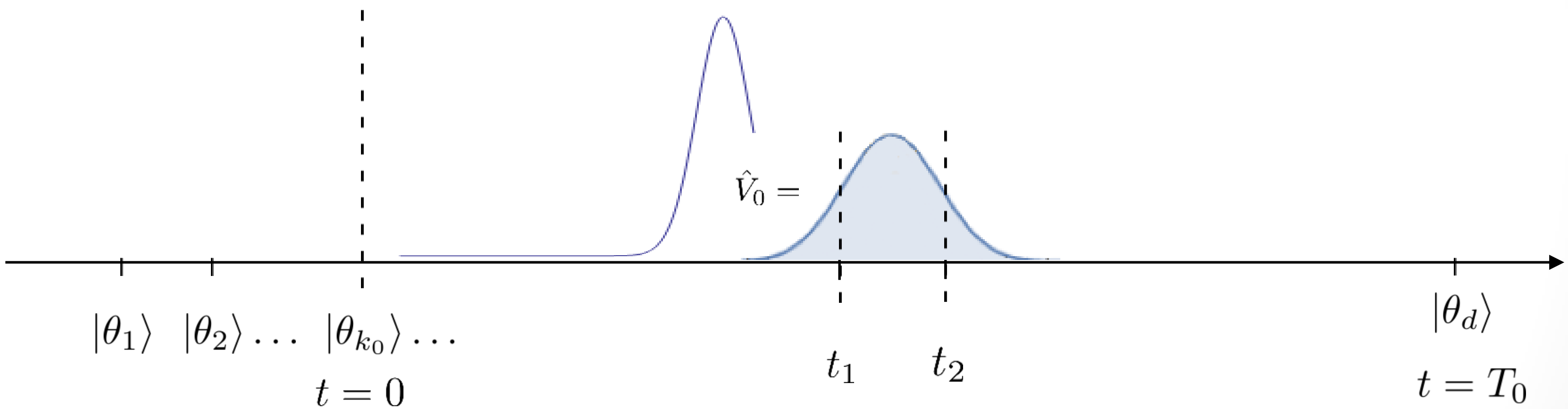

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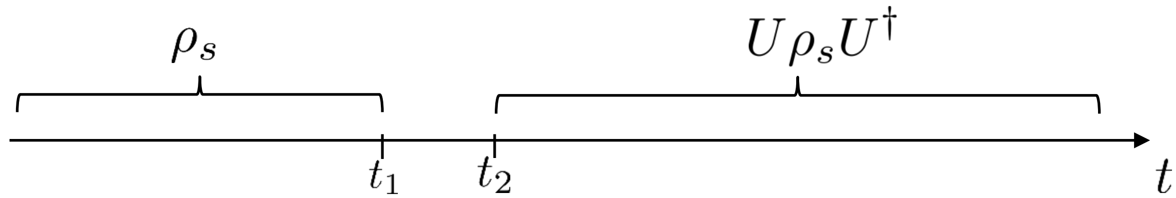

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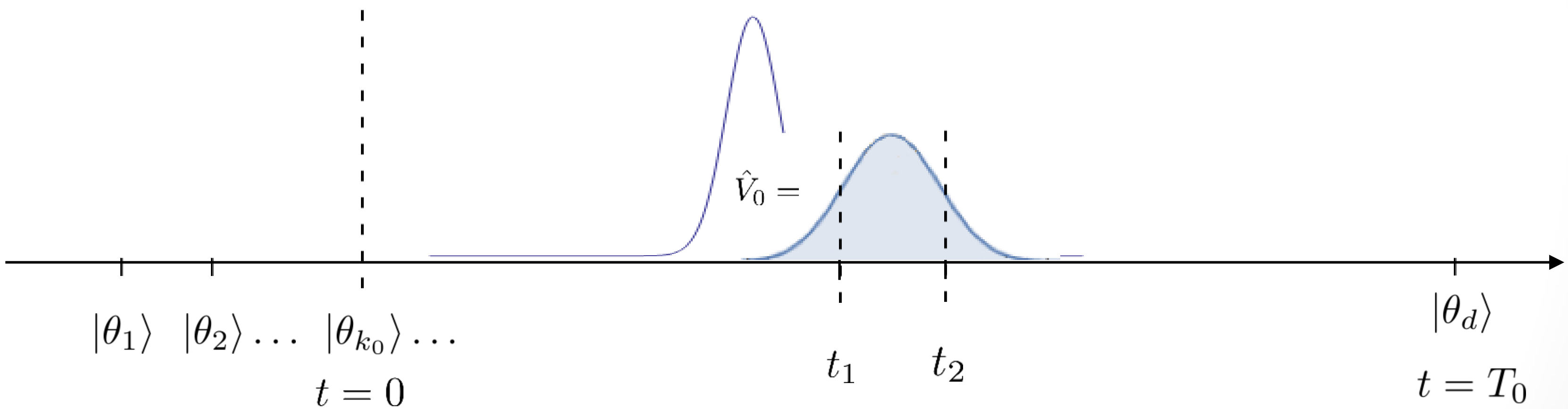
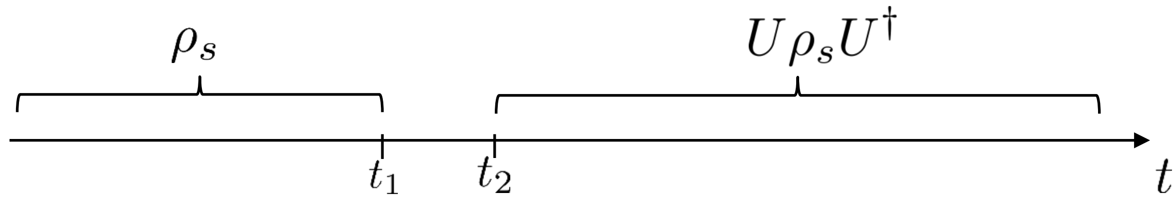


Correction term $\mathrm{F}(\rho'_s(t), \rho_s(t)) \leq \sqrt{d_s}\, poly(d)\, e^{-c\sqrt{d}}$

$t \in [0, t_1] \cup [t_2, T_0]$

$\mathrm{F}(\rho'_c(T_0), \rho_c(0)) \leq poly(d)\, e^{-c\sqrt{d}}$

Thermodynamic Consequences



$$U = e^{\hat{H}_s^{\text{int}}}$$

$$\rho'_{sc}(t) = \big(\rho_s \otimes |\Psi(k_0)\rangle\langle\Psi(k_0)|\big)(t), \qquad \hat{H}_c \otimes \mathbb{1}_s + \hat{V}_0 \otimes \hat{H}_s^{\text{int}}$$

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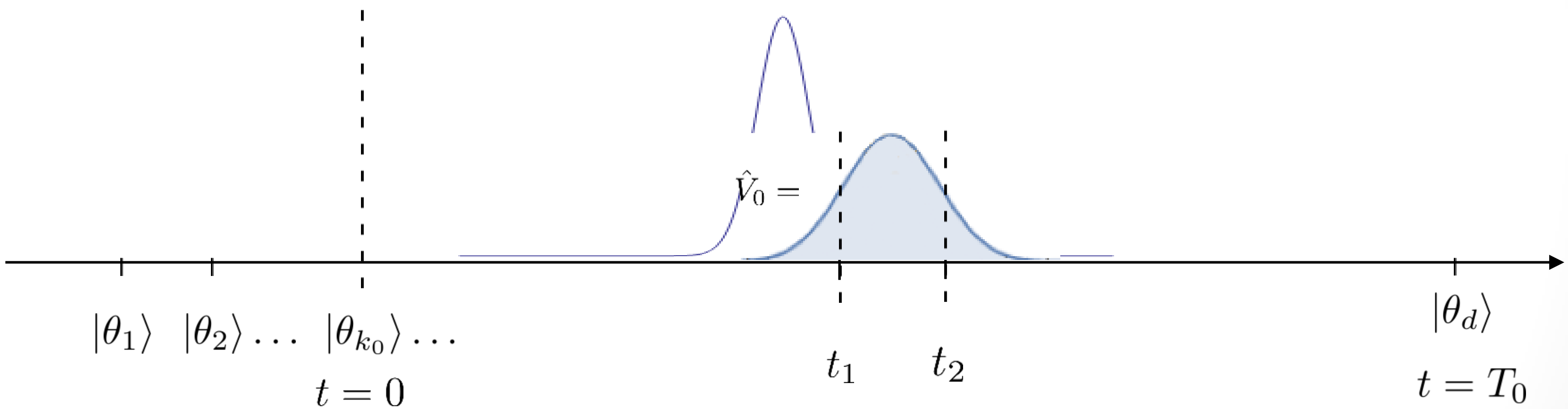

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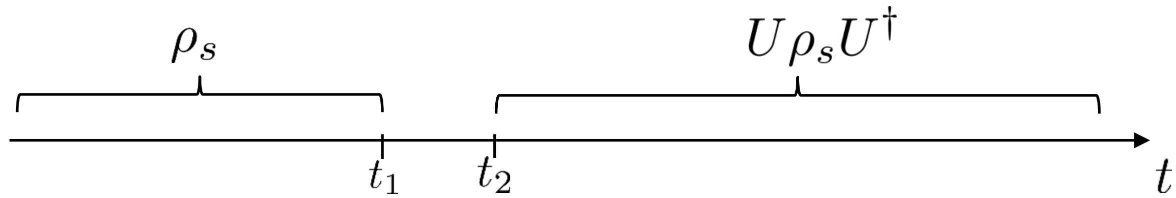

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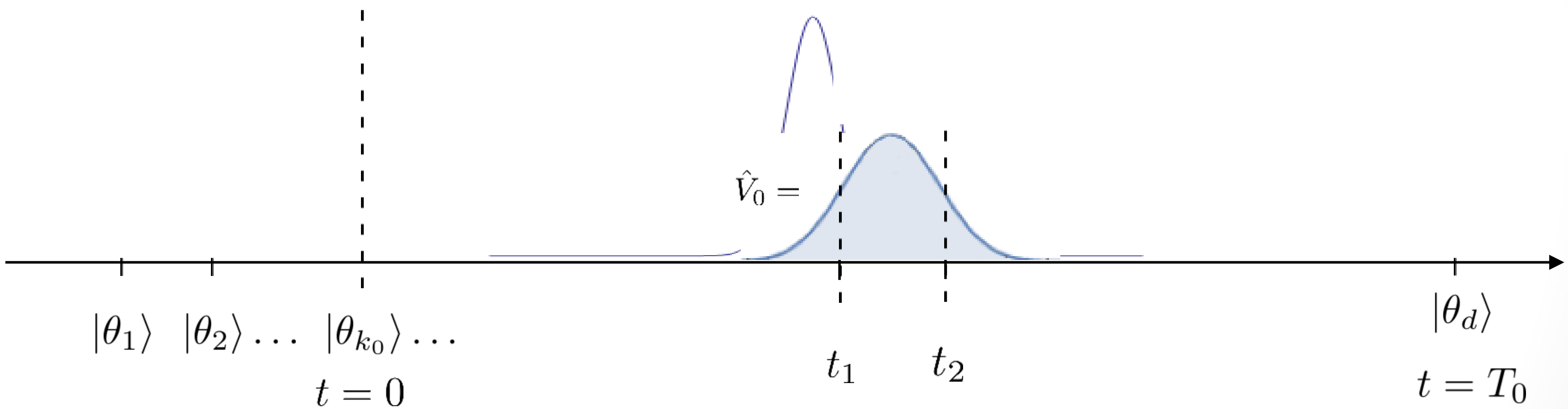

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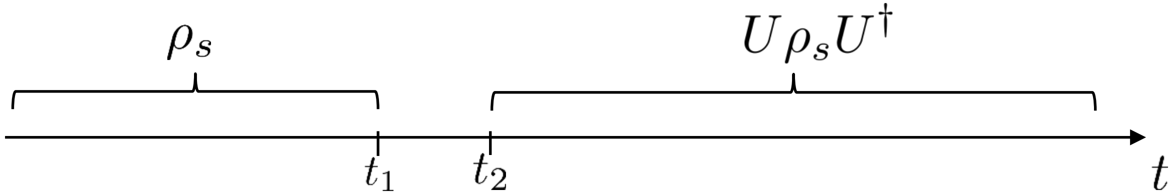

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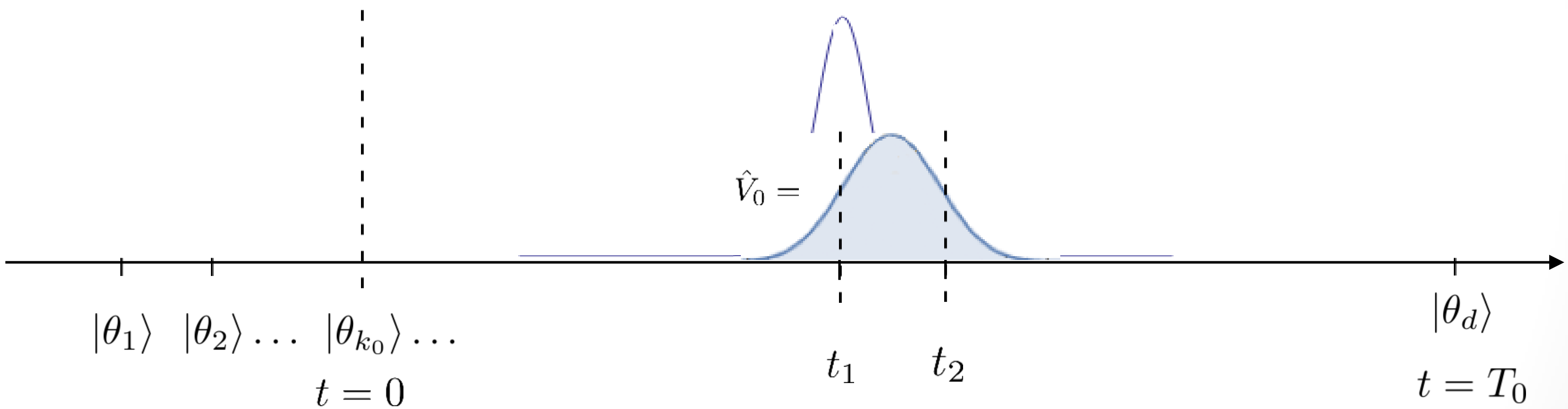

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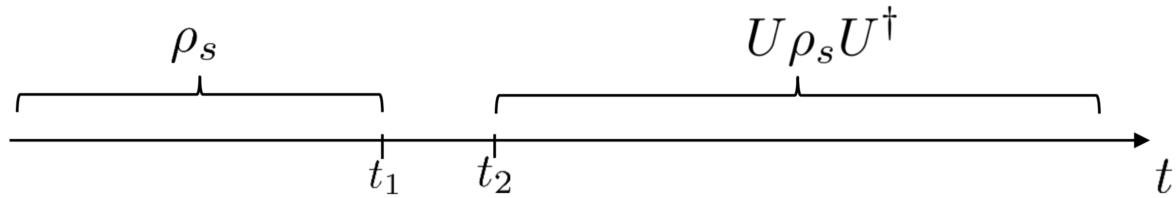

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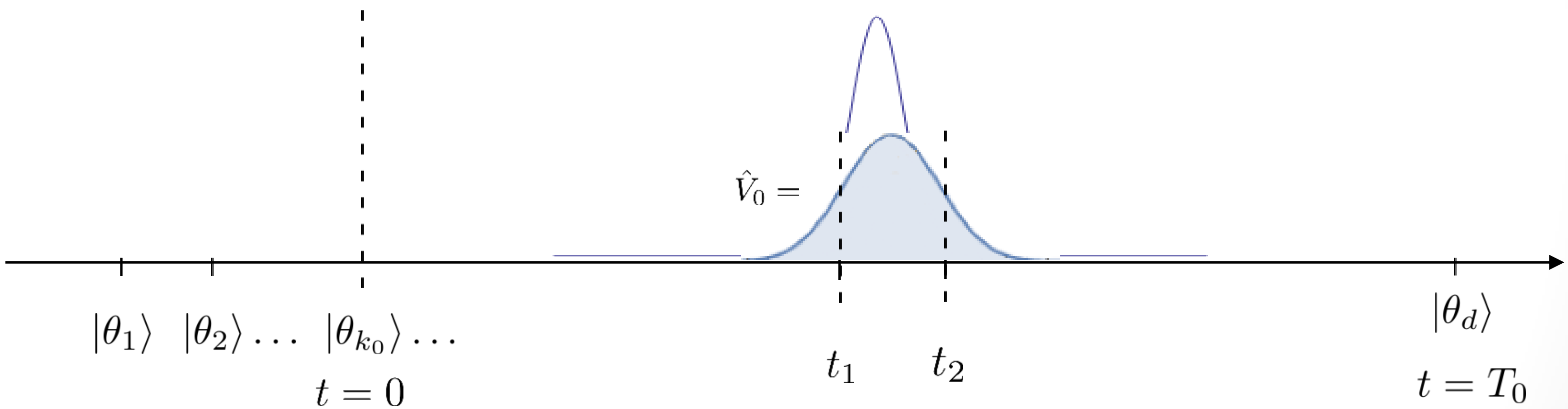

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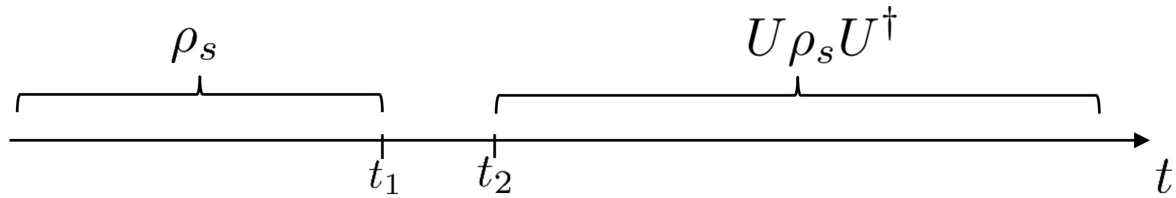

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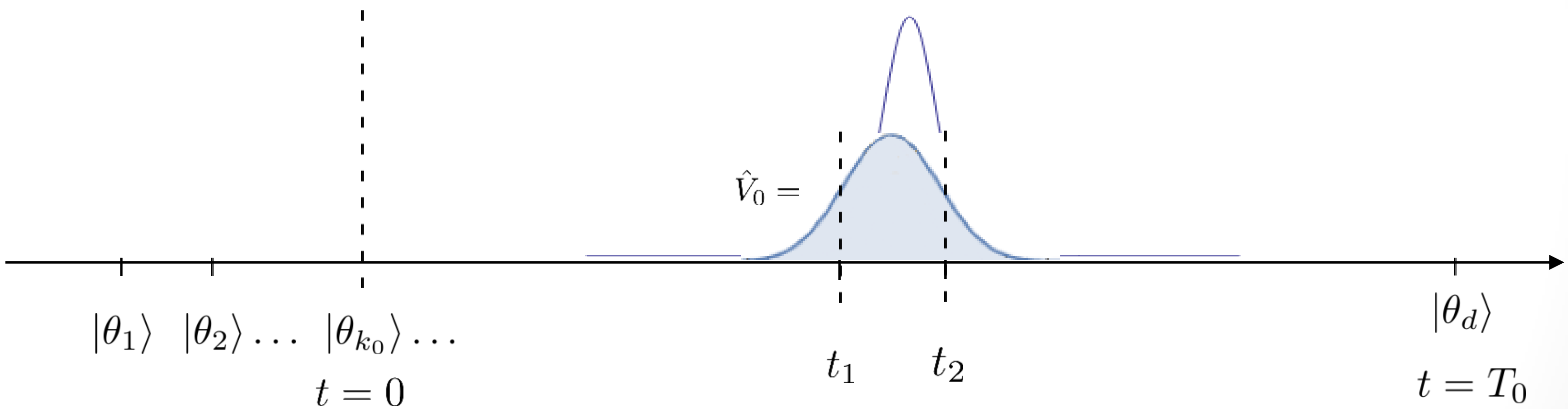

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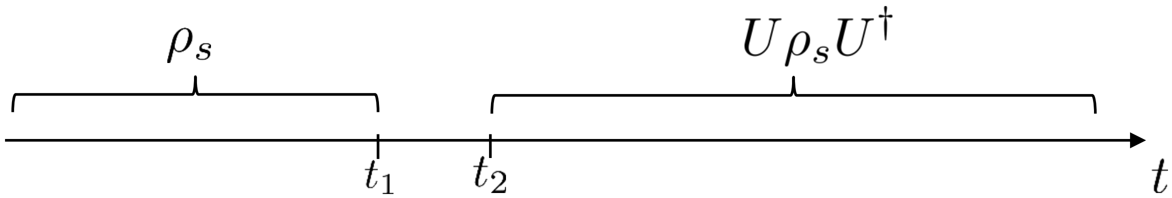

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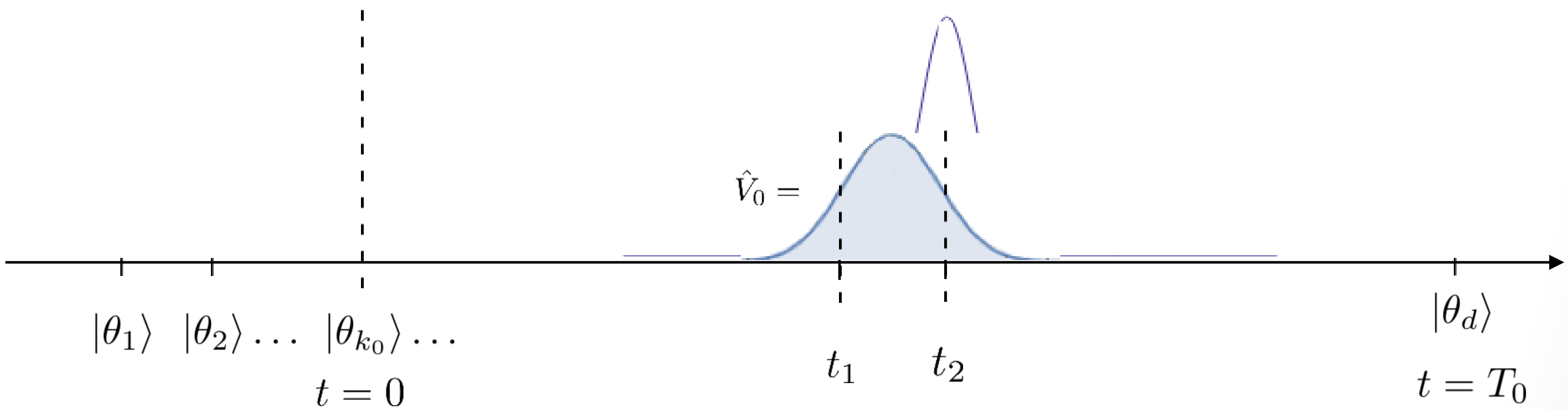

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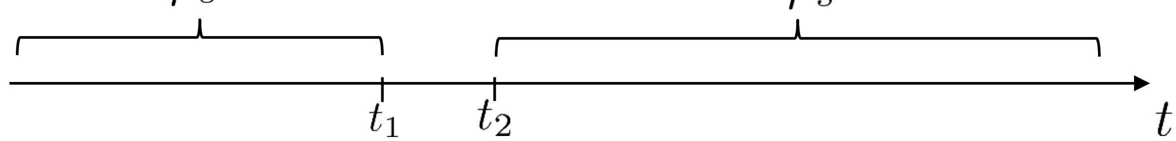

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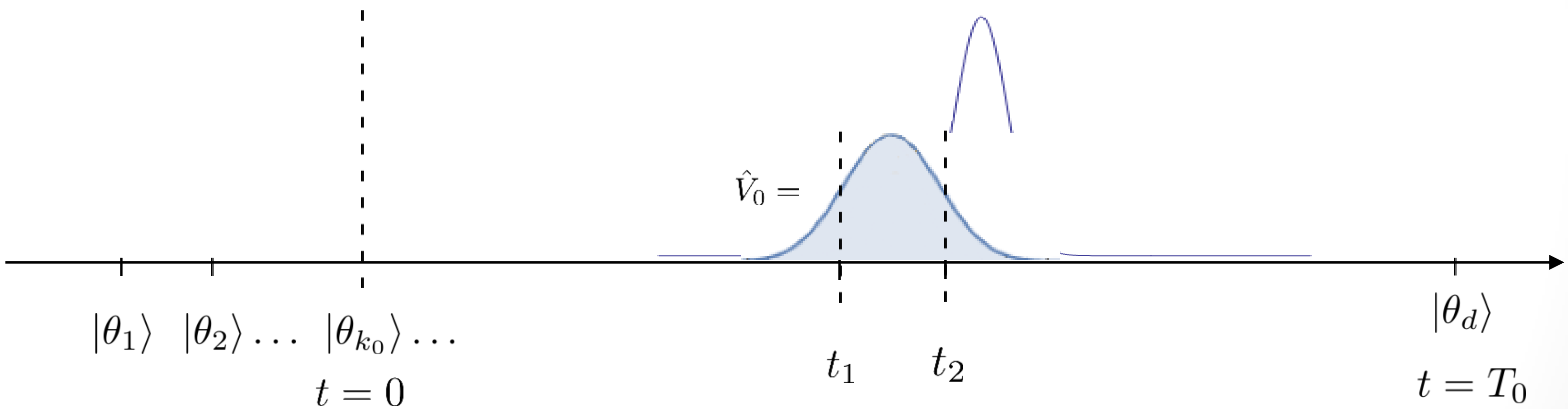

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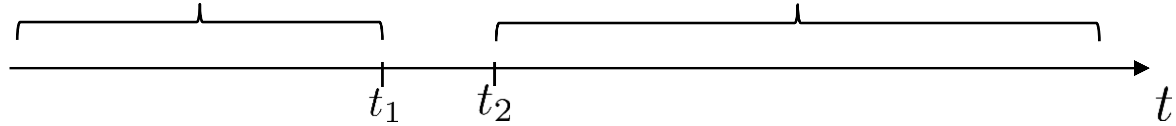

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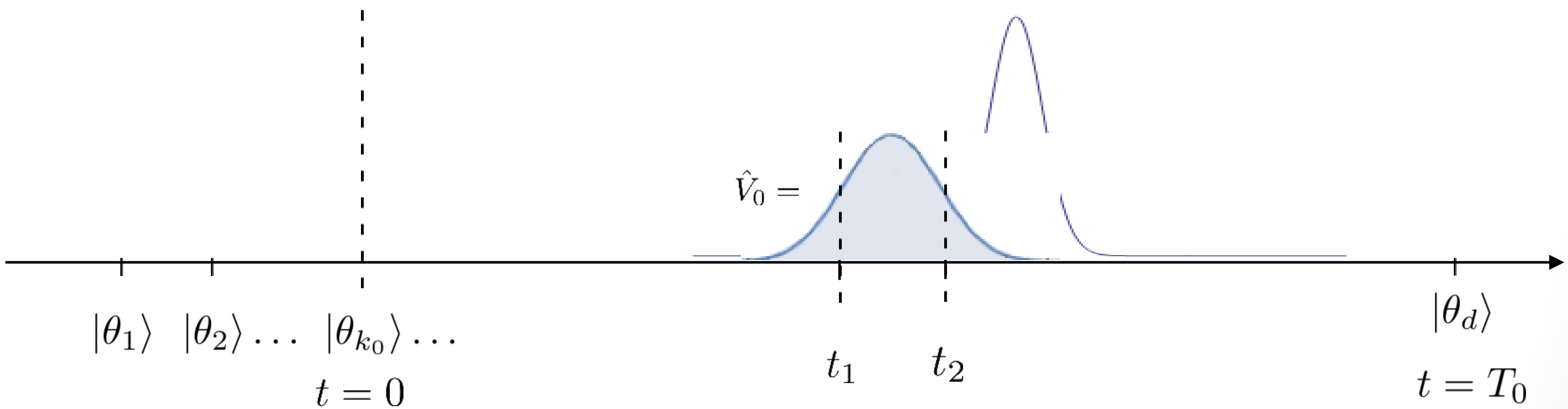

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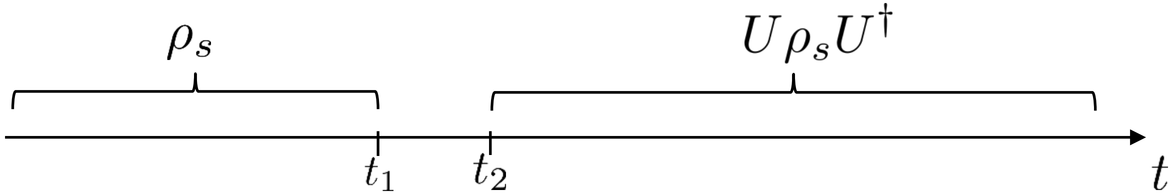

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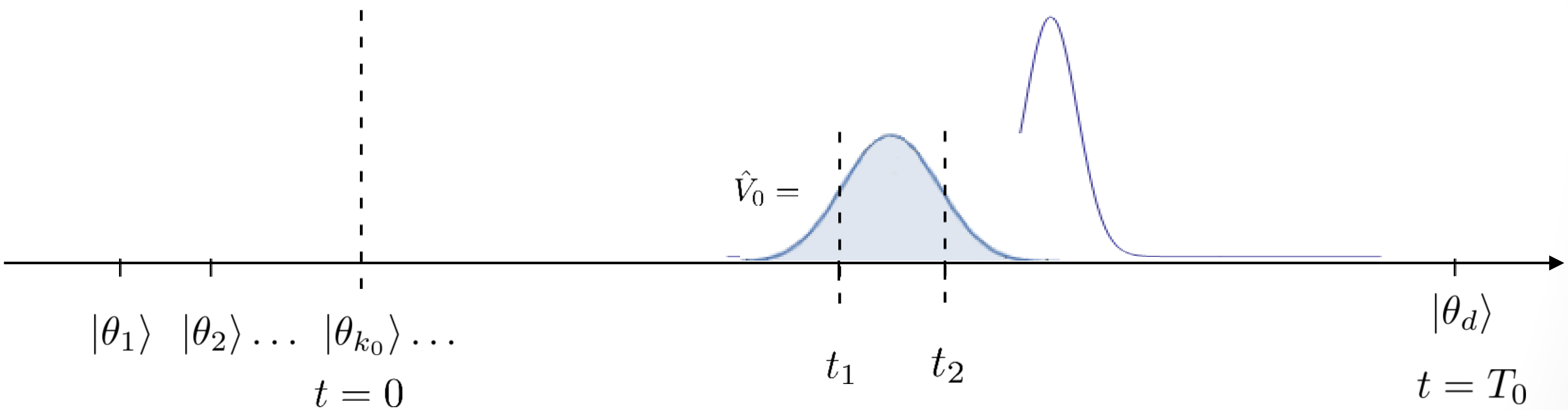

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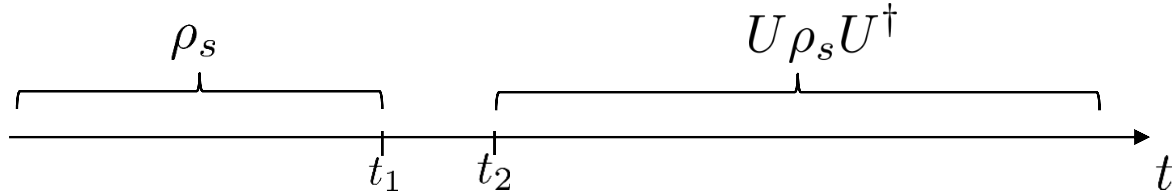

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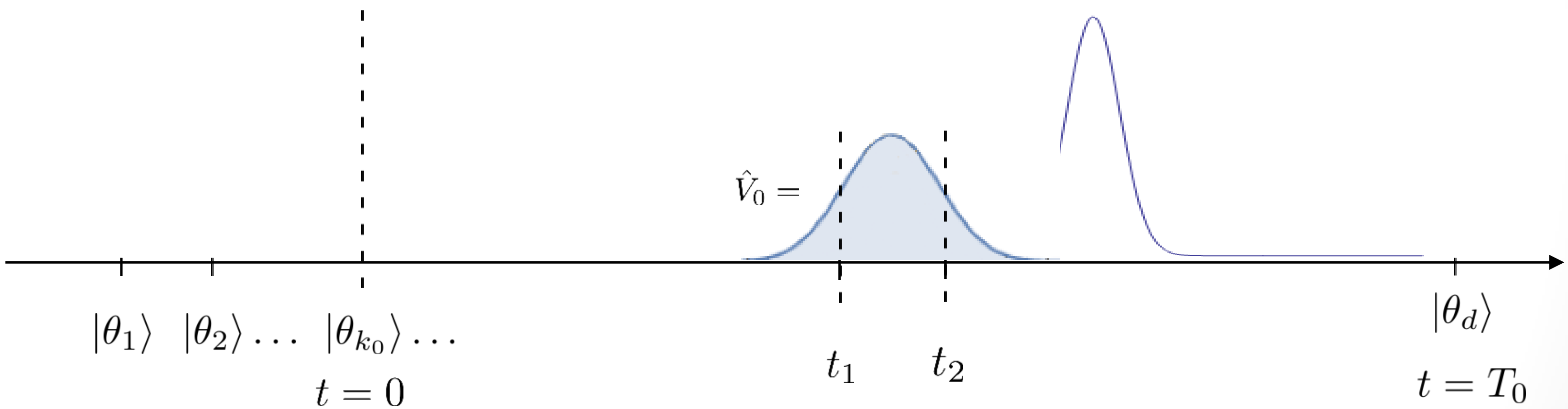

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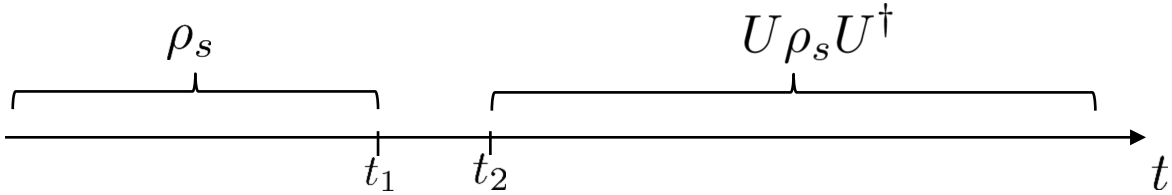

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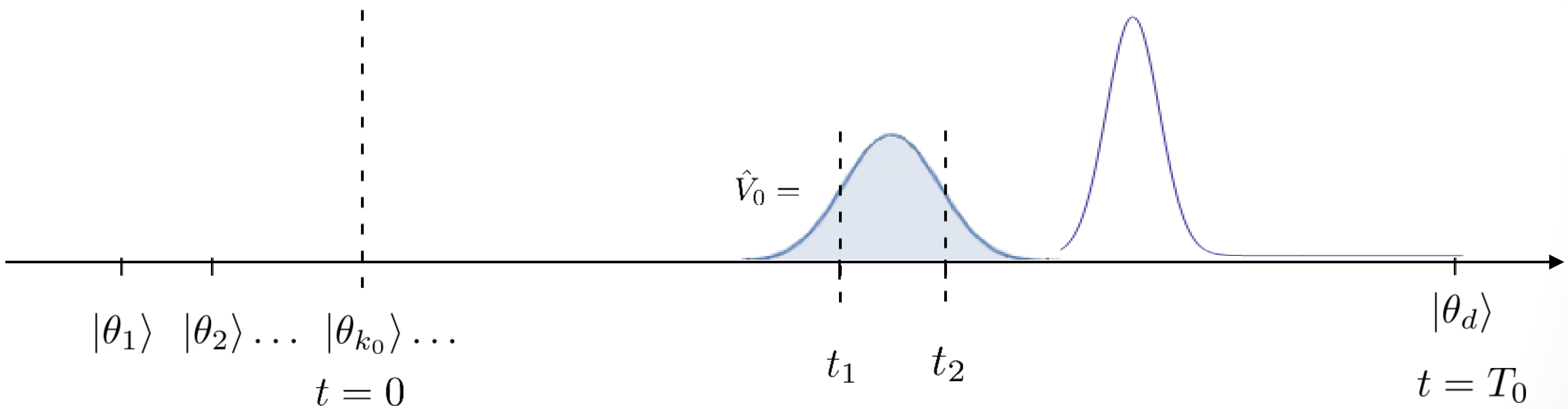

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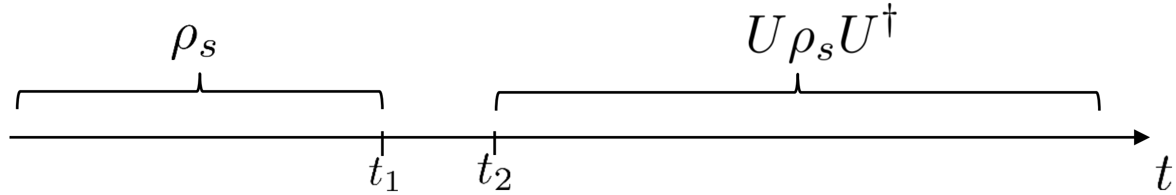

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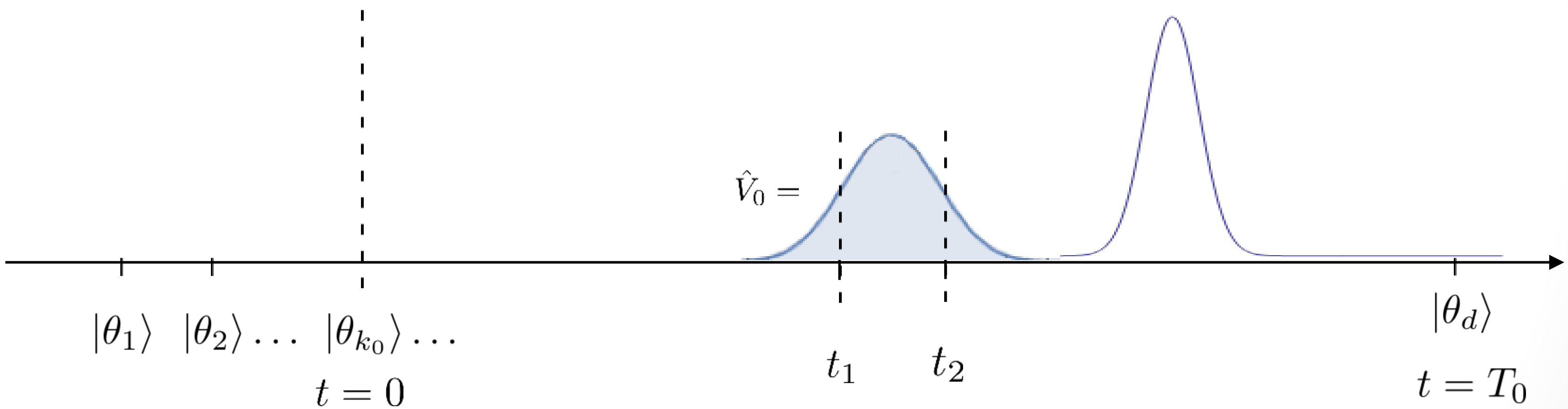

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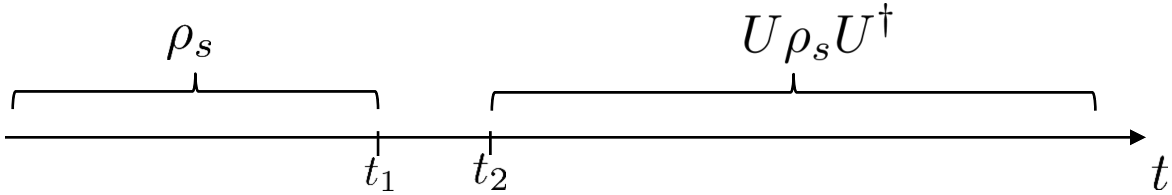

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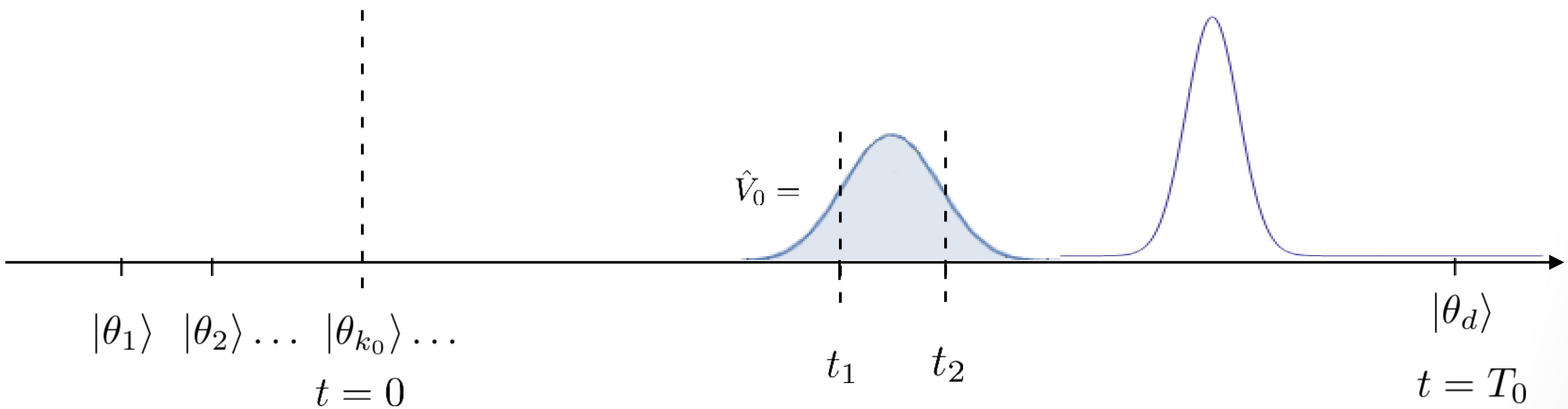

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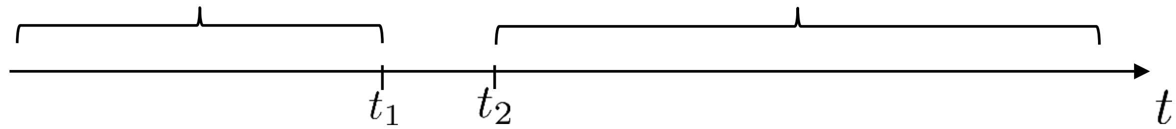

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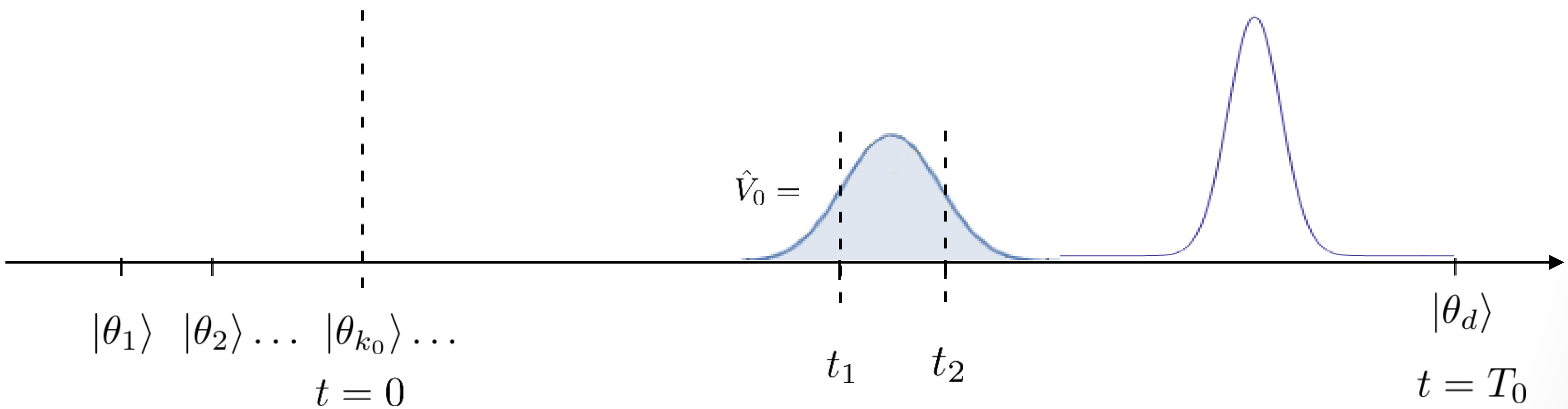

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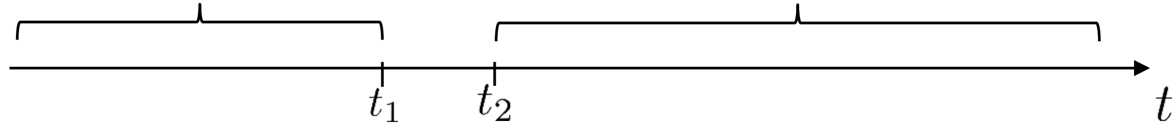

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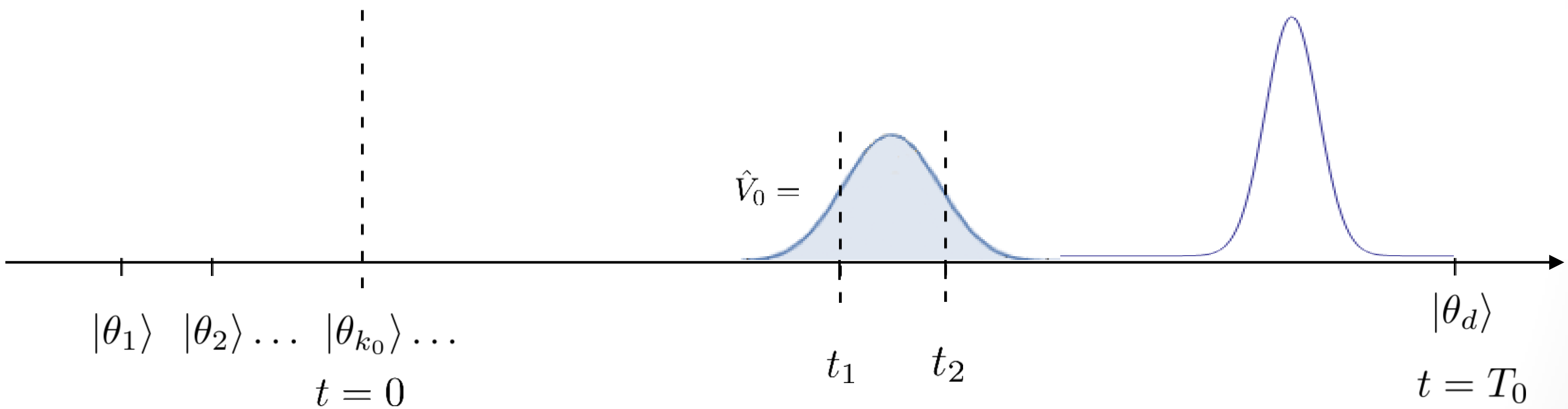

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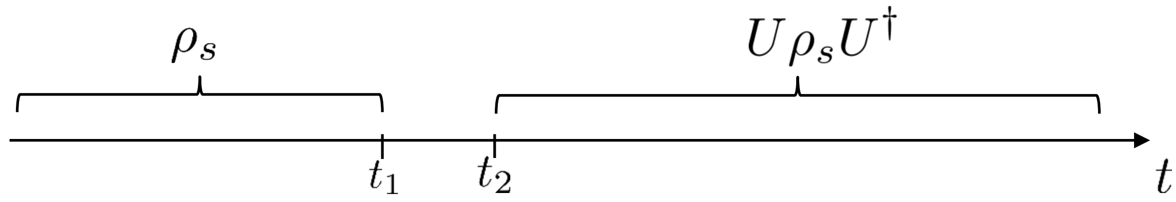

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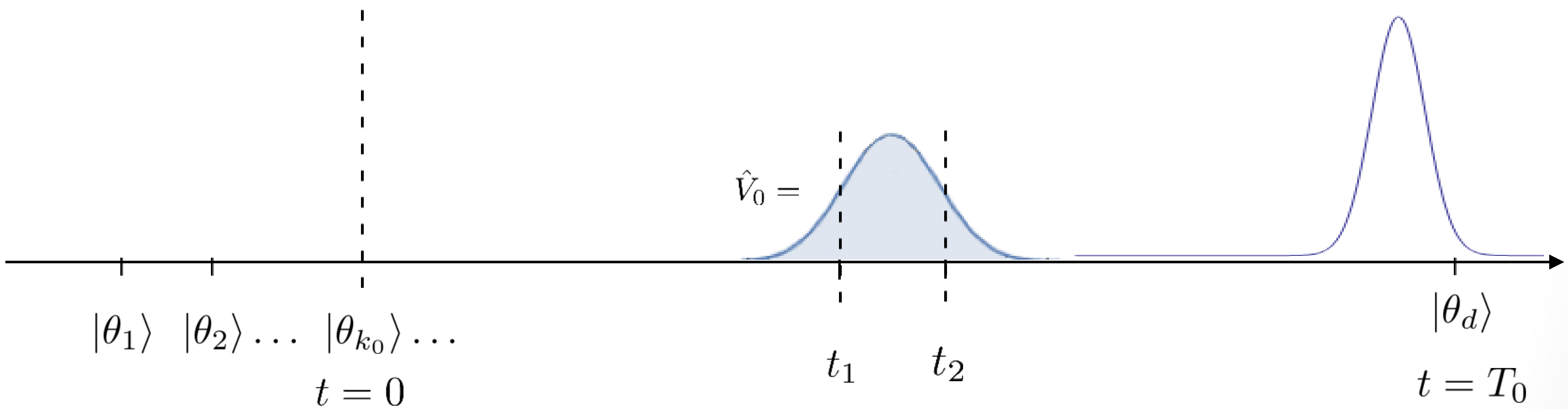

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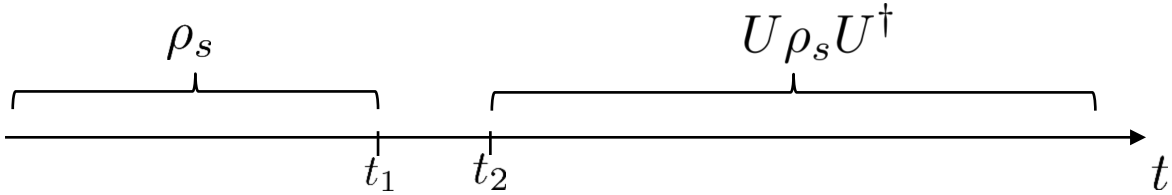

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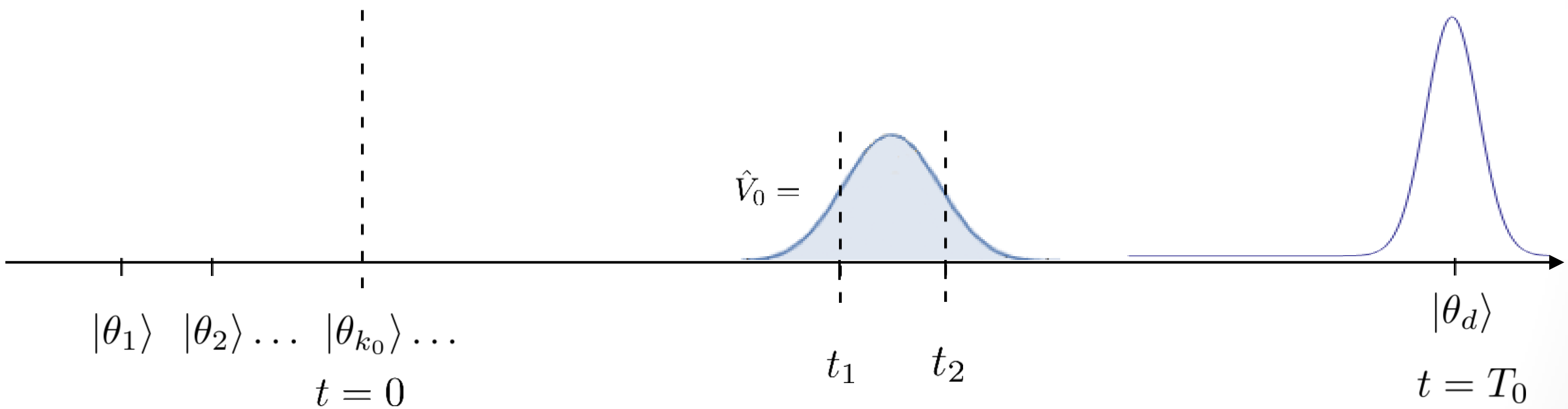

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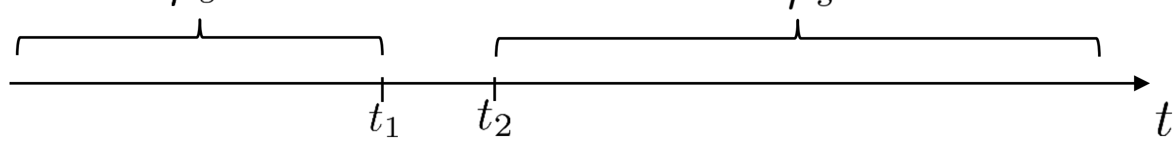

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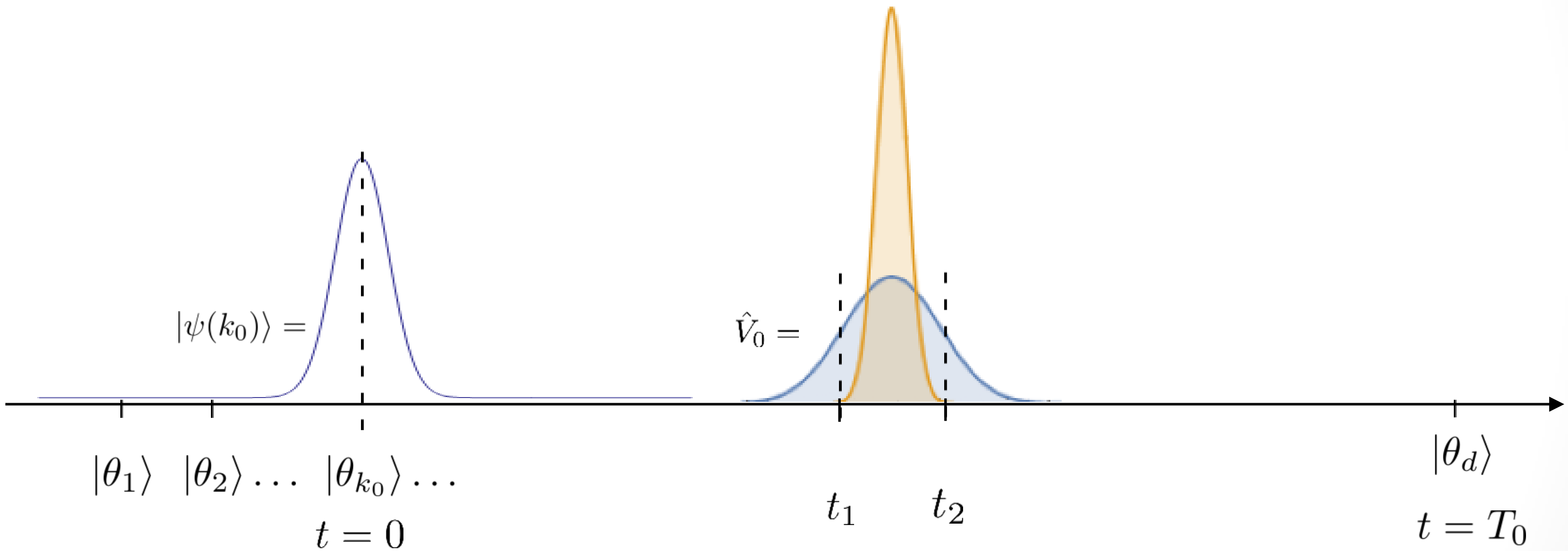

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Conclusion and Outlook   ArXiv 1607.04591

- **Unitary operations** are a **basic building block** in Q. thermodynamics.
  - Thermal engines (e.g. many talks)
  - Fluctuations relations
  - 2^{nd} laws of Quantum thermodynamics [Brandao et. al. PNAS (2015)]

- **Cost** (of unitary implementation & clock backreaction) is **exponentially small in** clock **energy** and **dimension**.